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MODELLING OF THE MULTIPLE-TOROIDICITY STELLARATOR/TORSATRON MAGNETIC FIELDS

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1. Introduction

It is well-known that for current stellarators/ torsatrons and heliotrons, especially for those with toroidal or modular field coils (TF or MF coils), a complex three-dimensional nature of the magnetic field is typical. Along a field line, the magnitude of such a magnetic field requires the following expansion in Fourier series:

$$B/B_0 = \sum_{j=0}^{\infty} \sum_{N=-\infty}^{\infty} \varepsilon_{j,N}(r)\cos(j\theta - NM\phi). \qquad (1)$$

Here r, θ and φ are the magnetic coordinates [1], where r is the flux surface label normalized by the plasma radius a_p, θ and φ are the poloidal and toroidal angle-like variables, respectively; N is the toroidal mode number; j is the poloidal (helical) mode number; M is the principal number of field periods over the device, and B_0 is the value of B at the axis.

It has been found as early as two decades ago [2,3] that magnetic fields of classical stellarators, torsatrons and heliotrons have the multiple-helicity character. This means the presence in Eq. (1) of $\varepsilon_{j,N}$ terms with only two toroidal mode numbers, N=0,1, and different poloidal mode numbers j. For current stellarator configurations with modular or toroidal field coils, distant satellite harmonics with high-order $N \geq 2$ toroidal mode numbers are also present in decomposition (1) (Fig. 1). So, strictly speaking, magnetic fields of these devices have both the multiple-helicity and the multiple-toroidicity character. By numerical computations done in Ref. 4 it has been demonstrated that in a number of configurations with discrete TF and/or MF coils, accounting of the high-order $N \geq 2$ distant satellite harmonics in expansion (1) can change considerably the value of the effective helical field ripple. This reflects specific changes in the structure of the secondary magnetic wells along field lines due to the distant harmonics effect [5]. Therefore, for such configurations, the high-order $N \geq 2$ harmonics cannot be ignored in computations (Figs. 2,3).

The presence of additional distant satellite harmonics in the magnetic field decomposition of a stellarator with MF or TF coils and impossibility to suppress these harmonics completely put a number of questions which need answers:

1) What are the new physical effects caused by the high-order $N \geq 2$ harmonics?

2) Is it possible to control transport and confinement properties of a stellarator configuration with the help of the proper chosen amplitudes of distant satellite harmonics? What values of currents in TF or MF coils could generate distant harmonics, favourable from the view point of particle confinement?

The search for unswers to the above mentioned questions can be simplified on the base of the so-called "bounce-averaged" approach. To use this approach, it is desirable to have analytical expressions for adiabatic invariants of charged particle motion. The main difficulty, unfortunately, which creates a number of barriers in extending of the bounce-averaged approach to the case of a multiple-toroidicity magnetic field consists in impossibility of expression of the adiabatic invariants of charged particle motion, in this case, as a sum of elliptic integrals and elementary functions [6]. Owing to this fact, to investigate the transport and confinement properties of a multiple-toroidicity stellarator configuration, a number of specific numerical methods have been developed [7-10]. A time-consuming of these methods, as their main deficiency, involves a number of difficulties in the clarification of the physical gist of the TF or MF coils effects on charged particle confinement.

The key problem for all current stellarator devices is a search for the simpler magnetic-field models, which reflecting adequately real features of the magnetic field behavior along a field line, could reduce the adiabatic invariants to elliptic integrals. Such a model representation of the magnetic field, when exists, gives a number of advantages, in comparison with more time-consuming computations, using the actual magnetic fields (1). So, the main purpose of the present paper is to develop a specific approach suitable for creating of the accurate magnetic-field model of any multiple-helicity and multiple-toroidicity stellarator/torsatron magnetic field.

2. Creating of magnetic-field models

Combining in Eq. (1) terms $\varepsilon_{j,N}$ with the same toroidal mode number N and all different poloidal mode numbers j, it is possible to rewrite this equation in the following form: $B/B_0 = B_T + B_H$, where the first term,

$$B_T = \sum_{j=0} \varepsilon_{j,0}\cos(j\theta), \qquad (2)$$

describes the toroidal component of the magnetic field.

$$B_H = \sum_{N=1}^{\infty} \varepsilon_{NH}(r,\theta)\cos(\chi_N - NM\phi) \qquad (3)$$

is the ripple component of the magnetic field. Riple amplitudes ε_{NH} in Eq.(3) are calculated, using

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harmonics of the initial Fourier-decomposition (1), as follows: $\varepsilon_{NH} = \sqrt{C_N^2 + D_N^2}$, (4)

where $C_N = \varepsilon_{0,N} + \sum_{j=1} \left[\varepsilon_{j,N} + \varepsilon_{j,-N}\right]\cos(j\theta)$, (5)

$$D_N = \sum_{j=1} \left[\varepsilon_{j,N} - \varepsilon_{j,-N}\right]\sin(j\theta), \qquad (6)$$

and $\cos\chi_N = C_N / \varepsilon_{NH}$. (7)

In the case of the classical stellarator, the above mentioned expression for the magnetic field is reduced to the well-known simpler form,

$$B/B_0 = B_T + \varepsilon_H (r,\theta) \cos(\chi - M\phi), \qquad (8)$$

in which the second term is the helical component of the magnetic field with the effective helical ripple amplitude $\varepsilon_H = \varepsilon_{1H}$, and the helical ripple phase is $\chi = \chi_1$.

As has been shown in Ref. 5,the high-order N ≥ 2 harmonics, affecting considerably the depths of the local ripple wells along field lines, can also change the number of these wells over the device length (the number of the magnetic-field periods). Let us determine the periodicity of the B_H component of the magnetic field. Determining the TF ripple amplitude value corresponding to the toroidal mode number N as $\langle\varepsilon_{NH}\rangle = \frac{1}{2\pi}\int_0^{2\pi} \varepsilon_{NH}\, d\theta$, assume that among all $\langle\varepsilon_{NH}\rangle$ values there exists one of them, namely $\langle\varepsilon_{jH}\rangle$, with the largest amplitude, i.e. in some interval of values of the flux surface radii, the following inequality is correct for all numbers i ≠ j: $\langle\varepsilon_{jH}\rangle \geq \langle\varepsilon_{iH}\rangle$. If this takes place, then the main periodicity of the B_H component of the magnetic field is equal to 2π/P, where P = jM (generally, j≠1). In this case, it is convenient to rewrite the full magnetic-field decomposition (1) in the following form:

$$B/B_0 = B_T + \varepsilon_{HH} (r,\theta,\phi) \cos(\xi - P\phi), \qquad (9)$$

where $\varepsilon_{HH} = \sqrt{E^2 + F^2}$; (10)

$$\cos\xi = E / \varepsilon_{HH} \ ; \qquad \sin\xi = F / \varepsilon_{HH} \ ;$$

$$E = \sum_{i=1} \left[C_i \cos\{(i-j)M\phi\} + D_i \sin\{(i-j)M\phi\}\right];$$

$$F = \sum_{i=1} \left[D_i \cos\{(i-j)M\phi\} - C_i \sin\{(i-j)M\phi\}\right].$$

From these expressions one can see that in the presence in Eq. (1) of the high-order N ≥ 2 harmonics, both the ripple amplitude (10) and ripple phase ξ are fast-oscillating functions of the toroidal angle. Through the first order in small parameters $\alpha_{ji} = \langle\varepsilon_{jH}\rangle \langle\varepsilon_{iH}\rangle$, the ripple amplitude (10) is determined, to a large degree, by the value of the main amplitude $\varepsilon_{jH} = \sqrt{C_j^2 + D_j^2}$. Additional components ε_{iH} presented in equation (10) for ε_{HH}, being much smaller, can make, however, some corrections to the value of ε_{HH} . For example, if the following relation is correct, $\sum_{i \neq j} \varepsilon_{iH} = \varepsilon_{jH}$, then $\varepsilon_{HH} = 2\varepsilon_{jH}$. As has been shown in Ref.11, a presence in Eq. (10) of periodical functions of the toroidal angle, affecting the depth of the secondary ripple wells, can also cause an appearance of additional ripple wells along field lines. These additional ripple wells (third-order wells) are much shallower, and the fraction of charged particles, which can be trapped here, is a comparatively small. In the low collision frequency transport regime (1/ν), such particles will have transitional orbits. Therefore, to investigate transport properties of a multiple-toroidicity stellarator in the 1/ν regime, it is permissible to ignore in computations the presence of small third-order ripple wells. Then, to express the cumulative effect of a number of distant harmonics on the secondary magnetic wells along field lines, it is permissible to average both the ripple amplitude (10) and ripple phase ξ over the main toroidal period of the magnetic field (2π/P).

After averaging, the magnetic field is expressed in the following form:

$$B/B_0 = B_T + \overline{\varepsilon_H} (r,\theta) \cos(\gamma - P\phi). \qquad (11)$$

Here $\cos\gamma = \gamma_c \,/\, \sqrt{\gamma_c^{\,2} + \gamma_s^{\,2}}$, where

$$\gamma_c = \frac{1}{2\pi}\int_0^{2\pi} \cos\xi \; d(P\phi); \quad \gamma_s = \frac{1}{2\pi}\int_0^{2\pi} \sin\xi \; d(P\phi);$$

$$\overline{\varepsilon_H} = \langle\varepsilon_{HH}(r)\rangle \sqrt{\gamma_c^{\,2} + \gamma_s^{\,2}} \ ; \qquad (12)$$

$$\langle\varepsilon_{HH}\rangle = \frac{1}{2\pi}\int_0^{2\pi} \varepsilon_{HH} \; d(P\phi). \qquad (13)$$

Additional valuable merit of the proposed magnetic-field model is the possibility to describe the character of the magnetic field with a complicated structure in terms of the parameters conventional for the multiple-helicity fields. It can be done, using the following aproximate expression for the effective helical field ripple (ε_H):

$$\varepsilon_H = \langle\overline{\varepsilon_H}\rangle (1 - \sigma \cos\theta), \qquad (14)$$

where $$\langle\overline{\varepsilon_H}\rangle = \frac{1}{2\pi}\int_0^{2\pi} \overline{\varepsilon_H} \; d(\theta), \qquad (15)$$

$$\sigma = -\frac{1}{\pi \langle\overline{\varepsilon_H}\rangle}\int_0^{2\pi} \overline{\varepsilon_H} \cos\theta \; d(\theta). \qquad (16)$$

After this second step of the averaging of the field ripple over the poloidal angle, an investigation of the distant harmonics effects on the particle confinement can be further simplified, for example, by making a comparison between the values of modulation parameter σ and the helical ripple amplitude given by expressions (14)-(16) and those derived in the simpler magnetic field representation where distant harmonics are ignored.

3. Summary

In the present paper, a specific approach suitable for creating of the accurate magnetic-field model of any multiple-helicity and multiple-toroidicity stellarator/torsatron magnetic field has been developed. It is shown, that the magnetic-field models derived in the paper can adequately reflect all main features of the behavior along field lines of complicated magnetic fields of current devices with TF or MF coils, such as the Wendelstein 7-AS stellarator, the TJ-2 heliac, Heliotron J, and so on. The use of these models can simplify both analytical and numerical investigations of transport and confinement properties of current stellarator-type devices [12].

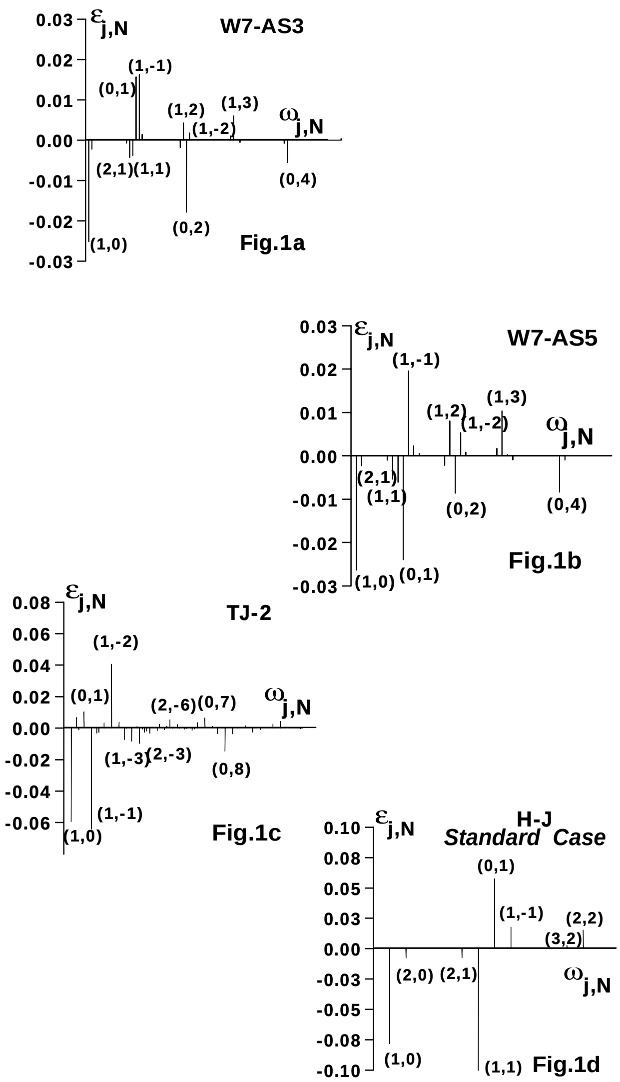

Fig.1. Magnetic field harmonics $\varepsilon_{j,N}$ according to the Fourier-decomposition (1) are shown as functions of the frequencies $\omega_{j,N} = |NM - \iota j|$ for the vacuum magnetic configurations of following stellarators/torsatrons: the Wendelstein 7-AS stellarator (W7-AS3, a), (W7-AS5, b); the TJ-2 heliac (c); the standard configuration of Heliotron J (d). The averaged normalized radius of the flux surface is equal to 0.5

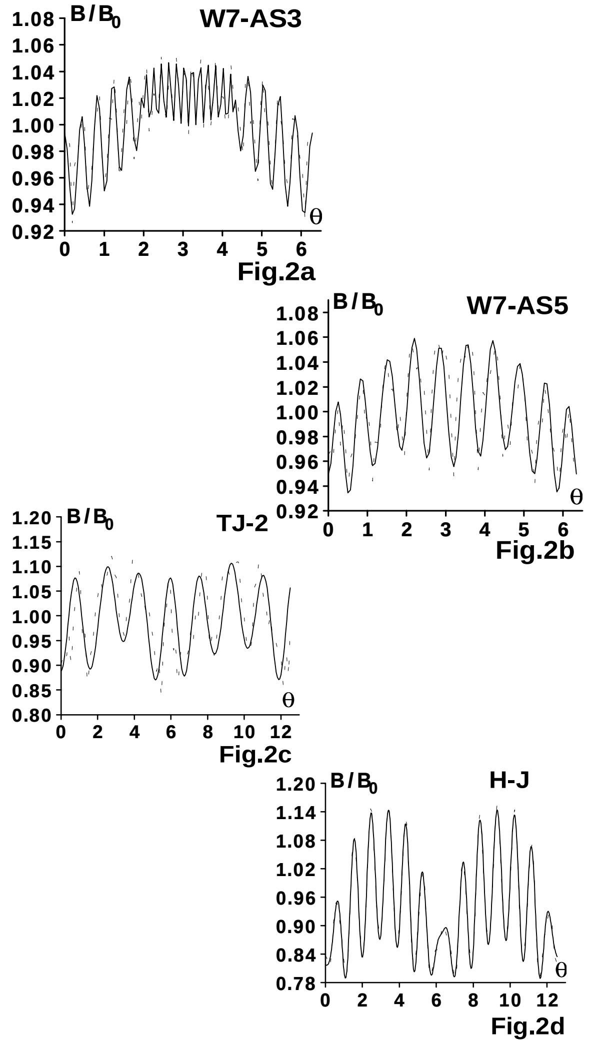

Fig.2. The variation of the magnetic field strength along a field line on the r = 0.5 flux surface, calculated with the help of the full Fourier-decomposition (1) (dotted lines) and using the magnetic-field models (18) (solid lines), is plotted for the same stellarator/torsatron configurations, as in Fig.1

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