UDK 539.3

M. Król

STOCHASTIC STABILITY OF A CLASS OF PARTIAL DIFFERENTIAL EQUATIONS OF THERMOELASTICITY

An example of stability analysis of a class of partial differential equations (in terms of Lyapunov functional) is presented. Applying Kozin's method to construction of Lyapunov functional the sufficient conditions of stochastic stability of the heat transfer in a strip-plate are established.

1. Introduction. Considerable progress has been made over the last four decades, in the study of the problem of the stochastic stability of partial differential equations. It has been initiated by the J. C. Samuels and Yu. M. Eringer [12] and studies by mathematicians and engineers in the vibration analysis of beams, plates, shells and heat transfer problems. Two basic models with stochastic parametric excitations have been developed in the literature, namely with stationary ergodic processes and white noise processes. For the models with stationary ergodic processes the stability analysis has been initiated by T. K. Caughey and A. H. Gray (jr) [6] and developed by F. Kozin [8, 9], F. Kozin and C. M. Wu [10]. A. Tylikowski has obtained several interesting results [16, 17].

The first work for the models with white noise processes has been published by U. G. Haussmann [7] who applied the obtained results to the study of a heat-transfer problem. T. Caraballo, K. Liu, and X. Mao [5, 11] generalized these results for instance. The new criteria of stability for string, stick and plate models with parametric white noise excitations have been obtain by A. Tylikowski [14, 15].

In this paper we will deal with problem of stability of the partial differential equation, which describe the heat-transfer in strip-plate. We use the equation of motion derived by Yu. M. Kolyano, Ya. S. Podstryhacz [4] and we assume that the parametric excitations are in the form of stationary ergodic process. We apply the Kozin's method to determine sufficient conditions of almost sure asymptotic stability.

2. Mathematical preliminaries. The common stability properties of stochastic systems that have been studied in the literature have generally been related to Lyapunov stability [13]. Recognizing that stability in the Lyapunov sense is merely a uniform convergence, which respect to the initial conditions, various concepts of stability for stochastic systems can be immediately defined by invoking one of the usual modes of probability theory. That is, for instance, convergence in probability and almost sure convergence.

In that follows, $u(t;x_0,t_0)$ will denote the n-dimensional vector solution at time t, with initial state x_0 at time t_0, $\|u\|$ will denote a suitable norm, such as an absolute value or Euclides norm, and we shell be testing the stability of the equilibrium solution $u \equiv 0$ of the partial differential equations

$$\frac{\partial u_i}{\partial t} = F_i(u,x,t)\,, \qquad i = 1,\ldots,M_1\,, \tag{1}$$

$$A_k(u,x,t) = 0\,, \qquad \{t : t \ge 0\}\,, \quad k = 1,\ldots,M_2\,. \tag{2}$$

That system has the following distinctive marks [14]:

1) occupies some region (coherent, open set) $\Omega \subset \mathbb{R}^N$ is N-dimensional Euclidean space with a boundary $B^{N-1} = \partial\Omega$, $x = [x_1,\ldots,x_N] \in \mathbb{R}^N$;

2) describes the set M of the function $u(x,t) = [u_1(x,t),\ldots,u_M(x,t)]$, which belongs to a function space of $X(\bar{\Omega}) \equiv X$, which is called phase space and satisfies the system of partial differential equations (1), (2) in region $C^{N+1} = \Omega \times \Delta \subset \mathbb{R}^{N+1}$, where F_i and A_k are the differential operators, which are described by continuous, differentiable functions and related to spatial values. The space X of the points may be the set of variable parameters, which characterize the condition of the system;

3) the function $u(x,t)$ which characterize the system are assumed to take the values

$$u_0 = u_0(x, t_0) \in X_0(\bar{\Omega}) \supset X(\bar{\Omega}) \tag{3}$$

in the plane $\Omega \times \{t = 0\} \subset \mathbb{R}^{N+1}$ (initial conditions) and the boundary conditions

$$u_1 = u_1(x, t) \in X_1(\bar{\Omega}) \supset X(\bar{\Omega}). \tag{4}$$

In stochastic case instead of equation (1) we consider the following stochastic differential equation

$$\frac{\partial u_i}{\partial t} = F_i(u, x, t, \omega), \qquad i = 1, \ldots, n, \tag{5}$$

where ω is an element of probability space (Γ, B, P). Similarly, the initial and boundary conditions are determined [14].

In this paper we will use the following definitions of stochastic stability [1, 2, 17].

Definition 1. (*Almost Sure Lyapunov Stability*). *The equilibrium solution of system* (5) *is said to be almost surely stable if*

$$P\left\{ \lim_{\|x_0\| \to 0} \sup_{t \ge t_0} \|u(t; x_0, t_0)\| = 0 \right\} = 1.$$

Definition 2. (*Almost Sure Asymptotic Stability*). *The equilibrium solution of system* (5) *is said to be almost surely asymptotically stable if definition* **1** *holds and*

$$P\left\{ \lim_{T \to \infty} \sup_{t \ge T} \|u(t; x_0, t_0)\| = 0 \right\} = 1.$$

2. The strip-plate equation of heat transfer. Let Θ be a bounded domain in $\mathbb{R}^d$, where $d \le 3$, with C^2 boundary. We will study the following heat transfer equation of a thickness $(Z = 2\delta = \text{const})$ isotropic homogenous infinite strip-plate, which is averages by a integrals characteristic of temperature [4]

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} - æ_*^2 T = \frac{1}{a}\frac{\partial T}{\partial \tau^*} + \frac{1}{c_q^2}\frac{\partial^2 T}{\partial \tau^{*2}}, \tag{6}$$

where $T = T(X, Y, \tau^*)$, $\tau^* \in \mathbb{R}^+$ (where $\mathbb{R}^+$ denotes the interval $[0, \infty)$), $X, Y \in \Theta$, $X \in [0, d_1]$, $Y \in [0, d_2]$.

In this equation we use the following notation: T is the temperature, X, Y, Z are the space coordinates, τ^* is the time, c_q is the propagation of speed of the heat, $æ_*^2 = \frac{\alpha_z}{\lambda_t \delta}$, $a = \frac{\lambda_t}{c_v}$ is the coefficient of thermal conductivity, c_v is the coefficient of volumetric heat contents, α_z is the coefficient of the given up the heat by flank, λ_t is the coefficient of heat conduction.

Now we assume the following initial conditions

$$T = 0\,, \qquad \frac{\partial T}{\partial \tau^*} = 0 \qquad \text{for} \qquad \tau^* = 0\,, \tag{7}$$

$$T = 0\,, \qquad \frac{\partial T}{\partial Y} = 0 \qquad \text{for} \qquad Y = 0 \quad \text{and} \quad Y = d_2\,. \tag{8}$$

We introduce in this equation the following notations

$$x = \frac{X}{k_x}\,, \qquad y = \frac{Y}{k_y}\,, \qquad z = \frac{Z}{k_z}\,, \qquad \tau = \frac{\tau^*}{k_\tau^*} \tag{9}$$

are dimensionless coordinates and time, where k_x, k_y, k_z are scale coefficients, and assume that

$$2\beta = \frac{c_q^2}{a}\,, \tag{10}$$

$$æ_*^2 = æ_0 + æ(\tau)\,, \tag{11}$$

where $æ(\tau)$ is a stationary ergodic stochastic process, whose samples are continuous functions with the probability one, $æ_0$ is a constant.

If we assume, that $k_x = d_1$, $k_y = d_2$ in coordinates (9) and using notation (10) and (11) in equation (6) we obtain

$$T_{\tau\tau} + 2\beta T_\tau - c_q^2 T_{xx} - c_q^2 T_{yy} + c_q^2 (k_0 + æ(\tau))T = 0\,, \tag{12}$$

where

$$T_{\tau\tau} = \frac{\partial^2 T}{\partial \tau^2}\,, \qquad T_\tau = \frac{\partial T}{\partial \tau}\,, \qquad T_{xx} = \frac{\partial^2 T}{\partial x^2}\,, \qquad T_{yy} = \frac{\partial^2 T}{\partial y^2}\,,$$

$$(x, y) \in \left(0, \frac{d_1}{k_x}\right) \times \left(0, \frac{d_2}{k_y}\right) = \Omega\,, \qquad z \in \left(-\frac{\delta}{k_z}, \frac{\delta}{k_z}\right), \qquad \tau \in [0, +\infty)\,.$$

3. Stochastic stability analysis. In this section we use the Kozin's method to the equation (6). We investigate the stability of the trivial solutions of this equation.

We set the initial conditions

$$T = 0\,, \qquad T_{xx} = T_{yy} = 0 \qquad \text{for} \qquad (x, y) \in \partial\Omega\,. \tag{13}$$

We assumed that $æ(\tau)$ is the stationary ergodic process with the differentiable realizations with probability 1. We shell study asymptotic stability of the trivial solution $T = T_\tau = 0$ of equation (12) via a Lyapunov functional approach, using Kozin's method [8].

We define the Lyapunov functional

$$V = \int_\Omega Q\left(T, \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial^2 T}{\partial x^2}, \frac{\partial^2 T}{\partial y^2}, \frac{\partial T}{\partial \tau}, \frac{\partial^2 T}{\partial \tau^2}\right) d\Omega\,,$$

where Q is a quadratic form of its variables.

We shell use the following approach. Upon expanding $T(x, y, \tau)$ into its modes, we have

$$T(x, y, \tau) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_{nm}(x, y, \tau) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{nm}(\tau) \sin(\pi n x) \sin(\pi m y)\,. \tag{14}$$

Substituting (14) into (12) yields the model equations

$$\ddot{W}_{nm}^2 + 2\beta \dot{W}_{nm} W_{nm} + c_q^2 ((\pi n)^2 + (\pi m)^2 + æ^2(\tau) + æ_0) W_{nm}^2 = 0\,,$$

$$n, m = 1, 2, 3, \ldots.$$

Using the methodology proposed by F. Kozin and C. M. Wu [10] we will deal with quadratic Lyapunov function for studying the almost sure stability properties of $W_{nm}(\tau)$ in the form

$$V_{nm}(\tau) = \left\| W_{nm} \quad \dot{W}_{nm} \right\| \times$$
$$\times \left\| \begin{matrix} 2\beta^2 + c_q^2\left((\pi n)^2 + (\pi m)^2 + æ^2(\tau) + æ_0\right) & \beta \\ \beta & 1 \end{matrix} \right\| \cdot \left\| \begin{matrix} W_{nm} \\ \dot{W}_{nm} \end{matrix} \right\| .$$

Taking into account the properties

$$(\pi n)^2 T_{nm} = \frac{\partial^2 T_{mn}}{\partial x^2}, \qquad (\pi m)^2 T_{nm} = \frac{\partial^2 T_{mn}}{\partial y^2},$$

we can apply conditions of the ortogonality of the functions $\sin(\pi n x)$ and $\sin(\pi m x)$ on $[0,1]$ for $n, m = 1, 2, 3, \ldots$ in order to obtain the desired Lyapunov functional

$$V(\tau) = \int_\Omega \left(T_\tau^2 + 2\beta T T_\tau + 2\beta^2 T^2 + c_q^2 T_x^2 + c_q^2 T_y^2 + c_q^2 æ_0^2 T^2\right) d\Omega . \tag{15}$$

The time-derivative of the functional (15) along the solution of the equation (12) is given by:

$$\frac{dV(\tau)}{d\tau} = 2\int_\Omega \Big[T_{\tau\tau}(T_\tau + T) + \beta T_\tau^2 + T T_\tau\left(2\beta^2 + c_q^2 æ_0^2\right) +$$
$$+ c_q^2 T_x T_{x\tau} + c_q^2 T_y T_{y\tau}\Big] d\Omega , \tag{16}$$

where

$$T_{x\tau} = \frac{\partial^2 T}{\partial x \partial \tau}, \qquad T_x = \frac{\partial T}{\partial x}, \qquad T_{y\tau} = \frac{\partial^2 T}{\partial y \partial \tau}, \qquad T_y = \frac{\partial T}{\partial y}.$$

Substitution $T_{\tau\tau}$, determined by equation (12) into (16), yields the time-derivative of the functional (15) in the form

$$\frac{dV(\tau)}{d\tau} = -\, 2\beta V(\tau) + 2U(\tau), \tag{17}$$

where new functional $U(\tau)$ has the form

$$U(\tau) = \int_\Omega \Big[\beta T_\tau^2 + 2\beta^2 T T_\tau + 2\beta^3 T^2 + \beta c_q^2 T_x^2 + \beta c_q^2 T_y^2 + \beta c_q^2 æ_0^2 T^2 -$$
$$- c_q^2 æ(\tau)\, T T_\tau - \beta T_\tau^2 - \beta c_q^2 T_x^2 - \beta c_q^2 T_y^2 -$$
$$- \beta c_q^2 æ^2(\tau)\, T^2 - \beta c_q^2 æ_0^2 T^2\Big] d\Omega . \tag{18}$$

Using the dependencies of integrating by parts as for every pair of elements T and T_τ:

$$\int_\Omega T_x T_{x\tau}\, d\Omega = -\int_\Omega T_{xx} T_\tau\, d\Omega ,$$

$$\int_\Omega T_x^2\, d\Omega = -\int_\Omega T T_{xx}\, d\Omega ,$$

$$\int_\Omega T_y T_{x\tau}\, d\Omega = -\int_\Omega T_{xx} T_\tau\, d\Omega ,$$

$$\int_\Omega T_y^2\, d\Omega = -\int_\Omega T T_{yy}\, d\Omega ,$$

and taking into account the fact, that the $V(\tau)$ satisfies the initial conditions (13), we can introduce the functional $U(\tau)$ in the form

$$U(\tau) = \int_{\Omega} \left[\left(2\beta^2 - c_q^2 æ^2(\tau)\right) T T_\tau + \beta \left(2\beta^2 - c_q^2 æ^2(\tau)\right) T^2 \right] d\Omega \,. \tag{19}$$

We look for a function λ, which satisfies the following inequality

$$U(\tau) \le \lambda V(\tau) \,. \tag{20}$$

The function λ is defined as a minimum of the ratio U/V with respect to permissible functions T and T_τ, which satisfy the initial conditions (13). Since the minimum is the particular case of the stationary point, we can apply the calculus of variations and we consider the variation problem $\delta(U - \lambda V) = 0$. After solving the equation of the variations we obtain

$$\int_{\Omega} \Big\{ \left[(2\beta^2 - c_q^2 æ^2(\tau)) T - \lambda (2T_\tau + 2\beta T) \right] \delta T_\tau + \\ + \left[\left(2\beta^2 - c_q^2 æ^2(\tau)\right) T_\tau + 2\beta \left(2\beta^2 - c_q^2 æ^2(\tau)\right) T - \\ - \lambda \left(2\beta T_\tau + 4\beta^2 T - 2c_q^2 T_{xx} - 2c_q^2 T_{yy} + 2c_q^2 æ_0^2 T\right) \right] \delta v \Big\} d\Omega = 0 \,. \tag{21}$$

Taking into account the independence of variations appearing in integrals (18) we find that the square brackets in (21) are equal zero

$$\left(2\beta^2 - c_q^2 æ^2(\tau)\right) T - \lambda (2T_\tau + 2\beta T) = 0 \,, \tag{22}$$

$$(2\beta^2 - c_q^2 æ^2(\tau))(T_\tau + 2\beta T) - \\ - \lambda \left(2\beta T_\tau + 4\beta^2 T - 2c_q^2 T_{xx} - 2c_q^2 T_{yy} + 2c_q^2 æ_0^2 T\right) = 0 \,. \tag{23}$$

We calculate T_τ from equation (22) and substitute it into equation (23). Then we obtain a partial differential equation for the function $T(x, y, \tau)$:

$$\left(2\beta^2 - c_q^2 æ^2(\tau)\right) \left[\frac{(2\beta^2 - c_q^2 æ^2(\tau)) T - 2\lambda\beta T}{2\lambda} + 2\beta T \right] - \\ - \lambda \left[2\beta \frac{(2\beta^2 - c_q^2 æ^2(\tau)) T - 2\lambda\beta T}{2\lambda} + \\ + 4\beta^2 T - 2c_q^2 T_{xx} - 2c_q^2 T_{yy} - 2c_q^2 æ_0^2 T \right] = 0 \,. \tag{24}$$

It is easy to notice, that the n-order approximation of the solution of (24) in the form

$$T = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_{nm}(\tau) \sin(\pi n x) \sin(\pi m y) \tag{25}$$

satisfies initial conditions (13). Substituting (25) into equation (24), we obtain an algebraic equation with respect to the variable λ for every pair (n, m). We denote these variables by λ_{nm}. As a function λ we select the maximum of variables λ_{nm}, which satisfy inequality (20).

Finally the function λ has the form

$$\lambda = \max_{n,m=1,2,\ldots} \left\{ \frac{2\beta^2 - c_q^2 æ^2(\tau)}{2\left(\beta^2 - c_q^2 \left[(\pi n)^2 + (\pi m)^2 + æ_0^2 \right]\right)^{1/2}} \right\} . \tag{26}$$

After substituting inequality (20) to equation (17), we obtain the differential inequality of functional $V(\tau)$. Solving this inequality, we obtain the upper estimation of the functional $V(\tau)$ (lemma 2.1 [3])

$$V(\tau) \le V(0) \exp\left[-2\left(\beta - \frac{1}{\tau}\int_0^\tau \lambda(s)\,ds\right)\tau\right].$$

If $æ(\tau)$ is an ergodic and stationary process and we replace the mean value of the temperature by the corresponding mean value of the set of realizations, we obtain the condition of almost sure asymptotic stability with respect to the following measure of temperature

$$\|T\| = \sqrt{V}\,, \tag{27}$$

where $\tau \to \infty$ in the form $\beta \ge E[\lambda]$.

To derive the sufficient conditions of the almost sure instability we look for a function η, which satisfies the inequality $U(\tau) \ge \eta V(\tau)$. Watching the derivation of the function can check it λ. The function η equals to the minimum of λ_{nm} given by (26) and suitable estimation of the functional $V(\tau)$ has the form

$$V(\tau) \ge V(0) \exp\left[2\left(\frac{1}{\tau}\int_0^\tau \eta(s)\,ds - \beta\right)\tau\right].$$

The condition of almost sure asymptotic instability for the stationary and ergodic process has the form

$$\beta \le E[\eta].$$

The obtained results can be summarized in the following criteria.

Criterion 1. *The trivial solution of the partial differential equation* (12) *is almost sure asymptotic stable with respect to the norm* (27) *if the following conditions are satisfied*

(***i***) *process* $æ(\tau)$ *is stationary and ergodic*;

(***ii***) $\beta \ge E[\lambda]$,

where

$$\lambda = \max_{n,m=1,2,\ldots} \left\{ \frac{2\beta^2 - c_q^2 æ^2(\tau)}{2\left(\beta^2 - c_q^2\left[(\pi n)^2 + (\pi m)^2 + æ_0^2\right]\right)^{1/2}} \right\}.$$

Criterion 2. *The solution of the partial differential equation* (12) *is almost sure asymptotic unstable with respect to the norm* (27) *if the following conditions are satisfied*

i) *process* $æ(\tau)$ *is stationary and ergodic*;

(***ii***) $\beta \le E[\eta]$,

where

$$\eta = \min_{n,m=1,2,\ldots} \left\{ \frac{2\beta^2 - c_q^2 æ^2(\tau)}{2\left(\beta^2 - c_q^2\left[(\pi n)^2 + (\pi m)^2 + æ_0^2\right]\right)^{1/2}} \right\}.$$

Example. We consider a particular case of system (6).

If we assumed that $\frac{\partial^2 T}{\partial X^2}$ is equal zero in equation (6), then this equation we can rewrite in the form

$$\frac{\partial^2 T}{\partial Y^2} - æ_*^2 T = \frac{1}{a}\frac{\partial T}{\partial \tau^*} + \frac{1}{c_q^2}\frac{\partial^2 T}{\partial \tau^{*2}}, \qquad (28)$$

where $T = T(Y, \tau^*)$, $\tau^* \in \mathbb{R}^+$ (where $\mathbb{R}^+$ denotes the interval $[0,\infty)$), $Y \in \Theta$.

Now we assume the following initial conditions

$$T = 0, \qquad \frac{\partial T}{\partial \tau^*} = 0 \qquad \text{for} \quad \tau^* = 0,$$

$$T = 0, \qquad \frac{\partial T}{\partial Y} = 0 \qquad \text{for} \quad Y \in (0, d_2).$$

Now we introduce the following notations

$$y = \frac{Y}{k_y}, \qquad z = \frac{Z}{k_z}, \qquad \tau = \frac{\tau^*}{k_\tau} \qquad (29)$$

are dimensionless coordinates and time, where k_y, k_z are scale coefficients, like in equation (6).

Using notation (10), (11) and (29) in equation (28) we obtain

$$T_{\tau\tau} + 2\beta T_\tau - c_q^2 T_{yy} + c_q^2(k_0 + æ(\tau))\, T = 0 \qquad (30)$$

where

$$T_{\tau\tau} = \frac{\partial^2 T}{\partial \tau^2}, \qquad T_\tau = \frac{\partial T}{\partial \tau}, \qquad T_{yy} = \frac{\partial^2 T}{\partial y^2},$$

$$y \in (0, d_2) = \Omega, \qquad z \in \left(-\frac{\delta}{k_z}, \frac{\delta}{k_z}\right), \qquad \tau \in [0, +\infty),$$

with the simplified initial conditions (13).

Now we shell look for the Lyapunov functional using the method describe above as defined in (15).

The functionals $V(\tau)$ and $U(\tau)$ for equation (30) have the form as follows

$$V(\tau) = \int_\Omega \left(T_\tau^2 + 2\beta T T_\tau + 2\beta^2 T^2 + c_q^2 T_y^2 + c_q^2 æ_0^2 T^2\right) d\Omega,$$

$$U(\tau) = \int_\Omega \left[\left(2\beta^2 - c_q^2 æ^2(\tau)\right) T T_\tau + \beta\left(2\beta^2 - c_q^2 æ^2(\tau)\right) T^2\right] d\Omega.$$

Again we wish to determine the λ so that satisfies the inequality

$$U(\tau) \le \lambda V(\tau)$$

and we apply the variational calculus to solve the problem

$$\delta(U - \lambda V) = 0. \qquad (31)$$

After applying the straightforward computations, we find the sequence of λ_m that satisfies (31) to be

$$\lambda = \max_{n,m=1,2,\ldots} \left\{ \frac{2\beta^2 - c_q^2 æ^2(\tau)}{2\left(\beta^2 - c_q^2\left[(\pi m)^2 + æ_0^2\right]\right)^{1/2}} \right\}.$$

4. Conclusions. The major conclusions are that the Lyapunov's method is an effective tool of solving the stability problem of strip-plate. The explicit criteria developed in the paper define the stability region in terms of the excitation process and physical characteristic of strip-plate. The analytical formulas defining the stability regions are obtained using the calculus of variations.

1. *Гихман И. И., Дороговцев А. Я.* Об устойчивости решений стохастических дифференциальных уравнений // Укр. мат. журн. – 1968. – **17**, № 6. – С. 3–21.
2. *Гихман И. И., Скороход А. В.* Стохастические дифференциальные уравнения. – Киев: Наук. думка, 1968. – 354 с.
3. *Хасъминский Р. З.* Устойчивость систем дифференциальных уравнений при случайных возмущениях их параметров. – Москва: Наука, 1969. – 367 с.
4. *Подстригач Я. С., Коляно Ю. М.* Обобщенная термомеханика. – Киев: Наук. думка, 1976. – 310 с.
5. *Caraballo T., Liu K.* On exponential stability criteria of stochastic partial equation // Stoch. Proc. Appl. – 1999. – **83**. – P. 289–301.
6. *Caughey T. K., Gray A. H. (Jr.)* On the almost sure stability of linear dynamic systems and stochastic coefficients // Trans. ASME. J. Appl. Mech. – 1965. – **32**. – P. 365–372.
7. *Haussmann U. G.* Asymptotic stability of the linear Ito equation in infinite solution // J. Math. Anal. Appl. – 1978. – **65**. – P. 219–235.
8. *Kozin F.* On almost sure stability of linear systems with random coefficients // J. Math. and Phys. – 1963. – **42**. – P. 59-67.
9. *Kozin F.* Stability of the linear stochastic systems // Lect. Notes in Math. – Berlin: Springer-Verlag. – 1972. – **294**. – P. 186–229.
10. *Kozin F., Wu C. M.* On the stability of linear stochastic differential equations // Trans. ASME. J. Appl. Mech. – 1973. – **40**. – P. 87–92.
11. *Liu K., Mao X.* Exponential stability of non-linear stochastic evolution equations // Stoch. Processes and their Appl. – 1998. – **72,** No. 2. – P. 173–193.
12. *Samuels J. C., Eringen Yu. M.* On stochastic linear systems // J. Math. and Phys. – 1959. – **78**. – P. 83–103.
13. *Skalmierski B., Tylikowski A.* Stabilność układów dynamicznych. – Warszawa: Wyd-wo Nauk. PWN, 1973. – 174 s.
14. *Tylikowski A.* Dynamic stability of rotating shafts // Ing.-Archiv. – 1981. – **50**. – P. 41–48.
15. *Tylikowski A.* Stability of non-linear plate // Trans. ASME. J. Appl. Mech. – 1978. – **45**. – P. 583–585.
16. *Tylikowski A.* Stochastyczna stateczność ciągłych układów dynamicznych // Zeszyty Nauk. Politechniki Śl. (Gliwice). – Automatyka. – 1972. – № 20.
17. *Tylikowski A.* Stochastyczna stateczność układów ciągłych. – Warszawa: Wyd-wo Nauk. PWN, 1991. – 230 s.

СТОХАСТИЧНА СТАБІЛЬНІСТЬ ДЕЯКОГО КЛАСУ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ТЕРМОПРУЖНОСТІ У ЧАСТИННИХ ПОХІДНИХ

Виведено умови стохастичної стабільності для рівняння термопружності для тонкої пластинки. Для цього згідно з методом Козіна використано функціонал Ляпунова. Як приклад встановлено умови стохастичної стабільності рівняння термопружності для напівнескінченного стержня.

СТОХАСТИЧЕСКАЯ СТАБИЛЬНОСТЬ НЕКОТОРОГО КЛАССА ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ТЕРМОУПРУГОСТИ В ЧАСТНЫХ ПРОИЗВОДНЫХ

Выведены условия стохастической стабильности для уравнений термоупругости для тонкой пластинки. Для этого согласно методу Козина использован функционал Ляпунова. В качестве примера установлены условия стохастической стабильности уравнения термоупругости для полубесконечного стержня.

Politechnika of Rzeszów, Rzeszów, Poland
Received
12.08.08