FRACTAL PROPERTIES OF DISPERSION CHARACTERISTICS OF SINUSOIDALLY RIPPLED PLASMA WAVEGUIDE

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Features of the solution of Maxwell's equations in sinusoidally rippled waveguide with the metal walls, filled with cold plasma, are considered. The dispersion equation which gives connection between longitudinal wave number *h* and frequency of fluctuations of a field *ω* in a waveguide is received. The "dense" spectrum which appears in the cross points of plasma modes with various numbers of spatial and radial harmonics is considered. Hausdorff's dimension for the case of equidistant distribution of forbidden bands as a function of ripple depth is constructed and the restriction for ripple depth (depth value when suggestion becomes not valid) is obtained.
PACS: 52.35. Hr

Infinite periodic sinusoidally rippled waveguide is considered in this work. It is supposed that waveguide is filled with cold collisionless plasma. Ions are considered to be reposed and not taking part in particle movement. Exterior magnetic field is assumed to be infinite and directed along *z*-axis of a waveguide. The general view of a waveguide is represented on the Fig. 1.

Dependence of distance between walls of a waveguide $X(z)$ on longitudinal coordinate *z* was chosen as $X(z)=R_0\left(1-\alpha\sin k_0 z\right)$, where $k_0=2\pi/D$, $\alpha\prec 1$ - the ripple depth, R_0 and D are the average radius and a spatial period of a waveguide accordingly.

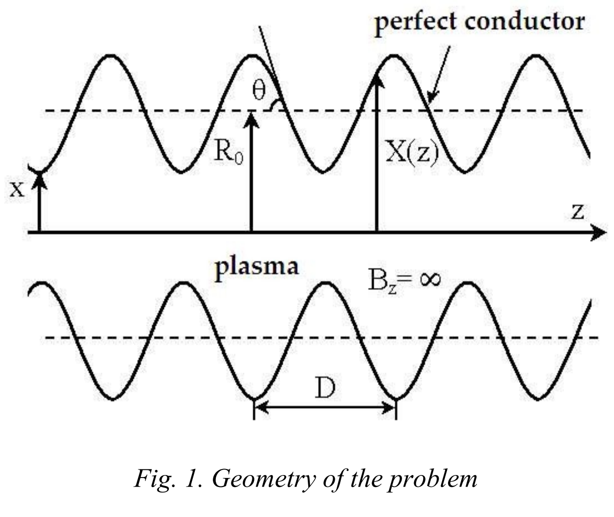

Fig. 1. Geometry of the problem

In a case of planar symmetric waveguide system of Maxwell equations breaks up to two subsystems, relevant to existing of *E* and *H* waves. In this work *E*-wave is considered.

It is assumed that electromagnetic field may be presented in the following view:

$$F(x,z,t)=\sum_{n=-\infty}^{+\infty} a_n F_n(x)\, e^{ik_n z-i\omega t} \quad ,(1)$$

where $F(x,z,t)$ - are the values of *E*-wave components, a_n and $F_n(x)$ are the *n*-th spatial harmonic amplitude and the distribution function in a plane of cross section of a waveguide; $k_n=h+nk_o$ - longitudinal wave number of *n*-th harmonic. The given presentation was supposed valid everywhere, including the space between waveguides humps.

It is known, that the dispersion equation expressing dependence of a longitudinal wave number *h* on frequency ω, follows from the requirement of equality to zero of the infinite Hill-type determinant [1].

The following view of infinite determinant elements is received:

$$\begin{gathered} ¿ \\ \times J_{n-m}\left(\alpha\kappa_n R(z)\right)\sin\kappa_n R(z), \\ \text{if } n-m=2\mathrm{p} \\ i\left(1+\varepsilon_{II}k_0(n-m)h_n/\kappa_n^2\right)\times \\ \times J_{n-m}\left(\alpha\kappa_n R(z)\right)\cos\kappa_n R(z), \\ \text{if } n-m=2\mathrm{p}+1\ , \\ ¿\left(1+\varepsilon_{II}k_0(n-m)h_n/\kappa_n^2\right)\times \quad ¿¿\{¿\{¿\{¿¿ \\ ¿ \\ ¿ \\ ¿ \\ ¿ \\ C_{mn}=¿¿ \\ ¿ \end{gathered} \qquad (2)$$

where $\varepsilon_{II}=1-\omega_p^2/\omega^2$, ω_p - electron plasma frequency, $\kappa_n=\sqrt{\varepsilon_{II}\left(\omega^2/c^2-k_n^2\right)}$ - radial wave number. We shall note, that C_{mn} values are received without expansion on a small parameter α, i.e. ripple depth is considered to be finite. For reception of the dispersion equation restriction on value of the determinant (the majorant) is received. In supposition that the majorant of determinant converges the dispersion equation for a wave of *E*-type of sinusoidally rippled plasma waveguide is received.

$$\prod_{n=-\infty}^{\infty} \sin\left(\kappa_n R_0\right) \times ¿$$
$$iG_1(\omega, h) \cdot ctg\left(\kappa_n R_0\right) +$$
$$+G_2(\omega, h)$$
$$¿$$
$$rli$$
$$1 + ¿|¿|¿ = P, \quad (3)$$
$$¿$$
$$¿$$
$$¿$$
$$¿ \times \prod_{n=-\infty}^{\infty} ¿$$
$$¿$$

where $G_1(\omega, h) = \frac{\varepsilon_{II} k_0 h_n}{\kappa_n^2} \alpha \kappa_n R_0$ and $G_2(\omega, h) = J_0\left(\kappa_n R_0\right)$. Equation (3) allows to construct a set of dispersion curves of sinusoidally rippled plasma waveguide assuming that $\alpha = 0$ (zero approximation). Then, taking $\alpha \neq 0$ a multiplicity of forbidden bands can be constructed. Forbidden bands originate in the cross points of various spatial and radial harmonics.

The analysis of equation (3) allows to spot the forbidden bands at cross points of dispersion curves of various radial and spatial modes. It is shown, that the forbidden band's width is proportional α^p, where p is the order of a forbidden band.

In the case of plasma waves ($\omega \prec \omega_p$) construction of dispersion curves gives in occurrence so-called "dense" spectrum for the first time submitted in [2] and shown on the Fig. 2 (according to [3]).

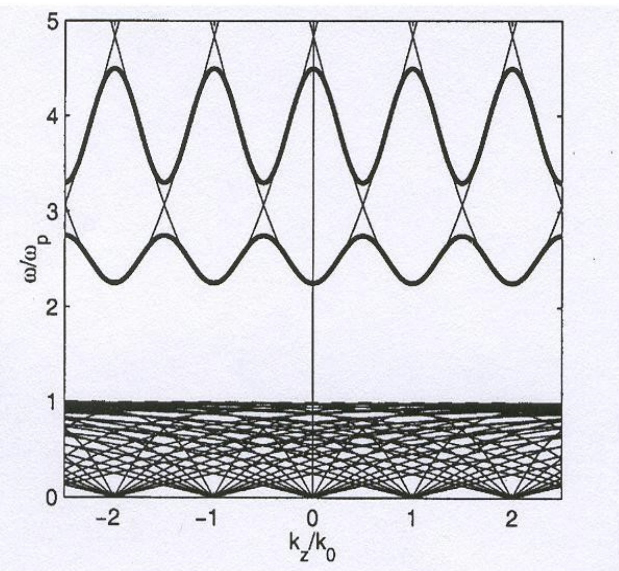

Fig. 2. "Dense" spectrum (appears below $\frac{\omega}{\omega_p} = 1$ *)*

One can see, that such structure has fractal properties (in particular, scaling self-similarity) [4, 5]. The amount of forbidden bands is infinite. Further calculation of higher numbers of spatial and radial harmonics gives forbidden bands which width can be compared to the width of forbidden bands of primary modes. For today some expedients of the description of similar object are offered. It is shown that in the case when all spatial and radial harmonics are taken into account, every point of ω-k_z plane blow ω_p becomes a solution of dispersion equation. The behaviour of such multiplicity of forbidden bands (cross points) is investigated in [3, 6, 7]. However, no conclusion can be made from these results. For today Hausdorff's dimension is calculated under suggestions that simplify calculation.

In this work Hausdorff's dimension is calculated under suggestion that forbidden bands formed in the cross points of modes with various numbers of spatial and radial harmonics are equidistantly spread at the segment of 0- ω_p . Hausdorff's dimension is calculated according to the following:

$$D_H = \frac{\ln\left(\frac{N'(l)}{N(l)}\right)}{\ln\left(\frac{l'}{l}\right)}, \quad (4)$$

where N and N', l and l' are the generalized number and generalized length on the preceding and current steps of fractal structure development respectively [5].

Calculated under stated suggestion and (4) Hausdorff's dimension is $D_H = {}^{-ln2}/_{ln\alpha}$. At the value $\alpha = 0,221$ forbidden bands begin to cover each other and performed suggestion becomes not right.

Authors consider that the problem of "dense" spectrum is vital for experimental and theoretical researches. Dispersion characteristics of periodic plasma waveguides still are the questions to discuss. It is necessary to construct the general theory of "dense" spectrum and to obtain experimental data to bring experiment to conformity with theory.

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ФРАКТАЛЬНЫЕ СВОЙСТВА ДИСПЕРСИОННІХ ХАРАКТЕРИСТИК СИНУСОИДАЛЬНО ГОФРИРОВАННОГО ВОЛНОВОДА, НАПОЛНЕННОГО ПЛАЗМОЙ

В.И. Лапшин, А.Ф. Стоянов, В.И. Ткаченко, И.В. Ткаченко

Рассмотрены особенности решения уравнений Максвелла в синусоидально гофрированном волноводе с металлическими стенками, наполненном холодной плазмой. Получено дисперсионное соотношение, которое дает связь между продольным волновым числом h и частотой колебаний поля ω в волноводе. Рассмотрен «плотный» спектр зон непрозрачности, возникающий в точках пересечения различных пространственных и радиальных мод. Получено значение Хаусдорфовой размерности в предположении эквидистантного распределения зон непрозрачности, также получено ограничение на глубину гофра, когда указанное предположение перестает быть справедливым.

ФРАКТАЛЬНІ ВЛАСТИВОСТІ ДИСПЕРСІЙНИХ ХАРАКТЕРИСТИК СИНУСОЇДАЛЬНО ГОФРОВАНОГО ХВИЛЕВОДУ, НАПОВНЕНОГО ПЛАЗМОЮ

В.І. Лапшин, О.Ф. Стоянов, В.І. Ткаченко, І.В. Ткаченко

Розглянуто особливості рішення рівнянь Максвелла в синусоїдально гофрованому хвилеводі з металевими стінками, наповненому холодною плазмою. Отримано дисперсійне співвідношення, що дає зв'язок між подовжнім хвильовим числом h і частотою коливань поля ω в хвилеводі. Розглянуто "щільний" спектр зон непрозорості, що виникає у точках перетину різних просторових та радіальних гармонік. Отримано розмірність Хаусдорфа у припущені, що зони непрозорості розповсюджені еквідістантно. Також отримано обмеження на глибину гофра, при якому зазначене припущення перестає бути справедливим.