HIGH HARMONICS OSCILLATOR RADIATION IN A PERIODIC STRUCTURE

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We discuss spectral and angular characteristics of radiation by a harmonic oscillator placed in a field of a crystal lattice. The main attention is paid to high harmonics which can not be described in a dipole approximation. Certain possibilities of X-rays generation using this effect are discussed as well.

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Intense monochromatic radiation of high frequency can find a lot of scientific and technical applications. In the majority, installations for generation of high quality short-wave radiation are rather costly, e.g. free electron laser, undulator devices and so on. The point of this paper is a discussion of the method in this field, suggested in [1].

The method implies electrons moving under influence of a harmonics electric filed in a field of a periodic structure where high harmonics of the diving field can be generated, especially under the strong Doppler effect.

The electron motion can be driven by a laser or by a HF oscillator. Inhomogeneity of the environment can be natural as in the case of a crystal or produced with the help of various grids and lattices.

The basic opportunity of high frequency generation will be estimated here using an elementary model, following that of the paper [1].

The electron motion will be considered in the field of the wave periodical in time $E(t) = E\cos\Omega t$. In a periodic structure created, for example, by a crystal lattice, the motion can be one-dimensional, as it is supposed to be below. In principle, the presence of a large magnetic field could provide that. We suppose that the wave comes along the normal of a crystalline plane. As it concerns the periodic structure we shall suppose that its potential varies along the particle motion as $U(x) = U_0 + \rho\cos\kappa x$. The general layout is presented in Fig.1.

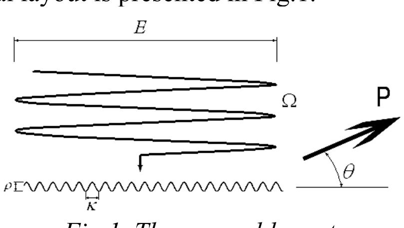

Fig.1. The general layout

We do not consider here the transverse electron motion although a transverse acceleration generally provides larger radiation power than a longitudinal one.

Quantitative characteristics of radiation power can be obtained, of course, with detail of the method realization only. In particular, spatial distribution of radiators is of importance, including influence of skin effect, the possibility of induced effects and so on. For these reasons we shall restrict ourselves by spectral-angular distribution of single particle radiation only.

In its essence the described picture is a modification of undulator radiation or of Smith-Purcell effect. The latter means that a high-power electron beam has to propagate along a short-period system. The main problem then is the damage of the structure by the beam.

Similar effect can be obtained with electrons oscillating under action of an external wave. In this case the electron velocity is a function of time. Note that only those parts of the trajectory where the velocity is almost equal to that of light (relativistic case) can provide the spatial coherency from various parts of the lattice. This is essential for getting a monochromatic radiation.

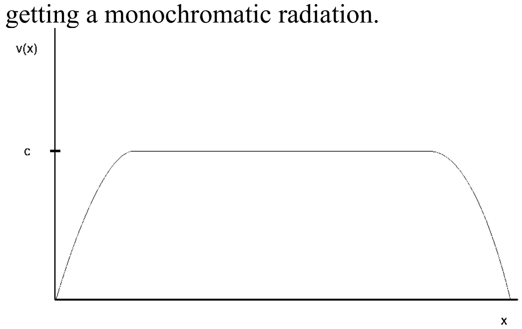

Fig.2. An oscillating velocity in case of a strong electric field of a wave

The relative power of various harmonics can be directly obtained from the harmonics of the vector potential. The latter can be presented as an integral along the particle trajectory

$$\vec{A}_\omega = e\frac{e^{ikR_0}}{cR_0}\int e^{i\omega\left(t-\frac{kr}{\omega}\right)}d\vec{r}\,. \qquad (1)$$

It is convenient in the one-dimension case to present the particle motion in a rather unusual way – i.e. as a dependence of time upon the particle coordinate. Then, the equation of motion of an electron in the periodic electric field and in the field of the periodic structure

$$\frac{d}{dt}\frac{\frac{\dot{x}}{c}}{\sqrt{1-\left(\frac{\dot{x}}{c}\right)^2}} = \frac{eE}{mc}\cos\Omega t + \frac{\rho\kappa}{mc}\sin\kappa x\,,$$

will take the form

$$\frac{c^2 t''}{\left(c^2 t'^2 - 1\right)^{3/2}} = -\frac{eE}{mc}\cos\Omega t - \frac{\rho\kappa}{mc}\sin\kappa x\,.$$

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Solving this equation with substitution into the integral (1) yields spectral-angular characteristics of radiation within the frames of the accepted model.

We solve Eq.(1) by perturbation methods supposing the field of the periodic structure is smaller than the driving field $\kappa\rho << eE$. Then the electron passes many periods of the lattice during one period of the external wave.

This approach still gives a rather complicated construction of the vector potential. However, for very high harmonic numbers the stationary phase method can be used for evaluations. The physical basis of the method is the fact that an extensive wave-particle energy exchange takes place at the moments when the phase slippage velocity is minimal. Then the argument of the exponent can be expanded in the vicinity of these points. The result can be writhed as

$$P = \left|A_{\omega}\right|^2 \omega = \frac{\pi}{2}\frac{e^2}{R_0^2}\frac{n^2}{\lambda^2 c^3}\frac{\rho^2}{m^2c^2\kappa^2}\frac{1}{\frac{eE}{mc^2}\sqrt{b^2-\alpha^2}}\cdot$$

$$\cdot\left(1+\frac{n^2}{\lambda^2\kappa^4}\left(\frac{eE}{mc^2}\right)^2 b^4\left(b^2-\alpha^2\right)\right)\frac{b^4}{\left(1-\frac{eE}{mc^2}\frac{1}{\lambda\kappa^2}b^2\sqrt{b^2-\alpha^2}\right)^2},$$

where $b^2 = \left(\cos\theta - \frac{\lambda\kappa}{n}\right)^2 - 1$; $\frac{1}{\alpha} = \frac{eE}{mc^2}\lambda$ – a wave strength parameter being a measure of the particle maximal energy.

The zeroes of the denominator determine the resonant harmonics

$$\frac{\lambda\kappa}{\sqrt{1+\alpha^2}+1} \le n \le \frac{\lambda\kappa}{\sqrt{1+\alpha^2}-1}. \qquad (2)$$

For integral numbers satisfying the inequality (2) one can find an angle θ at which the radiation intensity is maximal

$$\theta_{cr} = \arccos\left(\sqrt{1+\alpha^2} - \frac{\lambda\kappa}{n}\right).$$

The dependence of the radiated power upon the angle θ is presented in the Fig.3.

The inequality (2) means that the maximal harmonic number is equal to

$$n = \frac{\lambda\kappa}{\sqrt{1+\left(\frac{mc^2}{eE\lambda}\right)^2}-1}$$

The relative radiation power versus the harmonic number at the angle optimal for the 500-th harmonic is presented in Fig.4 for the same parameters as in Fig.3. One can see that for a fixed angle only one harmonic is effectively radiated.

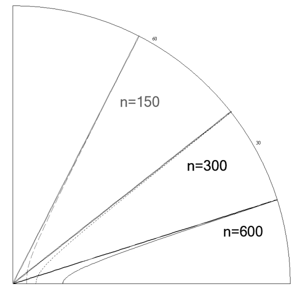

Fig.3. Dependencies of harmonics power on angle θ for the electric field strength of $E=10^8$ V/cm, the pumping wave length $\lambda = 10^{-2}$см, the structure period $2\pi/\kappa = 10\ \mu$ (the wave strength parameter =2)

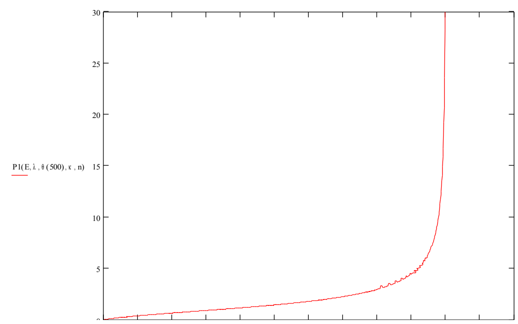

Fig.4. The relative radiation power versus the harmonic number at the angle optimal for the 500-th harmonic

The main results can be formulated as follows:

- The radiation spectrum consists of sharp lines.
- A spectral coherency is possible for large field strengths. So, the spatial coherency of bunched electrons is available in principle.
- The maximal harmonic number increases with the field strength increase.
- The absolute value of intensity requires more detailed model of the phenomenon.

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СПЕКТРАЛЬНО-УГЛОВЫЕ ХАРАКТЕРИСТИКИ ИЗЛУЧЕНИЯ ОСЦИЛЛЯТОРА В ПОЛЕ КРИСТАЛЛИЧЕСКОЙ РЕШЁТКИ

М.А. Горбунов, А.Н. Лебедев

Рассмотрены спектрально-угловые характеристики излучения гармонического осциллятора, находящегося в поле кристаллической решётки. Основное внимание уделяется высокочастотным гармоникам, не удовлетворяющим условию дипольности. Обсуждается возможность использования этого эффекта для генерации рентгеновского излучения.

СПЕКТРАЛЬНО-КУТОВІ ХАРАКТЕРИСТИКИ ВИПРОМІНЮВАННЯ ОСЦИЛЯТОРА В ПОЛІ КРИСТАЛИЧНОЇ ҐРАТКИ

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Розглянуто спектрально-кутові характеристики випромінювання гармонійного осцилятора, що перебуває в полі кристалічної ґратки. Основна увага приділяється високочастотним гармонікам, що не задовольняють умові дипольності. Обговорюється можливість використання цього ефекту для генерації рентгенівського випромінювання.