CONTROLLING THE DISTRIBUTION FUNCTION AT STOCHASTIC ACCELERATION OF CHARGED PARTICLES BY A REGULAR ELECTROMAGNETIC WAVE IN AN EXTERNAL MAGNETIC FIELD

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1 INTRODUCTION

The possibility to control the particle distribution function by the energy can be used, in particular, for development of tunable plasma sources of charged particles and X-rays on the basis of plasma particle stochastic acceleration (heating). In the present paper the results of one of perspective ways to realize such a control with nonlinear dynamics of charged particles in electromagnetic fields are considered.

Chaotic dynamics of the charged particle ensemble in a field of electromagnetic wave propagating at an angle to the constant magnetic field is studied. The stochastic instability develops as a result of overlapping nonlinear cyclotron resonances. From the analysis of the integral of motion, the set of equations has, it follows, that the maximum and average energy, gained by the particles interacting with a wave under conditions of developed dynamic chaos, is limited. It allows one to control the particle distribution function by the energy with changing the angle between the wave vector (propagation vector) and the external magnetic field.

2 STATEMENT OF A PROBLEM. BASIC EQUATIONS

We consider the motion of a charged particle in a constant externally applied magnetic field $H = \{0,0,H_o\}$ and in the field of an electromagnetic plane wave, propagating at the angle ϕ to the field H:

$$\begin{aligned} \vec{E} &= \mathrm{Re}\{E_O\vec{\alpha}\exp(i\vec{k}\vec{r} - i\omega t)\}, \\ \vec{H} &= \mathrm{Re}\{\frac{c}{\omega}[\vec{k}\vec{\alpha}]E_O\exp(i\vec{k}\vec{r} - i\omega t)\} \end{aligned} . \quad (1)$$

Here E_o is the wave amplitude, $\alpha = \{\alpha_x, i\alpha_y, \alpha_z\}$ is the polarization vector of the wave, and the wave vector $k = \{(\omega / c)N\sin\phi, 0, (\omega / c)N\cos\phi\}$ has only two components k_x and k_z. It can be always obtained by suitable choice of coordinates. N is the index of refraction. In dimensionless variables ($t \to \omega t$, $r \to r\omega / c$, $p \to p / mc$, $k \to kc / \omega$ the equations of charged particle motion can be reduced to the form:

$$\begin{aligned} \dot{\vec{p}} &= (1 - \vec{k}\vec{p} / \gamma)\mathrm{Re}(\vec{E}e^{i\psi}) + (\omega_h / \gamma)[\vec{p}\vec{h}] + \\ &+ \vec{k} / \gamma\ \mathrm{Re}\{(\vec{p}\vec{E})e^{i\psi}\} \end{aligned} , \quad (2)$$

$$\dot{\vec{r}} = \vec{p} / \gamma, \quad \dot{\psi} = \vec{k}\vec{p} / \gamma - 1,$$

where $\psi = kr - t$, $h = H / H_O$, $\omega_h = eH_O / mc\omega$, $\vec{E} = eE_O\vec{\alpha} / mc\omega$, $\gamma = (1 + \vec{p}^2)^{1/2}$ is the particle energy, p is its momentum. The set of Eqs. (2) possesses the integral of motion:

$$\vec{p} - \mathrm{Re}(i\vec{E}e^{i\psi}) + \omega_h[\vec{r}, \vec{h}] - \vec{k}\gamma = const . \quad (3)$$

For subsequent analysis it is convenient in (2), (3) to pass to new variables $p_\perp, p_z, \theta, \xi, \eta$ - guiding center coordinates, by formulas:

$$\begin{aligned} & p_x = p_\perp\cos\theta, p_y = p_\perp\sin\theta,\ p_z = p_z, \\ & x = \xi - p_\perp / \omega_h\sin\theta, y = \eta + p_\perp / \omega_h\cos\theta \end{aligned} . \quad (4)$$

and supposing that the amplitude of the electromagnetic wave $\varepsilon_o = eE_o / mc\omega$ is rather small and taking into account that the efficient interaction between the particle and the wave occurs if one of the resonance conditions is fulfilled:

$$k_z p_z + s\omega_h - \gamma = 0, \quad s = \ldots, -2, -1, 0, 1, 2, \ldots \quad (5)$$

Conditions of overlapping the nonlinear resonances.

Let us suppose that during the particle interaction with the wave its energy varies a little, i.e. $\gamma = \gamma_{0s} + \tilde{\gamma}_s, \tilde{\gamma}_s << \gamma_{0s}$, where γ_{0s} meets the resonant condition (5) and in view of the approximate integral of motion:

$$p_z - k_z\gamma = a = const, \quad (6)$$

a closed set of equations for $\dot{\theta}_s$ and $\tilde{\gamma}_s$ can be obtained from (2):

$$\dot{\tilde{\gamma}}_s = W_s\varepsilon_o\cos(\theta_s) / \gamma_{0s},\ \dot{\theta}_s = (k_z^2 - 1)\tilde{\gamma}_s / \gamma_{0s}, \quad (7)$$

$W_s = \alpha_x p_\perp sJ_s(\mu) / \mu - \alpha_y p_\perp J_s'(\mu) + \alpha_z p_z J_s(\mu)$, $\mu = k_x p_\perp / \omega_h$, $J_s(\mu)$ is the Bessel function, $J_s'(\mu)$ is its derivative over the argument.

Equations (7) are the equations of the mathematical pendulum. It is easy to find the width of the nonlinear isolated resonance from these equations:

$$\Delta\tilde{\gamma}_s = 2\sqrt{\left|\varepsilon_o W_s / (1 - k_z^2)\right|} . \quad (8)$$

From resonant conditions (5) and approximate integrals (6) we find the distance between the neighbouring resonances:

$$\delta\tilde{\gamma}_s = \omega_h / (1 - k_z^2) . \quad (9)$$

From expressions (8), (9) it is possible to write Chirikov's generalized criterion:

$$\varepsilon_o > \omega_h^2 / 16W_s(1-k_z^2), \qquad (10)$$

of development of a local instability of charged particle motion during particle interaction with the electromagnetic wave in the external magnetic field.

Let us suppose, that the particle is in resonance with the number $s = s^*$ and the amplitude of the field is those, that the stochastic instability of charged particle motion takes place. In the space $(\gamma, p_\perp, p_z)$ the particle motion is determined by approximate integral (7) and resonance conditions (5), which in the plane ($p_\perp, p_z$) become:

$$\frac{p_\perp^2}{\frac{s^2\omega_h^2}{(1-k_z^2)}-1}+\frac{\left(p_z-\frac{k_z s\omega_h}{(1-k_z^2)}\right)^2}{s^2\omega_h^2-(1-k_z^2)}=1,\ k_z^2\neq 1,$$

$$p_\perp^2-2s\omega_h p_z = s^2\omega_h^2-1,\quad k_z^2=1, \qquad (11)$$

$$p_\perp^2+s^2\omega_h^2(p_z-k_z s\omega_h/(1-k_z^2))^2=0,$$

$$s^2\omega_h^2=1-k_z^2.$$

The particle remaining on the integral (6) diffuses on resonances (11). To estimate the resonance width with a number which is greatly exceeding s^*, we find the constant a, which is the part of (6). By substituting in integral (6) the value of energy (5) we obtain:

$$a(s^*) = (1-k_z^2)p_{zs^*} - k_z s^*\omega_h. \qquad (12)$$

We find the value p_{zs^*} from the first equation (11):

$$p_{zs^*} = \frac{s^*\omega_h k_z}{1-k_z^2} \pm \sqrt{\frac{1}{1-k_z^2}\left\{\frac{s^{*2}\omega_h^2}{1-k_z^2}-1-p_{\perp s^*}^2\right\}}. \qquad (13)$$

Substituting p_{zs^*} from (13) into (12) we find:

$$a(s^*) = \pm\sqrt{\left\{s^{*2}\omega_h^2-(1-k_z^2)^2(1-p_{\perp s^*}^2)\right\}} \qquad (14)$$

Let us suppose now, that the particle has got in the vicinity of the resonance with the number $n >> s^*$. To find the width of this resonance it is necessary to calculate p_{zn} and $p_{\perp n}$. Using the resonance condition and the first equation from (10) we find:

$$p_{zs} = (a(s^*)+k_z n\omega_h)/(1-k_z^2),\ p_{\perp n} = \frac{n\omega_h}{k_z}. \qquad (15)$$

3 CONTROLLING THE DISTRIBUTION FUNCTION

For the wave with a polarization $\alpha = \{-\cos\phi, 0, \sin\phi\}$ (E-wave) the width of the nonlinear resonance decreases with the growth of n:

$$\Delta\tilde{\gamma}_n = 4\sqrt{\varepsilon_0 a\left(s^*\right)c_0 n^{-1/3}\sin^{-3}\phi}, \qquad (16)$$

(constant c_0=0.477) and criterion (10) can be represented as:

$$16\varepsilon_o c_o a(s^*)n^{-1/3}\sin\phi / \omega_h^2 > 1 \qquad (17)$$

From (17) it is seen, that at high n the criterion ceases to be fulfilled. It means, that the diffusion of particles into the high-energy region due to development of stochastic instability of charged particle motion is impossible, because of overlapping the nonlinear cyclotron resonances (5).

To show possibility of controlling the particle distribution function by energies with changing the angle ϕ between the wave vector k and the external magnetic field, for the ensemble of 1000 particles uniformly distributed on the phase ψ, the equations (2) were numerically solved. The refraction index $N = 2$. The angle ϕ is chosen so, that $k_z > 1$ ($\phi = 0.1\pi$). The initial energy of all particles was identical and was equal to $\gamma = 1.0001$. The initial value of the longitudinal pulse for all particles was chosen as zero. Polarization of the wave $\alpha = \{0, i, 0\}$, wave amplitude E_o=0.25, cyclotron frequency ω_h =0.5. Dynamics of the particle distribution function by energy in time, the time dependence of average particle energy and dispersion was investigated. Calculations was checked with the help of integral (6), which was conserved with precision no more than 10^{-6}.

Let us begin the analysis of results from Fig. 1, where for $\phi = 0.1\pi$ the resonant conditions (11) are represented (marked as s), integral (6) (marked as I) and the projection of hyperboloid ($\gamma^2 = 1 + p_\perp^2 + p_z^2$) on the plane ($\gamma, p_z$) at $p_\perp = 0$ (marked γ_m). γ_m is the minimum value of the energy, which the particle can have at given p_z, i.e. on the plane (γ, p_z) none can get to the area below γ_m. From Fig. 1 it is seen, that the resonant interaction of particles with the wave is possible only for resonances with the numbers s = 0, ±1, ±2, ±3.

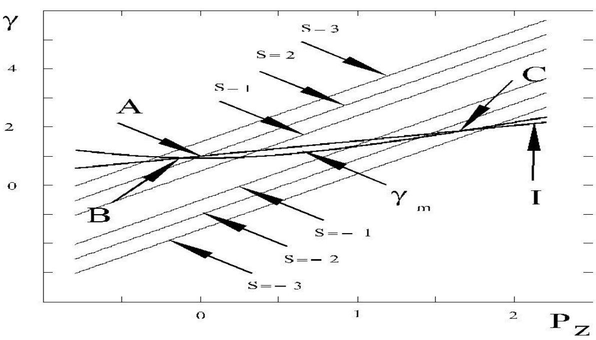

Fig. 1. Resonant conditions (11) marked s, integral (6) marked I and projection of hyperboloid $\gamma^2 = 1 + p_\perp^2 + p_z^2$ on the plane (γ, p_z) at $p_\perp = 0$ marked γ_m.

In an initial point of time all particles are in the point A. They are moving according to the integral I. At the chosen wave amplitude E_0=0.25, as noted below, the resonances s = 0, ±1, ±2 are overlapped, and dynamics of particles is stochastic, that leads to their diffusion by energy.

However, due to that particles are moving according to the integral I, which is above γ_m only between the points B and C, the particles are distributed in the segment [B, C]. It is shown below, that this distribution is uniform by energy, and its settling time is small.

The resonances with the numbers s and s+1 are overlapped, if the sum of their half-widths (8) is more than the distance between them (9). Amplitude of the field, at which it occurs, is:

$$\varepsilon_o = \omega_h^2 / 4\left|1 - k_z^2\right| * \left((p_{\perp s}|J'_s|)^{1/2} + (p_{\perp s+1}|J'_{s+1}|)^{1/2} \right)^2 , \quad (18)$$

where: $p_{\perp s} = \sqrt{\gamma_o^2 - 1 + (\gamma_o^2 - s^2\omega_h^2)(k_z^2 - 1)^{-1}}$, is determined from conditions of crossing of the integral I and resonance s. As the value $p_{\perp s}$ does not depend on the sign of s, and the module $|J_s'|$ is contained in (17), one can limit oneself to values $s \geq 0$. We find from (18) $\varepsilon_*^{(0,1)} = 0.027$, $\varepsilon_*^{(1,2)} = 0.102$ i.e., the resonances with the numbers s = 0, ±1, ±2 are overlapped, and dynamics of particles, which in the initial point of time were in the point A (Fig. 1), must be chaotic.

Numerical results of analysis of particle motion, (in the initial point of time particle was in the point A (Fig. 1) and had initial phase $\psi = 0$) are given in Fig. 2-4. In Fig. 2 the time dependence of the particle energy is represented.

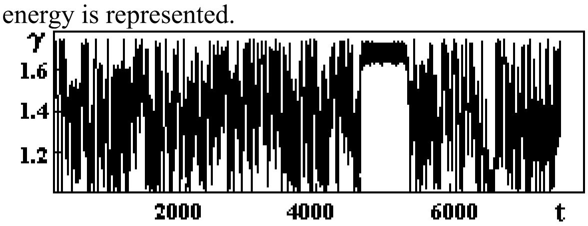

Fig. 2. Time dependence of the particle energy.

One can see, that particle dynamics is irregular, and its energy varies in finite limits $\gamma_{\min} = 1$, $\gamma_{\max} = 1.764$. From integral (5) and determination $\gamma = (1 + p_\perp^2 + p_z^2)^{1/2}$, we will find $\gamma_{\min}$ and $\gamma_{\max}$ values of the particle energy (point B and C in Fig. 1):

$$\gamma_{\max} = (k_z^2\gamma_o + \sqrt{k_z^2(\gamma_o^2 - 1) + 1})(k_z^2 - 1), \quad (19)$$

$$\gamma_{\min} = (k_z^2\gamma_o - \sqrt{k_z^2(\gamma_o^2 - 1) + 1})(k_z^2 - 1) .$$

The values $\gamma_{\min}$ and $\gamma_{\max}$, obtained in (19), are identical with the values obtained numerically.

In Fig. 3 the spectrum of the particle energy is represented.

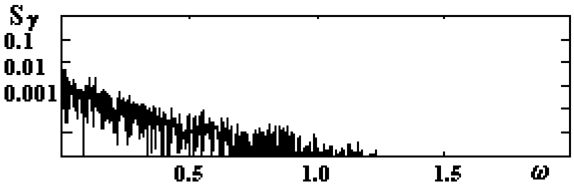

Fig. 3. Spectrum of the particle energy.

In Fig. 4 the time dependence of the maximum Lyapunov index L is shown. The value of the maximum Lyapunov index is 0.096 that indicates on the local instability of particle motion. The calculations which were carried out for other initial phases ψ have shown, that at the chosen field amplitude the particles dynamics is stochastic, and the particles are not captured in one of resonances s = 0, ±1, ±2.

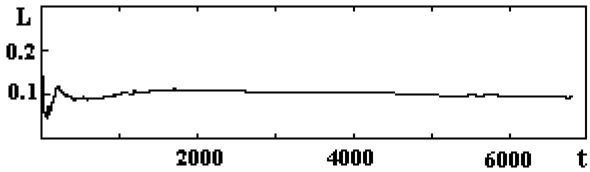

Fig. 4. Time dependence of the maximum Lyapunov index L.

The function of particle distribution by energy in the point of time $t = 250\,T$ (T - wave period) is represented in Fig. 5.

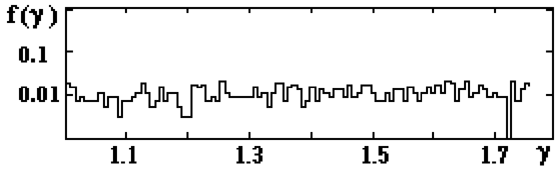

Fig. 5. Function of particles distribution by energy.

The value of the minimum energy is 1.0, and that of the maximum one is 1.764. It is identical with those values of $\gamma_{\min}$ and $\gamma_{\max}$, which are obtained analytically and numerically for one particle.

In Fig. 6 the time dependence of the average energy $<\gamma>$ (average over the ensemble) is shown.

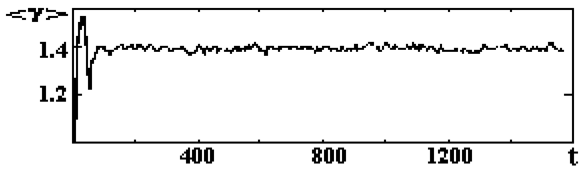

Fig. 6. Time dependence of the average energy <γ> (average over the ensemble).

It is seen, that for the time of about 30 periods, <γ> results to the stationary value: $(\gamma_{\min} + \gamma_{\max})/2 = 1.382$.

The time dependence of the dispersion σ^2 is represented in Fig. 7.

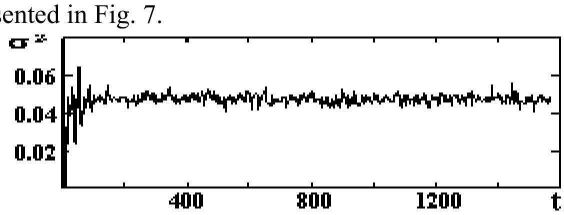

Fig. 7. Time dependence of the dispersion

$$\sigma^2 = <(\gamma - <\gamma>)^2> .$$

One can see that after reaching the stationary regime, the energy spreading does not change.

At the angle ϕ=0.375π k_z<1, integral (5) is nonclosed and can not limit the energy gaining by particles during stochastic acceleration that is confirmed by the numerical result.

Thus, we have showed, that there are possibilities to control the process of fast heating of particles (about hundreds periods of field) and to obtain the particle distribution function of a high quality in the given range of energies.

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