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Recovery of the Basis of Group π_{2n} Representation on its Subgroup $\pi_{n\times n}$ and Harriman's Theorem

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Восстановление базиса представления группы π_{2n} на её подгруппе $\pi_{n\times n}$ и теорема Гарримана

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Відновлення базису, що дає представлення групи π_{2n} на базисі її підгрупи $\pi_{n\times n}$, і теорема Гаррімана

The proof of Harriman's theorem [1] is given for arbitrary order reduced density matrix of both the clear, and the mixed states of fermions at once. Its essential parts are a Pauli exclusion principle, rotation group symmetry of spin functions and new commutation relations.

Теорема Гарримана [1] доказана для редуцированных матриц плотности чистого и смешанного состояний фермионов из принципа Паули, симметрии спиновых функций, новых следствий из перестановок штрихованных и не штрихованных переменных.

Теорема Гаррімана [1] доведена для редуцироних матриць густини чистого і змішаного станів ферміонів з використанням принципу Паулі, симетрії спінових функцій і нових наслідків, пов'язаних з переміщеннями штрихованих і нештрихованих змінних

1. Reduced density matrix

Reduced density matrix of arbitrary order *p* (RDM-p) [2, 3], including transitional RDM-p, for fermions states with spin projection values *M* and *M′* can be written in form [4]

$$\Gamma^{(p)\mu}\left(x_1,x_2,\ldots,x_p\middle|x_1',x_2',\ldots,x_p'\right) = \sum_{(\gamma)\omega\mu} R^{(p)\mu}_{(\gamma)\omega}\cdot \bar{C}^{(p)\mu}_{(\gamma)\omega} . \qquad (1)$$

Its spin and "conjugated" space components look like

$$\hat{T}^{(p)\mu}_{(\gamma)\omega} = \sum_t \sum_{\{i\}_t} \sum_{\{i'\}_{t-\mu}} \left\langle \sigma \middle| \mathrm{p}\{i\}_t \right\rangle \left\langle \mathrm{p}\{i'\}_{t-\mu} \middle| \sigma' \right\rangle \cdot$$

$$\cdot \left\langle p\{i\}_t \{i'\}_{t-\mu} \middle| p(\gamma)\omega\mu \right\rangle, \tag{2}$$

$$R^{(p)\mu}_{(\gamma)\omega} = \sum_t \sum_{\{i\}_t} \sum_{\{i'\}_{t-\mu}} (-1)^{\varepsilon\{i\}_t + \varepsilon\{i'\}_{t-\mu}} \left\langle p(\gamma)\omega\mu \middle| p\{i\}_t \{i'\}_{t-\mu} \right\rangle \cdot$$

$$\cdot L^{(p)}_{\mu t} \left(\{i\}_t * \{j\}_{p-t} \middle| \{i'\}_{t-\mu} * \{j'\}_{p-t+\mu} \right). \tag{3}$$

There are conjugated unitary transformation coefficients $\left\langle p\{i\}_t \{i'\}_{t-\mu} \middle| p(\gamma)\omega\mu \right\rangle$ and $\left\langle p(\gamma)\omega\mu \middle| p\{i\}_t \{i'\}_{t-\mu} \right\rangle$ are appeared in Eqs (2), (3). The contravariant spinors

$$\left\langle \sigma \middle| \mathrm{p}\{i\}_t \right\rangle = \alpha(i_1)\cdot\alpha(i_2)\cdots\alpha(i_t)\cdot\beta(j_1)\cdot\beta(j_2)\cdots\beta(j_{p-t}), \tag{4}$$

its conjugated covariant spinors and hermit spin tensors $\hat{T}^{(p)\mu}_{(\gamma)\omega}$ of a rank ω are ortho-normalized. Antisymmetry of wave functions of states is concluded in Eq. (1). Therefore for antisimmetrizer $\hat{A} = \hat{A}^2$ is true $\sum_{(\gamma)\omega\mu} R^{(p)\mu}_{(\gamma)\omega} \cdot \hat{T}^{(p)\mu}_{(\gamma)\omega} = \hat{A}\cdot \sum_{(\gamma)\omega\mu} R^{(p)\mu}_{(\gamma)\omega} \cdot \hat{T}^{(p)\mu}_{(\gamma)\omega} \cdot \hat{A}$. Both spin and space coordinates of particles are designated by its numbers $\{i\}_t \cup \{j\}_{p-t} = \{1, 2, \ldots p\}$.

2. Recovery of the basis of group π_{2n} representation on its subgroup $\pi_{n\times n}$

Unlike the one-particle contravariant spinors α and β, which connected to spin space irreducible representation (IR) $D^{½}$ of rotation group $SU(2)$ [5, 6], covariant spinors $\alpha^+(i)$ and $\beta^+(i)$ belong to the <u>equivalent</u> IR $D^{½}* = V^{½}\cdot D^{½}\cdot(V^{½})^+$ with unitary matrix [5, page 165]: $V^{½} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then the spinors $(V^{½})^+\cdot\begin{pmatrix} \alpha^+ \\ \beta^+ \end{pmatrix} = \begin{pmatrix} \beta^+ \\ -\alpha^+ \end{pmatrix}$ are transforming on space of IR $D^{½}$, as well as spin functions $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$. Therefore conjugated to Eq. (4) covariant spinors are equivalent to contravariant ones. After replacement every $\alpha^+(i)$ on a spinor $-\beta(i)$ and $\beta^+(j)$ respectively on $\alpha(j)$ its generate the basis of spinors $\langle\sigma|\mathrm{p}\ [i]_t\rangle$. It's belong the same space of a direct product $\prod_{i=1}^{p} D_i^{1/2}$ of p identical IR $D^{1/2}$ of group $SU(2)$, as contravariant spinors (see Eq. (4)),

$$\langle\sigma|\mathrm{p}\ [i]_t\rangle = (-1)^t \cdot \beta(i_1)\cdot\beta(i_2)\cdots\beta(i_{p-t})\cdot\alpha(j_1)\cdot\alpha(j_2)\cdots\alpha(j_{p-t})\ . \tag{5}$$

As result, the basis from products of kontra- and of covariant spinors, as the basis of spinors of a $2p$-rank from products of only contravariant spinors $\langle\sigma|p\{i\}_t\rangle\langle\sigma'|p[i']_{t'}\rangle$, belong to the same representation of rotation group. It allows properties of one basis, established on the ground of its group-theoretical reviewing, to transfer on properties of equivalent basis.

From theory of IR $[\lambda]=[p+\omega, p-\omega]$ for permutation group π_{2p} it follows, that the functions $\varphi_u = \hat{Y}_u\hat{Q}_{uv}\varphi$ and function $\varphi_v = \hat{Y}_v\varphi$, where permutation $\hat{Q}_{vu} = \hat{Q}_{uv}^{-1}$ substitutes numbers of coordinates in Young table of u on numbers from the Young table v [5, 6], assimilate one IR basis. Instead the permutation $\hat{Q}_{uv}$ connects functions φ_u and φ_v [6]

$$\hat{Q}_{uv}\varphi_v = \hat{Q}_{uv}\hat{Y}_v\varphi = \hat{Q}_{uv}\hat{Y}_v\hat{Q}_{uv}^{-1}\hat{Q}_{uv}\varphi = \hat{Y}_u\hat{Q}_{uv}\varphi = \varphi_u\ . \tag{6}$$

Call to mind, that both in lines and in columns of standard Young tables [5, 6] the number of coordinates are ranking from left to right and from top to down according to increase of their place number in the initial list. For list 1, 2, ..., p-ω, p-ω+1,...,p, 1′, 2′, p-ω′, p-ω+1′,...,p′ such Young table will be obtained, when the coulombs of Young scheme [p+ω, p-ω], where $\omega = 0, 1, \ldots, p$, are filling by numbers 1, 1′, 2, 2′,..., p-ω, p-ω′, p-ω+1,...,p, p-ω+1′,...,p′

1	2	…	i_m	…	p-ω	p-ω+1	…	p	(p-ω+1)′	…	p′
1′	2′	…	i'_m	…	(p-ω)′						

Fig. 1.

The number of standard Young tables [5, 6] equals to dimension of IR $\left[p+\omega, p-\omega\right]$

$$f_{\omega}^{2p} = \binom{2p}{p-\omega} - \binom{2p}{p-\omega-1} = \frac{(2p)!(2\omega+1)}{(p+\omega+1)!(p-\omega)!}, \tag{7}$$

A permutation may be represented by product not intersected cyclical permutations. Its do not leave invariant variable places from their list. Cycles are representing by product of transposition, which are not commute. Such transpositions commutate according to a rule [6]

$$(i,k)\cdot(k,l) = (i,l)\cdot(i,k) = (k,l)\cdot(i,l), \tag{8}$$

We prove now main <u>assertion</u> according to which it is possible to receive any function φ_u simmetrized on the standard Young table u for IR $\left[p+\omega, p-\omega\right]$ from function φ_v permuting its arguments only inside each of sets on *p* variable with the primed or not primed numbers separately, if in the second line of the "standard" table of Young v the variables of one type set only remain, as in the Young table v in a Fig. 1. Clearly, if transition from φ_v to φ_u demands rearrangement of primed variables with not primed, then in Eq. (6) $\hat{Q}_{uv} \in \pi_{2p}$.

Really, it is possible to separate exactly l_{uv} transpositions $\hat{Q}_{(l)uv} = \prod_{i=1}^{l_{uv}} (p+1-i, i')$ in $\hat{Q}_{uv} \in \pi_{2p}$, conserving after their operation on the table v correct regularity of not primed numbers concerning primed both in table lines and in table columns. The number pairs l_{uv} of permuted primed and not primed variables is restricted an inequality $l_{uv} \le \min\left\{p-\omega, [p/2]\right\}$, where $[p/2]$ – the whole part from *p/*2.

The factorization of permutations $\hat{Q}_{uv} = \hat{Q}_{(uv)} \cdot \hat{Q}'_{(uv)} \cdot \hat{Q}_{(l)uv}$ reduces our <u>assertion</u> to the proof of a possibility of replacement of each transposition $\left(i_m i'_k\right)$ in $\hat{Q}_{(l)uv}\hat{Y}_v$ by an identical operation on $\hat{Y}_v$ of permutations belonging to only a subgroup $\pi_p \times \pi'_p$:

$$\left(i'_k, i_m\right)\cdot \hat{Y}_v = \hat{Y}_{\left(i'_k, i_m\right)\cdot v} \cdot \left(i'_k, i_m\right) = 2\hat{A}_{i_k i_m} \hat{A}_{i'_k i'_m} \hat{Y}_v . \tag{9}$$

In equality (9) we always have $k < m$ and as in Eq. (6) Young projectors are equal

$$\hat{Y}_\nu = c\cdot \prod_{\nu=1}^{p-\omega} \hat{A}_{i_\nu i'_\nu}\, \hat{S}_{\left\{\{i\}_p\{i'\}_\omega\right\}_{p+\omega}}\, \hat{S}_{\{i'\}_{p-\omega}} \ , \tag{10}$$

$$\hat{Y}_{(i'_k,i_m)\nu} = c\cdot \prod_{\substack{\nu=1\\(\nu\neq k,m)}}^{p-\omega} \hat{A}_{i_\nu i'_\nu}\, \hat{A}_{i_k i_m}\, \hat{A}_{i'_k i'_m}\, \hat{S}_{\left\{\{\dots\}_{p+\omega}\setminus i_m, i'_k\right\}_{p+\omega}}\, \hat{S}_{\left\{\{\dots\}_{p-\omega}\setminus i'_k, i_m\right\}_{p-\omega}} \ . \tag{11}$$

There $\prod_\nu \hat{A}_{i_\nu i'_\nu}$ $\left(\hat{S}_{\{\dots\}_{p+\omega}}\ \hat{S}_{\{\dots\}_{p-\omega}}\right)$ is product of antisimmetrizers and of simmetrizers respectively, acting on the corresponding sets of variables, c is constant to ensure that $\hat{Y}_\nu^{\,2} = \hat{Y}_\nu$. Due equality (9) any operator $\hat{Q}_{uv}$ in (6) may be changed on equal operator but only from permutations belonging to subgroup $\pi_p \times \pi'_p$. It constructs group π_{2p} basis $\{\varphi_u\}$ using the subgroup operators. If $i_k, i_m \in \{\dots\}_{p+\omega}$, $i'_k, i'_m \in \{\dots\}_{p-\omega}$, then $\hat{A}_{i_k i_m}\hat{S}_{\{\dots\}_{p+\omega}}\hat{S}_{\{\dots\}_{p-\omega}} =$ $= \hat{A}_{i'_k i'_m}\hat{S}_{\{\dots\}_{p+\omega}}\hat{S}_{\{\dots\}_{p-\omega}} = 0$ and in a right-hand part of Eq. (9) instead of $\hat{A}_{i_k i'_k}\cdot\hat{A}_{i_m i'_m}\cdot\hat{Y}_\nu$ the transpositions $\left(-(i_k,i'_k)-(i_m,i'_m)\right)/4$ will stay, when the definition (10) for $\hat{Y}_\nu$ is used:

$$\hat{A}_{i_k i_m}\hat{A}_{i'_k i'_m}\,\frac{1}{2}\prod_{\substack{\nu=1\\(\nu\neq k,m)}}^{p-\omega}\hat{A}_{i_\nu i'_\nu}\cdot\left(-(i_k i'_k)-(i_m i'_m)\right)\cdot\hat{S}_{\{\dots\}_{p+\omega}}\hat{S}_{\{\dots\}_{p-\omega}} \ . \tag{12}$$

Here $\{\dots\}_{p+\omega}$ and $\{\dots\}_{p-\omega}$ are the same sets of variables, as in Eq. (10).

Take in mind the equality:

$$\hat{S}_{\{\dots\}_{p+\omega}}\ \hat{S}_{\{\dots\}_{p-\omega}} = (i_m i'_k)\cdot\hat{S}_{\left\{\{\dots\}_{p+\omega}\setminus i_m, i'_k\right\}_{p+\omega}}\ \hat{S}_{\left\{\{\dots\}_{p-\omega}\setminus i'_k, i_m\right\}_{p-\omega}}\ (i_m i'_k) \ , \tag{13}$$

and rules (8) we transform expression (12) into an operator

$$\hat{A}_{i_k i_m}\hat{A}_{i'_k i'_m}\,\frac{1}{2}\prod_{\substack{\nu=1\\(\nu\neq k,m)}}^{p\text{-}\omega}\hat{A}_{i_\nu i'_\nu}\left(-(i_k i_m)(i_k i'_k)-(i'_k i'_m)(i_m i'_m)\right)\cdot\hat{S}_{\left\{\{\dots\}_{p+\omega}\setminus i_m, i'_k\right\}_{p+\omega}}\ \cdot$$
$$\cdot\,\hat{S}_{\left\{\{\dots\}_{p-\omega}\setminus i'_k, i_m\right\}_{p-\omega}}\,(i_m i'_k), \tag{14}$$

In Eq. (14) first transpositions are retracted into suitable antisimmetrizers with a change of sign, and second are retracted into simmetrizers. Transposition $(i_m i'_k)$ will be commutated to left with all operators transforming its. As a result we obtain the left-hand part of identity (9). For the function $\varphi_M = \left[(n+M)!(n-M)!\right]^{-1/2} \cdot [\alpha...\alpha]_{n+M} [\beta...\beta]_{n-M}$ [6, 7] we always can to give the IR $[n+s, n-s]$ basis of group π_{2n} in the following form

$$\varphi_u^{[\lambda]} = \sum_{\hat{Q}_1 \in \pi_{n+M}} \sum_{\hat{Q}_2 \in \pi_{n-M}} C_{uv}\left(\hat{Q}_1 \hat{Q}_2\right) \cdot \hat{Q}_1 \hat{Q}_2 \cdot \hat{Y}_v \cdot \varphi_M , \qquad -s \le M \le s. \tag{15}$$

Here $C_{uv}\left(\hat{Q}_1 \hat{Q}_2\right)$ are simple numerical coefficients and u is a standard Young table.

3. The proof of the Harriman's theorem

For the given p and $\mu = M - M' = t - t'$ in Eq. (3) the number of "independent" space components RDM is determined both impossibility of build-up of tensors of a higher rank, than $\omega = p$, in p–partial spin space, and Harriman's theorem, proved for some cases [1]. It defines a possibility to receive all components $R_{(\gamma)\omega}^{(p)\mu}$ in Eqs. (1), (2) from "standard" component with the same ω and μ, which obtained according to a "standard addition schema of the spin moments (γ_o)" [5, 6] , that is equivalent Young table v in a Fig. 1. The obtained above results (see Eq. 15)) allow us to give the common proof of the Harriman's theorem.

To build the irreducible tensors Eq. (2) routinely the products of the conjugated spinors $\alpha\alpha^+$, $\alpha\beta^+$, $\beta\alpha^+$ and $\beta\beta^+$ must be changed by one-particle operators [8],

$$\hat{E} = \alpha \cdot \alpha^+ + \beta \cdot \beta^+, \quad \hat{S}_z = \hat{t}_1^{(0)} = \tfrac{1}{2}(\alpha \cdot \alpha^+ + \beta \cdot \beta^+), \tag{16a}$$

$$\hat{S}_+ = -\sqrt{2} \cdot \hat{t}_1^{(+1)} = \alpha \cdot \beta^+, \quad \hat{S}_- = \sqrt{2} \cdot \hat{t}_1^{(-1)} = \beta \cdot \alpha^+, \tag{16b}$$

$$Sp_\sigma \hat{E}^2 = 2, \quad Sp_\sigma \hat{t}_1^{(\mu)} \cdot \hat{t}_1^{(\mu)+} = \frac{\delta_{\mu\mu'}}{2}, \quad Sp_\sigma \hat{t}_1^{\mu} \cdot \hat{E} = 0. \tag{16c}$$

Here $\hat{E}$ is a unit operator, $\hat{t}_1^{(\mu)}$ are components of one-particle tensor of the first rank. Then the complete set of addition schemes {(γ)} may be realized by serial adding the operator $\hat{E}$ or $\hat{t}_1^{(\mu)}$ to already built irreducible tensor of smaller number of particles. As result, we can define the union list from p one-particle operators and from the intermediate values of tensor rank for addition schema (γ).

The operators $\langle\sigma|p\{i\}_t\rangle\langle p\{i'\}_{t'}|\sigma'\rangle$ and the tensor $\hat{T}^{(p)\mu}_{(\gamma)\omega}$ can be transformed on the spinors $\langle\sigma|p\{i\}_t\rangle\langle\sigma'|p\ [i']_{t'}\rangle$, as in Eq. (5), and $\hat{T}^{(2p)\mu}_{[\gamma]\omega}$ depending out of 2p of spin variables. Then last save the same addition schema $[\gamma] = (\gamma)$, the moment ω and its projection μ. It is true, "as from the permutation operators of spin coordinates of particles commute with the spin rotation operators [6] it follows that the dimension Eq. (7) of $[p+\omega, p-\omega]$ IR for groups π_{2p} and the number of addition schemes $\{[\gamma]\}$ of α, β spins with same complete values ω and μ are equal." Each such addition schema correspond the line of IR of permutation group π_{2p}. Therefore 2p-particle functions $\hat{T}^{(2p)\mu}_{[\gamma]\omega}$ can be numbered by addition schemes of one set $\{[\gamma]\}$. They form basis for IR $[\lambda_\omega]=[p+\omega, p-\omega]$ of symmetric group π_{2p} and according to Eq. (15) they can be obtained from a function $\hat{T}^{(2p)\mu}_{[\gamma_0]\omega}$. Let's find mutual conformity between spinor $\hat{T}^{(2p)\mu}_{[\gamma_0]\omega}$ and tensor $\hat{T}^{(p)\mu}_{(\gamma_0)\omega}$. It is evident the tensor $\hat{T}^{(p)\mu}_{(1^{p-\omega}\ldots)\omega}$ with a special addition schema $(\gamma_0) = (1^{p-\omega}\ldots)\omega$ gives this appropriation. It looks like

$$\hat{T}^{(p)\omega}_{(1^{p-\omega}\ldots)\omega} = 2^{-\frac{p-\omega}{2}}\hat{I}(1)\hat{I}(2)\cdots\hat{I}(p-\omega)\hat{S}_+(p-\omega+1)\cdots\hat{S}_+(p). \qquad (17)$$

It has the predicted values of ranks of the all intermediate tensors for any p and after using Eqs. (16), (5) it is equivalent to a function-spinor of a rank ω

$$\hat{T}^{(2p)(\omega)}_{[1^{p-\omega}\ldots]\omega} = 2^{-\frac{p-\omega}{2}}\cdot\prod_{i=1}^{p-\omega}(\alpha(i)\beta(i')-\beta(i)\alpha(i'))\prod_{j=p-\omega+1}^{p}\alpha(j)\alpha(j'). \qquad (18)$$

Function (18) is really simmetrized under the Young table [6], which columns are completed by numbers 1, 1′, 2, 2′, ..., p-ω, p-ω′, ..., as in a Fig. 1, and from Eq. (15) follows

$$\hat{T}^{(2p)\omega}_{[\gamma]\omega} = \sum_{\hat{Q},\hat{Q}'\in\pi_p\times\pi'_p} C^{(p)}_{(\gamma)\omega}(Q,Q')\cdot\hat{Q}\hat{Q}'\cdot\hat{T}^{(2p)\omega}_{[1^{p-\omega}\ldots]\omega} = \hat{\pi}^{(p)}_{(\gamma)}\hat{T}^{(2p)\omega}_{[1^{p-\omega}\ldots]\omega}. \qquad (19)$$

After we return to tensors Eq. (2), using Eqs. (16) in Eqs. (18), (19), the same expansion remains valid both for the highest component $\hat{T}^{(p)\omega}_{(\gamma)\omega}$ and for all components of the tensor

$\hat{T}^{(p)\mu}_{(\gamma)\omega}$ too, as they may be obtained from $\hat{T}^{(p)\omega}_{(\gamma)\omega}$ by spin projection decreasing operators $\hat{S}_-$

$$\left[\hat{S}_\pm, \hat{T}^{(p)\mu}_{(\gamma)\omega}\right]_- = \sqrt{\omega\left(\omega+1\right)-\mu\left(\mu\pm 1\right)}\cdot\hat{T}^{(p)\mu\pm 1}_{(\gamma)\omega}. \tag{20}$$

As operators $\hat{S}_\pm=\prod_{i=1}^{p}\hat{S}_\pm\left(i\right)$ commute with all permutation operators of particles and with $\hat{Q}\cdot\hat{Q}'\in\pi_p\times\pi_p$ too, we have general proof of “Garriman’s theorem” in spin space

$$\hat{T}^{(p)(\mu)}_{(\gamma)\omega}=\sum_{Q,Q'\in\pi_p\times\pi'_p} C^{(p)}_{(\gamma)\omega}\left(Q,Q'\right)\cdot\hat{Q}\hat{T}^{(p)(\mu)}_{\left(1^{p-\omega}\dots\right)\omega}\hat{Q}'^{+}=\hat{\pi}^{(p)}_{(\gamma)}\hat{T}^{(p)(\mu)}_{\left(1^{p-\omega}\dots\right)\omega}. \tag{21}$$

These tensors may be orthonormalized, as in (1), without change its obtained structure (21).

In this case applying expansion (2), we make up the identity (21) into linear relationships between coefficients $\left\langle \mathrm{p}\{i\}_t\,\{i'\}_{t-\mu}\middle|\mathrm{p}(\gamma)\omega\mu\right\rangle$ inside of Eqs. (2), (3). They contain same factors, as in Eq. (21),

$$\left\langle \mathrm{p}\{i\}_t\,\{i'\}_{t-\mu}\middle|\mathrm{p}(\gamma)\omega\mu\right\rangle=$$
$$=\sum_{Q,Q'\in\pi_p\pi'_p} C^{(p)}_{(\gamma)\omega}\left(Q,Q'\right)\left\langle \mathrm{p}\hat{Q}^{-1}\{i\}_t\,\hat{Q}'^{-1}\{i'\}_{t-\mu}\middle|\mathrm{p}\left(1^{p\text{-}\omega}\dots\right)\omega\mu\right\rangle. \tag{22}$$

Taken into account that the number sets in Eq. (22) and ones at sign factor in Eq. (3) are same we shall receive the connection of space components $R^{(p)\mu}_{(\gamma)\omega}$ with <u>standard</u> <u>component</u> RDM-p $R^{(p)\mu}_{(1^{p-\omega}\dots)\omega}$, which is similar to Eq. (21):

$$R^{(p)(\mu)}_{(\gamma)\omega}=\sum_{\hat{Q},\hat{Q}'\in\hat{\pi}_p\times\hat{\pi}_p}\left(-1\right)^{q+q'}C^{(p)}_{(\gamma)\omega}\left(Q,Q'\right)\cdot\hat{Q}R^{(p)(\mu)}_{\left(1^{p-\omega}\dots\right)\omega}\hat{Q}'^{+}=\hat{\pi}^{(p)}_{(\gamma)}\cdot\hat{R}^{(p)(\mu)}_{\left(1^{p-\omega}\dots\right)\omega},. \tag{23}$$

where $\hat{\pi}^{(p)}_{(\gamma)}\in\pi_p\times\pi'_p$, $(-1)^{q+q'}$ is sign factor for permutation $Q\cdot Q'$. Moreover, it is true that

$$R^{(p)\mu}_{(\gamma)\omega}=Sp_\sigma\left\{\Gamma^{(p)\mu}\left(x_1,\dots,x_p\middle|x'_1,\dots,x'_p\right)\cdot\hat{T}^{(p)(\mu)}_{(\gamma)\omega}\right\}, \tag{24}$$

if spin tensors (21) are orthonormalized. This simple process always can be applied to obtain exact result type Eq. (23) for any RDM. It completes generally proof of Harriman’s theorem.

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