

# SYMMETRIC SURFACE WAVES AT NON-UNIFORM MAGNETIZED STRUCTURES WITH AZIMUTH EXTERNAL MAGNETIC FIELD

*Yu.A. Akimov, M.P. Azarenkov, V.P. Olefir, A.E. Sporov*

*Department of Physics and Technology, Kharkiv National University, Kharkiv, Ukraine,*

*E-mail: olefir@pht.univer.kharkov.ua; Fax: (0572)353977; Tel: (0572)350509*

This report is devoted to the investigation of dispersion properties and axial field structure of the axial-symmetric high-frequency electromagnetic  $E$ -waves that propagate in coaxial magnetized waveguide structure, partially filled with by radially non-uniform plasma. On the axis of the waveguide there is a metal rod of radius  $R_1$ , through which the direct current flows. This current creates a radially non-uniform azimuth magnetic field. The plasma layer occupies the region  $R_1 < r < R_2$ . Between plasma and external metal wall of the waveguide (radius  $R_3$ ) is vacuum gap. The plasma is considered in a hydrodynamic approach as cold collisionless medium. The influence of the geometric parameters of waveguide structure, plasma non-uniformity, direction and value of the direct current on phase characteristics, axial field structure and wave power flux of axial-symmetric  $E$ -waves is considered.

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## 1. INTRODUCTION

The plasma waveguides are widely used in devices of plasma electronics, gas discharges etc. [1]. At present the cylindrical plasma waveguide structures with metal rod on system axis are subject of intensive theoretical and experimental research [2, 3]. The direct current, flowing through the rod, creates azimuth magnetic field that can significantly influence on properties of eigen waves in waveguide structures. It is possible to control the wave properties, wave field spatial distribution and flux of its energy by changing value and direction of the current.

## 2. DISPERSION EQUATION

Let's consider waveguide structure consisting of metal rod (the region  $0 < r < R_1$ ), in which the direct current  $j_z$  flows. This current creates radially non-uniform azimuth magnetic field  $H^0$ . The interior, exterior plasma layer radiuses and radius of the metal waveguide are equal  $R_1, R_2, R_3$  accordingly. In the region  $R_2 < R_3$  there is a vacuum gap. Let's consider the radial profile of plasma layer density in following form:

$$n(r) = n_0[1 - a(r - R_1)/(R_2 - R_1)], \quad a = \Delta n/n_0, \quad (1)$$

where  $\Delta n$  is the difference of plasma density values on interior and exterior radiuses, and  $a$  is non-uniformity parameter. In the considered case the plasma permittivity tensor looks like:

$$\begin{pmatrix} \varepsilon_1 & 0 & -i\varepsilon_2 \\ 0 & \varepsilon_3 & 0 \\ i\varepsilon_2 & 0 & \varepsilon_1 \end{pmatrix}, \quad (2)$$

where  $\varepsilon_1 = 1 - \Omega^2(r)/(\omega^2 - \omega_e^2(r))$ ,  $\varepsilon_3 = 1 - \Omega^2(r)/\omega^2$ ,  $\varepsilon_2 = \Omega^2(r)\omega_e(r)/[(\omega^2 - \omega_e^2(r))\omega]$ , and  $\Omega(r), \omega_e(r)$  are electron plasma and cyclotron frequencies that depend on  $r$  respectively.

Let's consider the slow axial symmetric surface  $E$ -wave with components  $E_z, E_r, H_\phi$ . It is using the Maxwell equations together with boundary conditions that consist in continuity of axial components of wave electric field  $E_z$  and azimuth components of magnetic

one  $H_\phi$  at medium interfaces it is possible to obtain the following dispersion equation:

$$\left\{ F \left[ \frac{\varepsilon_1}{\chi^2} + \frac{\varepsilon_2 k_3}{\chi^2} G \right] - \frac{1}{\chi_v^2} G \right\} \Big|_{r=R_2} = 0, \quad (3)$$

where  $F = f(r)/f'(r)$ ,  $G = g(r)/g'(r)$ , dash means the derivative of appropriate functions with argument, and  $\chi^2 = k_3^2 - k^2 \varepsilon_1(r)$ ,  $\chi_v^2 = k_3^2 - k^2$  characterize the surface wave field penetration depth into plasma and vacuum respectively. The function  $g(r)$  characterizes the radial distribution of component  $E_z$  in plasma and is determined from the following equation:

$$\partial^2 g(r)/\partial r^2 + a_1(r)\partial g(r)/\partial r + a_2(r)g(r) = 0, \quad (4)$$

where  $a_1(r) = 1/r + \partial(\varepsilon_1/\chi^2)/\partial r$ ,  $a_2(r) = k_3 \varepsilon_2 / (r \varepsilon_1) + \chi^2 / \varepsilon_1 \partial(k_3 \varepsilon_2 / \chi^2) / \partial r - \chi^2 - \varepsilon_2 k^2 / \varepsilon_1$ , and function  $f(r) = [K_0(\chi_v r)I_0(\chi_v R_3) - I_0(\chi_v r)K_0(\chi_v R_3)]$  characterizes the radial distribution of  $E_z$  component of the electric field in vacuum. Here  $K_0, I_0$  are McDonald function and modified Bessel one of zero order respectively.

Since the waveguide structure is filled with radial non-uniform plasma that is located in external radial non-uniform azimuth magnetic field, the investigation of the dispersion equation (3) is possible only by numerical methods. For this purpose we define the following dimensionless variables: frequency  $\mu = \omega / \Omega$  ( $r = R_1$ ), axial wavenumber  $\tilde{k}_3 = k_3 R_1$ , radius  $\rho = r \Omega$  ( $r = R_1$ )/ $c$ , cyclotron frequency  $u = \omega_e / \Omega$  ( $r = R_1$ ), current  $J = e j_z / (2mc^3)$ , and  $\tilde{\chi} = \chi R_1$ ,  $\tilde{\chi}_v = \chi_v R_1$ .

In these variables the dispersion equation is reduced to the following form:

$$\left\{ \tilde{F} \left[ \frac{\varepsilon_1}{\tilde{\chi}^2} + \frac{\varepsilon_2 \tilde{k}_3}{\tilde{\chi}^2 \rho_1} \tilde{G} \right] - \frac{1}{\tilde{\chi}_v^2} \tilde{G} \right\} \Big|_{\rho=\rho_2} = 0, \quad (5)$$

where  $\tilde{F} = F \Omega$  ( $r = R_1$ )/ $c$ ,  $\tilde{G} = G \Omega$  ( $r = R_1$ )/ $c$ .

For numerical integration of the equation (5) the Gear method was used.

Note that in such structures the feature of SW is its non-mutuality because its properties depend on directions of wave propagation and of direct current. The influence of waveguide structure parameters on properties of co-directional waves (propagating in the current direction) and counter waves is different.

### 3. CO-DIRECTIONAL SW

One of the features of considered waveguide structure is the existence of high-frequency (HF) and low-frequency (LF) co-directional waves ( $J > 0$ ) in it (fig. 1). In the case, when the axial magnetic field is absent, the low-frequency SW disappears.

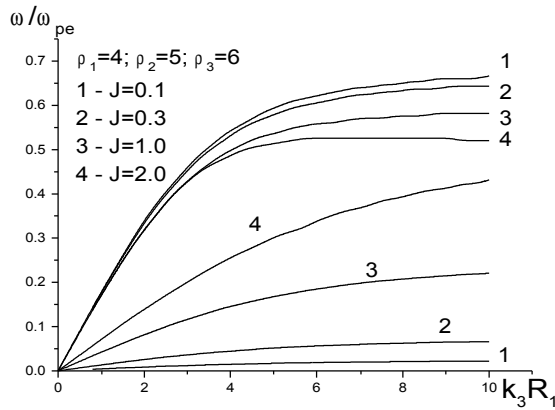


Fig. 1. The influence of direct current on co-direction SW in the case of uniform plasma layer

The carried out analysis has shown that for narrow waveguide structure the increase of current value in metal rod from value  $J = 0.5$  up to  $J = 5$  results in decrease of LF wave frequency approximately ten times. As against narrow waveguide, for wide one the growth of current  $J$  results on the one hand to increase of LF frequency and on the other hand to decrease of HF waves frequency. As a result of it, increase of current  $J$  leads to that frequencies of HF and LF waves come close to each other. Thus there is a change of HF wave dispersion: at the large wavenumber values wave becomes backward.

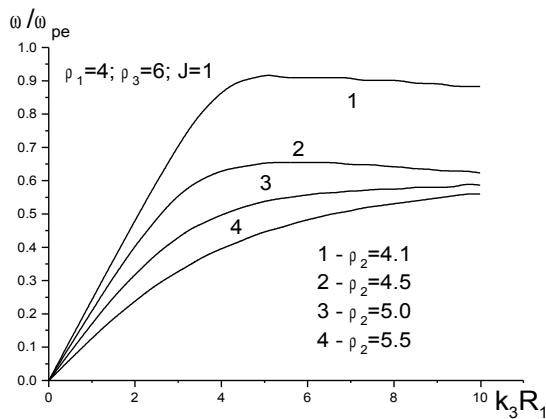


Fig. 2. The influence of the plasma layer thickness on dispersion of the co-directional HF waves

The influence of the plasma layer thickness on dispersion of the co-directional HF waves is shown in fig. 2. The increase of layer thickness at fixed metal radius can result in decrease of wave frequency. So, for example, for the narrow layer (the curve 1) in the region of enough short wave lengths ( $\tilde{k}_3 > 5$ ) HF wave is backward (in this case its phase and group velocities are opposite). With increase of the plasma layer thickness (the curve 4), decreases the vacuum gap thickness, and the character dispersion co-directional HF wave changes. The metal most strongly influences on HF wave in the region of enough large phase velocities.

The plasma layer thickness complicated influences on dispersion LF wave (fig. 3). So, as against co-directional wave (fig. 2), the increase of exterior radius of plasma layer  $\rho_2$  results in the beginning in growth of its frequency. And then, when the vacuum gap thickness becomes enough small, (curves  $\rho_2 = 5.97$ ,  $\rho_2 = 5.99$ ), the influence of metal on LF wave becomes essential and the frequency co-directional LF wave sharply decreases.

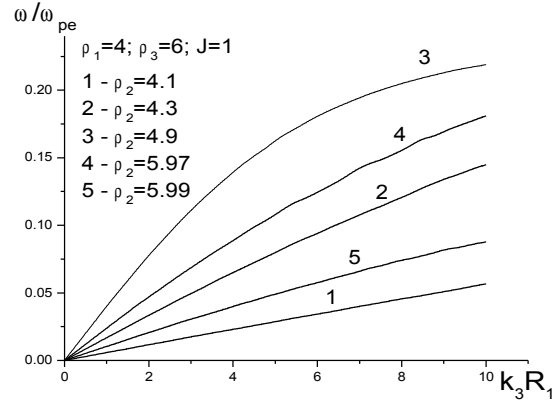


Fig. 3. The influence of the plasma layer thickness on dispersion of the co-directional LF waves

It is shown that the increase of metal waveguide radius  $\rho_3$  at fixed system parameters results in frequency increase of HF and LF co-directional waves.

### 4. COUNTER SW

The change of current direction results in change of wave dispersion (LF wave disappears). Thus mutual direction of wave propagation and current is of great importance and is determined by signs of  $k_3$  and  $\epsilon_2$  in  $a_2$  (4). Thus, considered waves are non-mutual.

The influence current  $J$  that flows through metal rod in the case of radially uniform plasma was investigated, when wave propagation direction and current direction are opposite ( $J < 0$ , counter wave). It is shown that the increase of absolute value  $J$  in this case results in growth of the normalized HF wave frequency  $\mu$ .

As well as in the case of co-directional wave (fig.2), the increase of plasma layer thickness at fixed metal radius results in decrease of the wave phase velocity (fig.4).

The numerical analysis shows that the increase of metal waveguide radius  $\rho_3$  at fixed parameters results in increase of counter wave frequency. The influence of metal

on counter wave is more essential than on co-directional one.

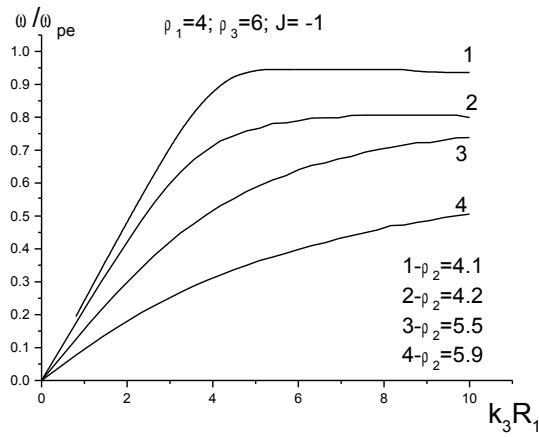


Fig. 4. The influence of the plasma layer thickness on dispersion of the counter waves

### 5. PLASMA DENSITY NON-UNIFORMITY

The influence of plasma density non-uniformity on co-directional HF wave at current  $J = 1$  is shown in fig.5. The curve 1 corresponds to a uniform plasma density profile. The growth of non-uniformity parameter  $a$  (1) results in some increase of the normalized wave frequency  $\mu$ .

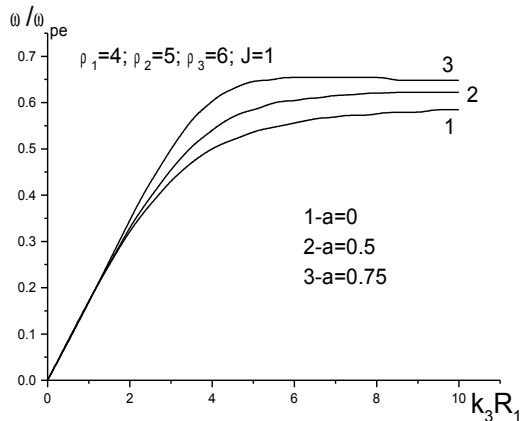


Fig. 5. The influence of plasma density non-uniformity on HF co-directional wave dispersion

The research of LF SW properties in radial non-uniform plasma has shown that the increase of radial plasma density non-uniformity results in decrease of LF co-directional wave frequency, as against HF wave.

The influence of radial plasma density non-uniformity on counter wave dispersion is similar to the case of HF co-directional SW.

Thus, the SW phase characteristics in structures with external azimuth magnetic field depend on radial average plasma density (as well as in the case of an axial magnetic field).

### 6. SW FIELDS STRUCTURE

The analysis of radial field structure for various system parameters has shown that the considered waves are surface (fig. 6). Note that at the certain system parameters

the localization of electric wave field at one plasma layer interface and magnetic wave field at another one is observed. However, as the wave magnetic field amplitude by order less than electric one, the energy of SW is concentrated near plasma-vacuum interface.

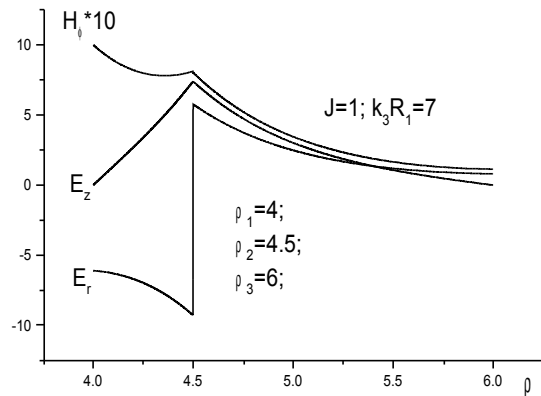


Fig. 6. The radial field structure of HF co-directional wave

### 7. CONCLUSIONS

It is shown that in waveguide structure considered high frequency (HF) and low frequency (LF) surface waves propagate. The feature of these waves is their non-mutuality. Their properties depend on directions of wave propagation and of direct current. The influence of waveguide structure parameters on properties of co-directional waves and counter waves is different.

The influence of current direction and its value on the wave dispersion is investigated. It is shown that the effective control of E-wave dispersion properties by the value and direction of the current is possible. The decrease of the metal radius results in decrease of the HF and LF wave phase velocity. This influence for counter wave is more essential than for co-directional one.

The plasma layer width influence on the dispersion of co-directional waves is studied. The increase of plasma layer width results in decrease of the wave frequency. It is obtained that at enough narrow vacuum gap the character of wave dispersion can be changed.

The influence of the radial plasma non-uniformity on properties of co-directional and counter HF and LF waves is investigated. It is shown that in the case when the plasma density decreases with growth of the radius the increase of radial non-uniformity results in growth of the phase velocity of HF waves and decrease of LF ones.

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