CYLINDRICAL WIGGLER BASED ON THE SOLENOID LOADED WITH DIFFERENT MAGNETIC PROPERTIES RINGS

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The electrodynamics analysis of topography of constant magnetic fields originating in spatially periodic system is carried out. The system under investigation is the solenoid inside which rings from materials with different magnetic properties are periodically placed. The system of equations connecting amplitudes of spatial harmonics of a field is obtained. This system is parsed numerically. Requirements are determined when in the spatial structure of a magnetic field the dominating role is played by the main harmonic. The amplitudes of longitudinal and transverse components of the modulated magnetic field as a function of the ring sizes, magnetic and electric properties of ring materials are investigated. The amplitudes of the fields obtained in the full consideration of electrodynamics are compared with the amplitudes of a periodic magnetic field, obtained in an impedance approximation is carried out.

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I. INTRODUCTION

Spatially periodic media are widely applied as slowing down structures in the accelerating technique [1,2], in the high-current electron beams focusing schemes [3], in powerful UHF electron devices [4]. One of new applications of periodic magnetic media is the spatial modulation of an electronic beam in a collective ion accelerator [5]. For these purposes the rings from materials with different magnetic properties, located in the homogeneous magnetic field, can be used. Magnitude and modulation of the magnetic field, created by such a system, in practice is defined by experimental measuring results [6]. In the present work precise electrodynamics calculation of a cylindrical wiggler on the basis of the solenoid loaded with rings having different magnetic properties is carried out

II. THE SET OF EQUATIONS

We shall conventionally divide the investigated volume into three areas in a cross-section: area I - the vacuum drift channel, area II - rings, III - exterior vacuum area. On the boundary of areas II and III the azimuth current slowly varying with frequency ω and creating in absence of rings the magnetic field with the strength amplitude equal to H_0 is given. For simplification of the analysis we shall neglect the thickness of solenoid walls. We shall divide area II into two parts: A and B, differing by values of permittivity and permeability.

The magnetic field originating in such a system, can be found by the method of partial areas when solving Maxwell equations for H- wave in each of the mentioned areas and matching the solutions at conjoint boundaries. The longitudinal component of the induction density vector B_z and transversal component of the magnetic intensity vector H_r are defined from the equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B_z}{\partial r}\right)+\mu(z)\frac{\partial}{\partial z}\left(\frac{1}{\mu(z)}\frac{\partial B_z}{\partial z}\right)+k^2\varepsilon(z)\mu(z)B_z=0$$

$$\frac{\partial}{\partial z}\left(\frac{1}{ik\varepsilon(z)}\left(\frac{\partial H_r}{\partial z}-\frac{1}{\mu(z)}\frac{\partial B_z}{\partial r}\right)\right)=ik\mu(z)H_r,$$

where $k^2=\omega^2/c^2$. In the areas I and III $\varepsilon=1$, $\mu=1$ and in the IIA area $(0\le z\le a)$ $\varepsilon=\varepsilon_1$, $\mu=\mu_1$ and in the IIB area $(-b\le z\le 0)$ $\varepsilon=\varepsilon_2$, $\mu=\mu_2$.

Due to the periodicity of the structure we shall search for solutions in the one period $L_w=a+b$. As equation (1) supposes variables separation we search for the solution in the form: $B_z=\sum_n R_n(r)\cdot Z_n(z)$. From (1) we have obtained the equations for the functions $R_n(r)$ and $Z_n(z)$:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR_n(r)}{dr}\right)+k_{rn}^2R_n(r)=0$$
$$\mu\frac{d}{dz}\left(\frac{1}{\mu}\frac{dZ_n(z)}{\partial z}\right)+k_{zn}^2Z_n(z)=0,$$

where $k_{rn}^2=-k_{zn}^2+k^2\varepsilon\mu$ are defined from requirements of periodicity of a problem.

In areas I and III we search for the solution as:

$$B_z^I=\sum_{n=-\infty}^{n=+\infty}C_{1n}J_0(k_{rn}r)exp(ik_{zn}z),$$
$$B_z^{III}=\sum_{n=-\infty}^{n=+\infty}C_{3n}H_0^I(k_{rn}r)exp(ik_{zn}z),$$

where $k_{zn}=\frac{2\pi n}{L}+k_3$, and $k_{rn}=\sqrt{k^2-k_{zn}^2}$. In the area II the solution looks like

$$B_z^{IIA}=\sum_{n=0}^{+\infty}\left(C_{2n}J_0(k_{rn}r)+D_{2n}N_0(k_{rn}r)\right)$$
$$\left(A_{21}^n exp(ik_{1zn}z)+B_{21}^n exp(-ik_{1zn}z)\right),$$
$$B_z^{IIB}=\sum_{n=0}^{+\infty}\left(C_{2n}J_0(k_{rn}r)+D_{2n}N_0(k_{rn}r)\right)$$
$$\left(A_{22}^n exp(ik_{2zn}z)+B_{22}^n exp(-ik_{2zn}z)\right),$$

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where k_{rn} and respectively $k_{1zn} = \sqrt{k^2\varepsilon_1\mu_1 - k_{rn}^2}$ and $k_{2zn} = \sqrt{k^2\varepsilon_2\mu_2 - k_{rn}^2}$ are defined from the boundary conditions

$$B_z^{IIA}\Big|_{z=0} = B_z^{IIB}\Big|_{z=0}, \quad H_r^{IIA}\Big|_{z=0} = H_r^{IIB}\Big|_{z=0},$$

$$B_z^{IIA}\Big|_{z=a} = B_z^{IIB}\Big|_{z=a}, \quad H_r^{IIA}\Big|_{z=a} = H_r^{IIB}\Big|_{z=a}$$

and requirements of periodicity

$$B_z^{IIB}\Big|_{z=a} = B_z^{IIB}\Big|_{z=-b} \times exp(-ik_3L),$$

$$H_r^{IIB}\Big|_{z=a} = H_r^{IIB}\Big|_{z=-b} \times exp(-ik_3L).$$

Substituting (5) in boundary conditions (6) and (7) we obtain the characteristic equation

$$\cos k_3L = \cos k_{1zn}a\cos k_{2zn}b -$$

$$\frac{\left(k_{1zn}\mu_2\right)^2 + \left(k_{2zn}\mu_1\right)^2}{2k_{1zn}k_{2zn}\mu_1\mu_2}\sin k_{1zn}a\sin k_{2zn}b.$$

Solving (8) with respect to k_{rn} we obtain two set of solutions k_{1rn} and k_{2rn}. The general solution in the second area is defined as:

$$B_z^{IIA} = \sum_{n=0}^{+\infty}\Big(\left(C_{21}^nJ_0(k_{1rn}r) + D_{21}^nN_0(k_{1rn}r)\right)$$

$$\left(FA1(k_{1rn})exp(ik_{11zn}z) + exp(-ik_{11zn}z)\right) +$$

$$\left(C_{22}^nJ_0(k_{2rn}r) + D_{22}^nN_0(k_{2rn}r)\right)$$

$$\left(FA1(k_{2rn})exp(ik_{12zn}z) + exp(-ik_{12zn}z)\right)\Big),$$

$$B_z^{IIB} = \sum_{n=0}^{+\infty}\Big(\left(C_{21}^nJ_0(k_{1rn}r) + D_{21}^nN_0(k_{1rn}r)\right)\times$$

$$\left(FA2(k_{1rn})exp(ik_{21zn}z) + FB2(k_{1rn})exp(-ik_{21zn}z)\right) +$$

$$\left(C_{22}^nJ_0(k_{2rn}r) + D_{22}^nN_0(k_{2rn}r)\right)\times$$

$$\left(FA2(k_{2rn})exp(ik_{22zn}z) + FB2(k_{2rn})exp(-ik_{22zn}z)\right)\Big),$$

where $FA1(k_{1,2rn}) = \dfrac{A_{21}^n}{B_{21}^n}$, $FA2(k_{1,2rn}) = \dfrac{A_{22}^n}{B_{21}^n}$,

$FB2(k_{1,2rn}) = \dfrac{B_{22}^n}{B_{21}^n}$.

The constants figuring in solutions (4) and (8) are determined from the radial boundary conditions

$$H_z^I\Big|_{r=ra} = H_z^{II}\Big|_{r=ra}, \quad B_r^I\Big|_{r=ra} = B_r^{II}\Big|_{r=ra},$$

$$H_z^{II}\Big|_{r=rb} = H_z^{III}\Big|_{r=rb} + G, \quad B_r^{II}\Big|_{r=rb} = B_r^{III}\Big|_{r=rb}.$$

Thus, the obtained set of equations (11), allows one to find fields in all partial areas. In the aspect of resultant expressions for field component awkwardness we omit them.

III. NUMERICAL CALCULATIONS

We carried out numerical calculations for parameters of experimental installation "Agate" [7]: $a = 2cm$, $b = 2cm$, $R_1 = 2.5cm$, $R_2 = 3,3cm$, $H_0 = 1kOe$, $\mu_1 = 5000$, $\mu_2 = 1$, $\sigma_1 = 1,0\times10^{15}s^{-1}$, $\sigma_2 = 3,6\times10^{15}s^{-1}$, $\varepsilon_{1,2} = 1 + 4\pi i\sigma_{1,2}/\omega$, $\omega = 200rad/s$, $k_3 = 100k$.

In fig.1 and fig.2 the calculation results of the cylindrical wiggler magnetic field for the mentioned parameters are shown. The infinite sums in the field expressions were substituted for finite sums ($-N \le n \le N$) in calculations.

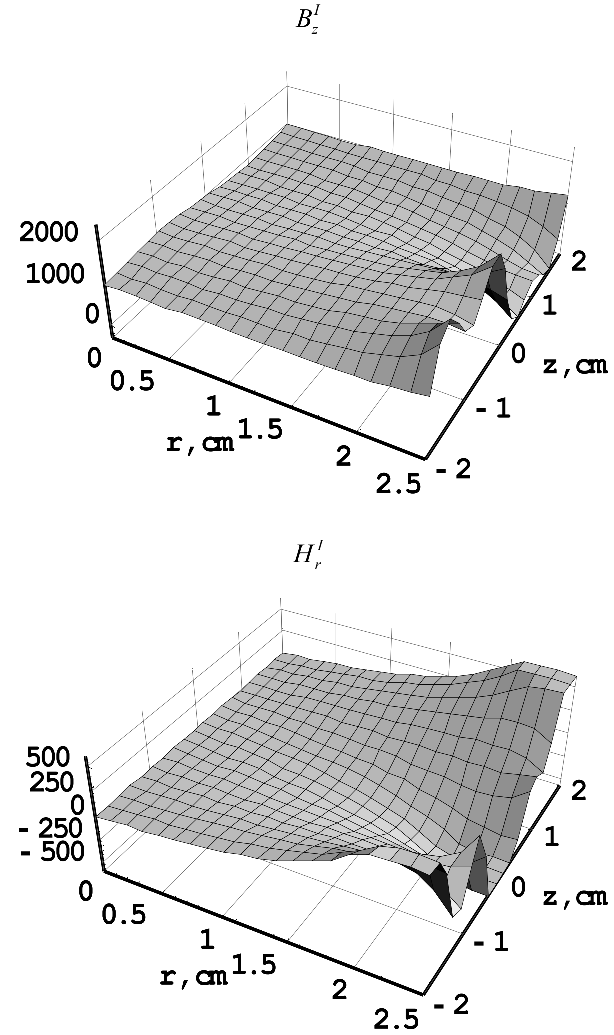

Fig.1. Structure of the cylindrical wiggler magnetic field in area I (drift chamber)

In Fig.1 the two-dimensional structure of the magnetic field created by a periodic system of iron and aluminum rings is shown. The quantitative pattern of the magnetic field topography corresponds to the qualitative ideas about the magnetic field behavior depending on a selected view point that confirms correctness of the carried out analytical and numerical calculations.

In Fig.2 the longitudinal topography of axial and radial components of the magnetic induction vector on the axis of the system and on the radius $r = 1.6cm$ are shown. As results from graphs, apart $1,6\ cm$ from the waveguide axis (at the surface of the thin electronic beam which is going into the second section of the accelerating installation "Agate" [7]) the depth of modulation is about 35%. The base contribution to magnetic field modulation is given with a harmonic $n = 1$. On the axis of the system the modulation depth is about 10%.

We have carried out calculations for other values of longitudinal and transversal sizes of rings. In numerical calculations, we have selected, the slow changes of the current in the solenoid, period of the structure and inte-

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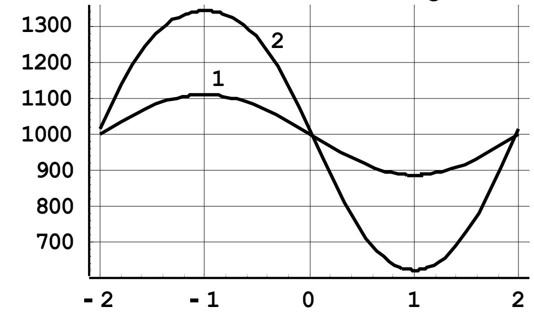
rior radius of the rings exerts the essential influence on the magnitude of the magnetic field modulation. The modulation depth grows with the period increase, and decreases with the interior radius of the rings increase.



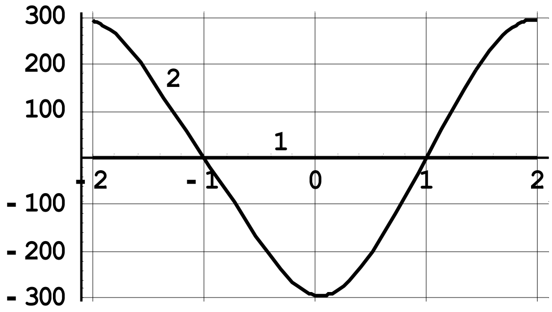


Fig.2. Axial and radial components of the induction density vector on the axis of our system $r = 0$ *(curves 1) and on a surface of the beam* $r = r_b$ *(curves 2) for one period of the structure.* $r_b = 1{,}6\ cm$

IV. SUMMARIES

1.The set of equations describing the magnetic field of the cylindrical wiggler is obtained. The wiggler comprises a periodic system of rings from the magnetic and nonmagnetic materials located in the solenoid, the current in which varies under the monochromatic law, generally, with the arbitrary frequency.

2.Numerical calculation of the magnetic field topography for different geometrical sizes of the wiggler is carried out. In the low-frequency approximation, the period of the structure and interior radius of the rings exerts the essential influence on the magnitude of the magnetic field modulation.

3.For parameters of the collective ion accelerator [7] the magnitude of longitudinal magnetic field modulation on the axis of the drift chamber is about 10%, and on the beam radius is about 35%.

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ЦИЛИНДРИЧЕСКИЙ ВИГЛЕР НА ОСНОВЕ СОЛЕНОИДА, НАГРУЖЕННОГО КОЛЬЦАМИ С РАЗЛИЧНЫМИ МАГНИТНЫМИ СВОЙСТВАМИ

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Проведен электродинамический расчет топографии постоянных магнитных полей, создаваемых в пространственно-периодической системе, представляющей собой соленоид, внутри которого периодически размещаются кольца из материалов с различными магнитными свойствами.

ЦИЛІНДРИЧНИЙ ВИГЛЕР НА ОСНОВІ СОЛЕНОЇДА, НАВАНТАЖЕНОГО КІЛЬЦЯМИ З РІЗНИМИ МАГНІТНИМИ ВЛАСТИВОСТЯМИ

Д.Ю. Копейченко, І.М. Оніщенко, Г.В. Сотніков

Проведено електродинамічний розрахунок топографії постійних магнітних полів, що виникають у просторово-періодичній системі, яка являє собою соленоїд, періодично навантажений кільцями з різними магнітними властивостями.