NUCLEAR REACTIONS

NON-MODEL DETERMINATION OF THE Δ(*1232*) PARAMETERS

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The Δ(1232) resonance and pole parameters are determined from the data of πN elastic scattering analysis in the framework of a non-model approach.

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In [1] the significant discrepancies in the pole parameters of the P_{33} πN scattering amplitude, namely in the absolute value and phase of the corresponding to the Δ(1232) resonance residue, were discussed using a realistic resonance model. Here we continue the discussion trying to perform model-independent evaluation of the resonance and pole parameters.

The P_{33} amplitude of the elastic πN scattering supposed to be purely elastic in the Δ(1232) excitation region. Corresponding element of S matrix depends on the total energy *W* and can be written in the following general form through real K matrix:

$$S(W) = \frac{1 + iK(W)}{1 - iK(W)}, \qquad (1)$$

for one-channel case the K matrix element can be written in the terms of the phase shift δ_{33} as

$$K(W) = \tan\delta_{33}(W). \qquad (2)$$

It is well known that the right-hand side of Eq. (2) has a pole at $W_0 \cong 1232$ MeV and decreases as $\sim q^3 \to 0$ if *W* goes to its value at the πN threshold (W_0 is the point where the phase shift δ_{33} passes through value 90° and q is the c. m. momentum). These feathers have such a solid experimental and theoretical basement that introducing them explicitly in parameterization of the K matrix element cannot be treated as some kind of a real model restriction:

$$K(W) = \frac{q^3(W)}{q^3(W_0)} \frac{\Gamma_0}{W_0 - W} F(W) \qquad (3)$$

In Eq. (3) Γ_0 is the experimental width of the Δ (1232) resonance, and function *F*(*W*) contains all dynamics of the P_{33} amplitude aside from the threshold behavior and the pole property. In the framework of any specific resonance description *F*(*W*) presents the energy dependence of the experimental width. For example, in [2,3] one can find many model variants of *F*(W). In any phenomenological model with explicit background on level with the resonance interaction the total K matrix element also can be presented in form (3) with *F*(*W*) depending on the background parameters. Quite similar situation takes place in more complicated dynamical models ([4], for example).

In searching the non-model description of the P_{33} amplitude we use power series for *F*(*W*) up to some maximal degree *n*:

$$F(W) = 1 + c_1(W - W_0) + c_2(W - W_0)^2 + \dots . \qquad (4)$$

Actually, this approach is a model-independent base for determination of the resonance and pole parameters of the P_{33} amplitudes in region of the Δ(1232) excitation, if the used series has sufficient converging near the point W_0. Using the K matrix gives an advantage of treating the most simple series expansion with real coefficients. In addition, in the complex W plane a circle of a fixed radius covers the maximal number of experimental points when its center is situated on the real axe. An interval of real axes from $\sim(W_0 - \Gamma_0/2)$ up to $\sim(W_0 + \Gamma_0/2)$ seems be the most preferable in the role of the corresponding mathematical vicinity, as in this case the region of convergence in the complex plane *W* reaches the pole position on the second Riemann's sheet.

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obtained from the SAID system (***http://said.-phys.vt.edu***). Data from different energy intervals $W_1 \dots W_2$ were fitted by χ^2 method with using Eqs. (2), (3) and (4) for calculation of the phase shift δ_{33}. As all but SM99s solutions are given without errors we have used an arbitrary error 0.25° for each point. So, W_0, Γ_0 and coefficients $c_{1\dots n}$ are free parameters ($n \le 4$). Parameters W_0, Γ_0, coordinates of the pole in the complex *W* plate Re W_p, Im W_p, absolute value $|res|$ and phase φ of the residue are presented in the table. *n* in (4) was restricted by the maximal value at which the fit is meaningful. The numbers of used points N and the χ^2 per number of degree of freedom are indicated, too.

As it follows from the first five lines of the table, all parameters, mentioned above, are practically the same for *n* equal 3 and 4. This situation is illustrated by the

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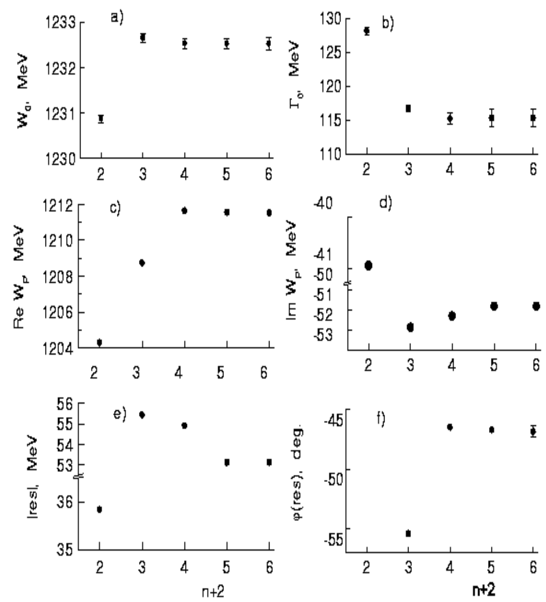
figure, too. Results of fitting on the narrower energy interval confirm this observation. The small shifts in values of being the most sensitive residue parameters give some measure of real errors. It is interesting to note that for SM99 fits on the full energy interval W from 1100 MeV to 1350 MeV give practically the same results as the fits with the data in vicinity of W_0. These answers are in accordance with the resonance model ([1]). The most plausible rounded values of obtained parameters are Γ_0=115.3 MeV; Re W_p=1211.5 MeV; Im W_p=-50.8 MeV; |res|=53.1 MeV; φ=46.7°.

Analyses	W_1, MeV	W_2, MeV	N	n	χ^2/d.f.	W_0, MeV	Γ_0, MeV	Re W_p, MeV	Im W_p, MeV	|res|, MeV	φ, deg.
SP99	1180	1260	17	0	.83D+02	1230.88	128.15	1204.29	-40.93	35.84	-61.32
			17	1	.40D+00	1232.66	116.72	1208.73	-51.89	55.44	-55.42
			17	2	.17D-02	1232.53	115.19	1211.65	-51.33	54.92	-46.50
			17	3	.21D-03	1232.53	115.32	1211.56	-50.84	53.11	-46.73
			17	4	.23D-03	1232.53	115.32	1211.53	-50.84	53.11	-46.87
SP99	1160	1280	25	0	.21D+03	1229.14	119.77	1204.57	-39.52	34.85	-59.45
			25	1	.21D+01	1232.68	118.64	1208.08	-51.66	54.40	-56.21
			25	2	.13D-01	1232.55	115.17	1211.64	-51.50	55.30	-46.67
			25	3	.27D-03	1232.53	115.28	1211.60	-50.92	53.40	-46.63
			25	4	.22D-03	1232.53	115.30	1211.51	-50.92	53.37	-46.98
SP99s	1180	1260	5	0	.15D+03	1230.51	132.19	1202.92	-41.41	36.11	-62.40
			5	1	.28D+01	1232.49	114.82	1209.21	-52.32	56.92	-54.72
			5	2	.54D+00	1231.83	112.21	1214.34	-48.64	49.39	-36.62
SP99s	1160	1280	7	0	.30D+03	1230.01	135.91	1201.51	-41.79	36.32	-63.40
			7	1	.59D+01	1232.89	116.12	1209.03	-53.59	59.06	-55.67
			7	2	.11D+01	1232.14	112.18	1213.60	-50.72	54.42	-40.64
			7	3	.53D+00	1232.04	113.87	1211.87	-47.85	44.98	-45.38
SP99	1100	1350	51	2	.33D+00	1232.63	116.33	1210.78	-51.84	55.65	-48.97
			51	3	.63D-02	1232.55	115.22	1211.66	-51.09	53.89	-46.52
			51	4	.20D-03	1232.53	115.30	1211.53	-50.91	53.34	-46.89
KA84	1180	1260	17	2	.50D+00	1231.27	116.35	1212.18	-51.89	55.56	-39.55
	1160	1280	25	2	.88D+00	1231.32	118.15	1208.68	-52.15	55.89	-50.23
			25	3	.80D+00	1231.40	117.86	1208.41	-54.10	62.73	-51.82
KP80	1180	1260	10	1	.79D+00	1230.94	116.62	1207.00	-49.78	51.51	-55.53
			10	2	.68D+00	1230.91	115.12	1210.53	-50.04	52.29	-44.68
KP80	1160	1280	13	1	.20D+01	1231.00	118.21	1206.46	-50.50	52.44	-56.36
			13	2	.46D+00	1230.92	115.36	1209.81	-50.14	52.52	-46.99
			13	3	.52D+00	1230.93	115.31	1209.84	-50.32	53.15	-46.93
KP80	1100	1350	23	2	.16D+01	1231.01	116.90	1208.45	-50.93	53.76	-50.80
			23	3	.67D+00	1230.92	115.31	1209.79	-49.63	50.68	-47.17
			23	4	.69D+00	1230.94	115.10	1210.07	-49.95	51.63	-46.29



The resonance and pole parameters for solution SM99 vs total number of free parameters n+2

Good convergence of the procedure discussed for solution SM99 can be partially conditioned by a form of the energy-dependent parameterization. Nevertheless such convergence for SM99s as for solutions from previous analyses is not reached. This can be considered as an argument in favor of additional measurements of elastic scattering in the first resonance excitation region.

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