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BARKER-LIKE SYSTEMS OF SEQUENCES AND THEIR PROCESSING

New systems of binary sequences, that give the similar correlation properties after signal processing as that of the Barker sequences, are suggested and analyzed. The author considers processing of such systems, as well as ways of their application to radio systems and their comparison with complementary sequences.

Keywords: Barker sequences, complementary sequences, correlation properties, sidelobe suppression, signal processing.

Barker sequences (codes) are generally well known in telecommunication systems (DSSS technology, synchronization) and radar technology. They are characterized by low sidelobes of normalized autocorrelation function (**ACF**) $|R^{SL}(\tau)| \leq 1/N$ (N is the length of sequence) [1]. Binary Barker sequences (elements $a_i \in \{\pm 1\}$) are only known for lengths N = 2; 3; 4; 5; 7; 11; 13. There are no these sequences for odd lengths $N > 13$, but it is unknown about an existence of these sequences for even lengths $N > 4$ [2, p. 109]. Ternary Barker sequences (elements $a_i \in \{0, \pm 1\}$) are also known up to length $N = 31$ [3, p. 23]. There is a known kind of sequences called "Generalized Barker Sequences (Codes)", they are polyphase sequences which consist of elements $a_i \in \{\exp(j \cdot 2\pi k/M)\}$, $k \leq M$ [4]. Polyphase Barker sequences are known up to length $N = 77$ [5].

The problem boils down to a controversy concerning the existence of sequences or systems of sequences with a greater length that would make it possible to obtain after their processing an ACF-signal with a low value of sidelobes and a narrow central-lobe (as that of the Barker sequences). One of the solutions of this problem is the complementary sequences [6]. They are pairs of sequences of lengths $N = 2^n \cdot 10^k \cdot 26^m$ ($n \geq 0$, $k \geq 0$, $m \geq 0$; except for the case $n = k = m = 0$) with a property that the result of adding of sidelobes of their ACF is equal to zero. Complementary sequences are used in pulse compression radar systems, telecommunication systems IEEE 802.11b/g and other applications. Although the complementary sequences are a good technical solution, the optimal ACF structure synthesis problem still hasn't been solved in general, and there may be some other kinds of sequences that can, for instance, provide better noise stability of radio systems and consistency with some digital modulation techniques, e.g., QAM.

This research is a continuation of the research described in [7] and concerns the problem of synthesis of the optimal ACF structure. The goal of this study is to suggest new systems of binary sequences (generation rules for such sequences) and their application in signal processing, which would make it possible to obtain ACF-signals with the maximum normalized absolute value of sidelobes $1/N_{\max}$ (as that of the Barker sequences), where $N_{\max}$ is the maximum length of sequence in the system of sequences.

Systems of Barker-like Sequences

Systems of sequences being suggested consist of the Barker sequence $a = \{-1; 1; -1; -1; -1\}$ (length $N = 5$) and binary sequences of lengths $N = 20 \cdot 2^q$, q = 0, 1, 2, 3, …, which can be obtained by using the generation rule

$$a_i = \begin{cases} -1, & i = 1; \\ (-1)^m, & i = 2m+1; \\ (-1)^n a_{2n-1}, & i = 2n; \\ \begin{cases} a_{2n} & \text{for subtype } A, \\ -a_{2n} & \text{for subtype } B, \end{cases} & i = N+1-2n; \\ \begin{cases} -a_{2n-1} & \text{for subtype } A, \\ a_{2n-1} & \text{for subtype } B, \end{cases} & i = N+2-2n; \\ m = \overline{1, (N/4-1)};\ n = \overline{1, N/4}. \end{cases} \quad (1)$$

Such systems of sequences allow to obtain, after the joint signal processing (considered below), an ACF-signal with the maximum normalized absolute value of sidelobes $1/N_{\max}$, which would match to the maximum value of sidelobes in a Barker sequence of length $N_{\max}$, if it existed. Therefore, such systems of sequences may be called "Barker-like systems of sequences".

Such system can contain subtype A or subtype B sequences (signs of some ACF sidelobes depend on the subtype, but their absolute values do not depend on the subtype).

Generation rule (1) generally gives binary sequences of lengths $N = 4k$, $k = 1, 2, 3, \ldots$, but in our case only lengths $N = 20 \cdot 2^q$, $q = 0, 1, 2, 3, \ldots$ are required.

The ACF of sequences that have been obtained by (1) is partly presented for certain τ values and $N \geq 12$ in

$$R(\tau) = \begin{cases} N, \tau = 0; \\ \begin{cases} 1 \text{ for subtype } A, \\ -1 \text{ for subtype } B, \end{cases} \tau = 1,\ 3; \\ 0,\ \tau = 2 + 4m,\ m = 0, \left(\frac{N}{4} - 1\right); \\ N - 16,\ \tau = 4. \end{cases} \quad (2)$$

It is important that the central lobe of ACF $R(0) = N$ is always separated from the first high sidelobe by low sidelobes (0 or ±1); high sidelobes are concentrated between zero sidelobes (at $\tau = 2 + 4m$, $m = 0, 1, 2, 3, \ldots$).

The structure of sequences which have been obtained by (1) is shown in **Fig. 1**.

Signal processing and ways of using Barker-like systems of sequences

Signal processing and ways of application of the suggested systems of sequences are based on the following property of the systems: the result of multiplication of ACF-signals (at the outputs of matched filters), that exist at the same time interval and match to these sequences, constitutes a signal that is similar to the ACF-signal with a narrow central-lobe (equals to the duration of the

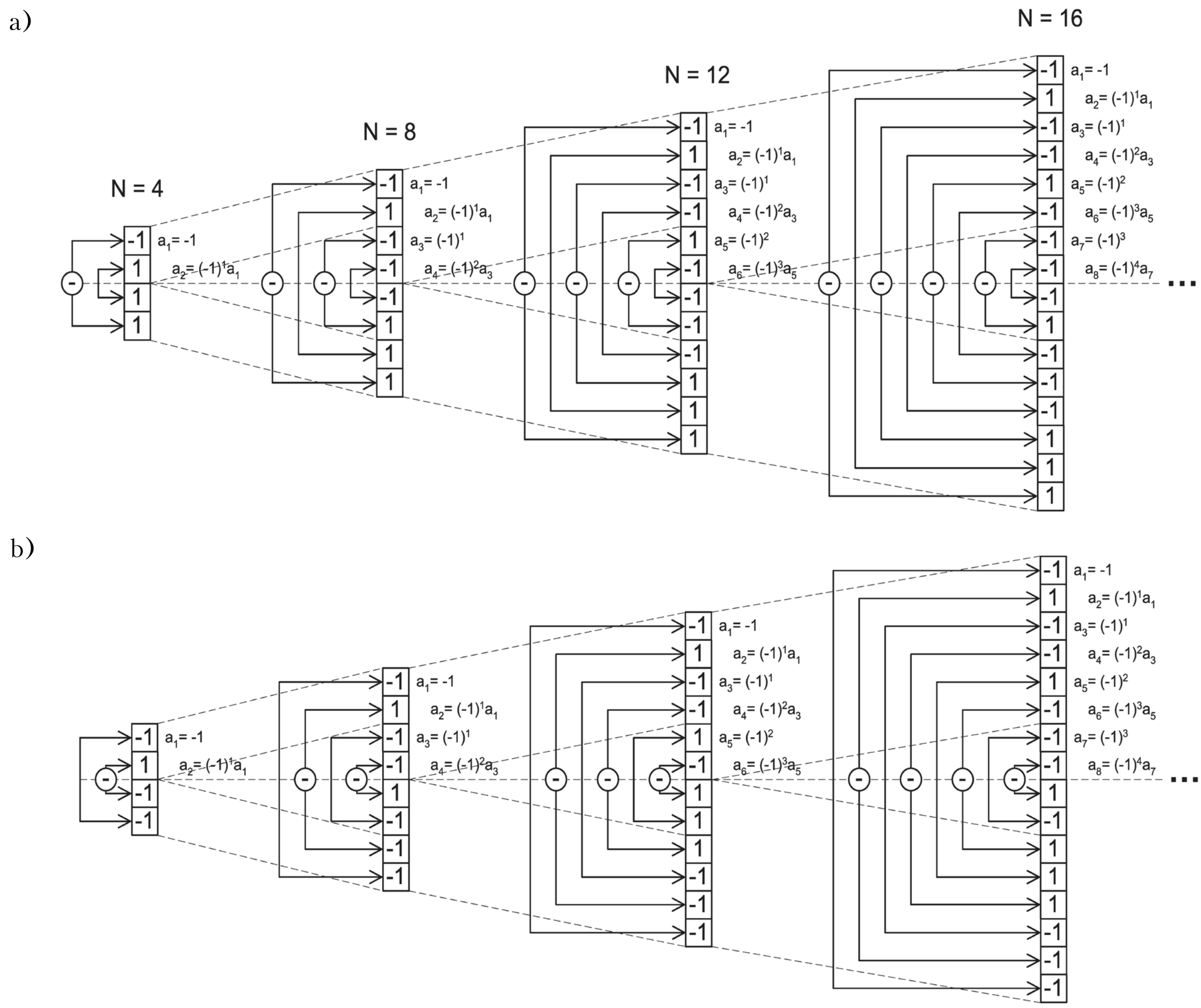

Fig. 1. Structure of suggested subtype *A* (*a*) and subtype *B* (*b*) sequences

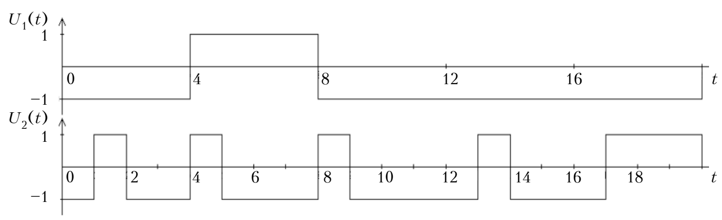

Fig. 2. Signals in the considered example of Barker-like system of sequences

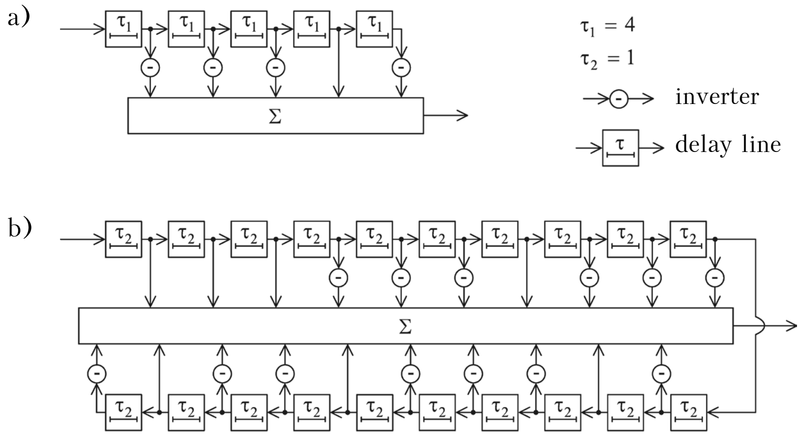

Fig. 3. Matched filters for sequences N_1=5 (*a*) and N_2=20 (*b*)

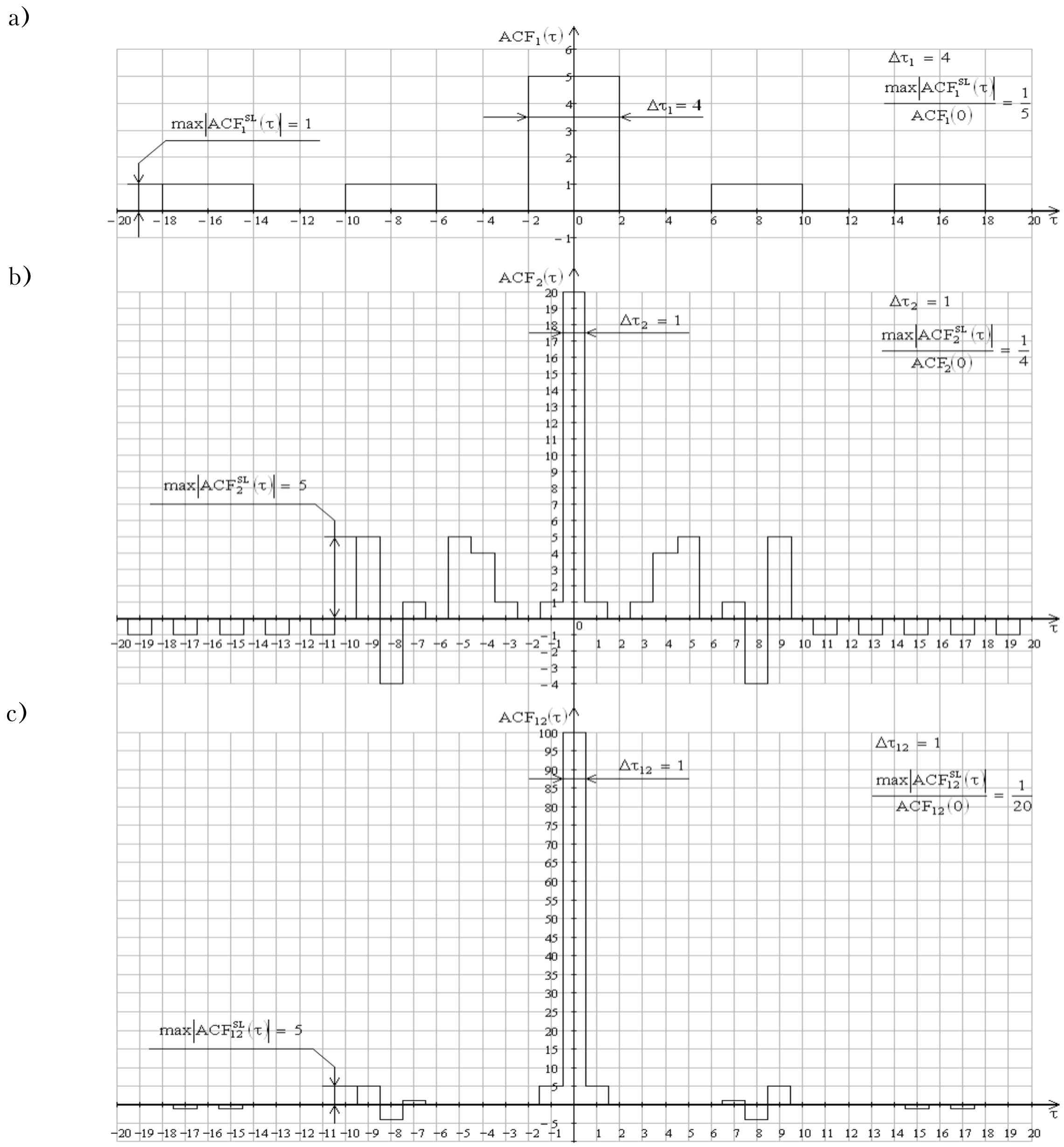

Fig. 4. Forms of signals after MF for sequences N_1=5 (*a*) and N_2=20 (*b*), and the result of their multiplication (*c*)

shortest element of sequences in all the system) and low sidelobes (normalized absolute values do not exceed $1/N_{max}$).

Let us take a detailed look at signal processing by using the example of the Barker-like system consisting of two sequences {−1; 1; −1; −1; −1} (N_1 = 5, Barker sequence) and {−1; 1; −1; −1; 1; −1; −1; −1; 1; −1; −1; −1; −1; 1; −1; −1; −1; 1; 1; 1} (N_2 = 20, sequence of subtype *A*). In **Fig. 2** are shown signals for those sequences (the same time interval, e. g. 20 μs, is used). Matched filters (**MF**) for signals of each sequence are shown in **Fig. 3**, and ACF-signals after the matched filters and the result of their multiplication (central-lobes of ACF-signals are reduced to the zero moment) are illustrated in **Fig. 4**.

In the case of the suggested systems of sequences the multiplication of ACF makes it possible to combine their advantages (low sidelobes of ACF_1 and narrow central-lobe of ACF_2) and neutralize their disadvantages (wide central-lobe of ACF_1 and high sidelobes of ACF_2). It is possible because the ACF of suggested sequences has a comb structure: the central-lobe is always separated from the first high sidelobe by low sidelobes (0 or ±1) — this results in the appearance of a narrow central-lobe after multiplication. High sidelobes are also separated from one another by zero sidelobes. Thus low and high sidelobes of different ACFs are partially or totally cross-thinned — this leads to partial or total sidelobe suppression.

Characteristics of some of the suggested Barker-like systems of sequences (data are confirmed by computer modeling) are given in **Table 1**.

Theoretical justification of the fact, that the maximum normalized absolute value of ACF sidelobes is $1/N_{max}$ for any possible number of sequences in a system, is expected in further research.

Using the Barker sequence of length N = 5 in the suggested systems is important for the considered case, because if other basic binary Barker sequence (e.g., N=13) and system (e.g., N_1=13 and N_2=52) were used, the resultant maximum normalized absolute value of sidelobes wouldn't be $1/N_{max}$ (for the case N_1=13 and N_2=52 it would be 7/169, which is more than $1/N_{max} = 1/N_2 = 1/52$).

In **Fig. 5** is shown the signal processing system for Barker-like systems of sequences. Delay lines $(2^{q+1} - 0.5)\tau_{q+2}$ in this system are used to align in time the centers of the central lobes.

In **Table 2** an example of possible modulation scheme for DSSS technology is shown. In this example we have used the system of sequences N_1=5 and N_2=20 and QPSK modulation with Gray coding.

In our example, for correct operation of the signal processing system in a DSSS-transmitter, to transmit

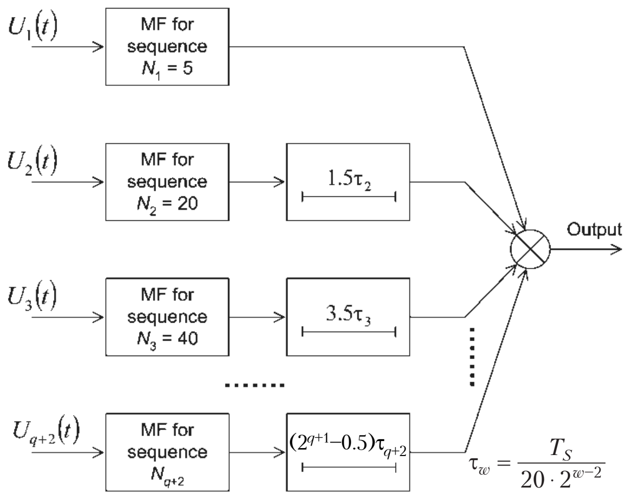

Fig. 5. Signal processing of Barker-like systems of sequences

Table 1

Characteristics of some of the suggested Barker-like systems of sequences

Sequences in the system	Central-lobe width of ACF in the result of multiplication	Maximum normalized value of sidelobes of ACF in the result of multiplication	Example of the suitable modulation
N_1 = 5 and N_2 = 20	$T_S/20$	1/20	QPSK
N_1 = 5, N_2 = 20 and N_3 = 40	$T_S/40$	1/40	8-PSK
N_1 = 5, N_2 = 20, N_3 = 40 and N_4 = 80	$T_S/80$	1/80	16-PSK or 16-QAM
N_1 = 5, N_2 = 20, N_3 = 40, N_4 = 80 and N_5 = 160	$T_S/160$	1/160	32-QAM
N_1 = 5, N_2 = 20, N_3 = 40, N_4 = 80, N_5 = 160 and N_6 = 320	$T_S/320$	1/320	64-QAM

T_S – signal duration.

Table 2

Example of using of the considered sequences in DSSS

Bit	1																				0																			
Chips (N_1=5)	−1				1				−1				−1				−1				−1				1				−1				−1				−1			
Chips (N_2=20)	-1	1	-1	-1	1	-1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	-1	1	1	1	1	-1	1	1	-1	1	1	1	-1	1	1	1	1	-1	1	1	1	-1	-1	-1
QPSK phases	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{7\pi}{4}$	$\frac{7\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{7\pi}{4}$	$\frac{5\pi}{4}$	$\frac{5\pi}{4}$	$\frac{5\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$

Table 3

Comparison of Barker-like systems of sequences with complementary sequences

Parameter	Barker-like systems of sequences	Complementary sequences
The main principle of signal processing	Multiplication of results of matched filtering of each sequence	Adding of results of matched filtering of each sequence
Quantity of sequences in the system	$L \geq 2$	$L = 2$
Lengths of sequences in the system	$N_1 = 5$ (Barker) $N_w = 20 \cdot 2^{w-2}$, $w = 2, \ldots, L$	$N_1 = N_2 = 2^n \cdot 10^k \cdot 26^m$, $n \geq 0$, $k \geq 0$, $m \geq 0$, except for the case $n = k = m = 0$
Maximum normalized value of sidelobes of ACF in the result of signal processing	$1/(20 \cdot 2^{L-2})$	0
Central-lobe width in the result of signal processing	T_S/N_L	T_S/N_1
Suitable modulation	2^L-level shift keying modulation (e.g., 64-QAM for $L = 6$)	4-level shift keying modulation (e.g., QPSK)

a zero bit, it is necessary to invert only one chip flow, which matches to the sequence $N_2 = 20$; the chip flow for the sequence $N_1 = 5$ is constant and doesn't depend on the bit value. Generally, in such systems may be inverted a certain odd number of chip flows, but not more than the total number of these flows.

Comparison of Barker-like systems of sequences with complementary sequences

One of the nearest analogs of the suggested Barker-like sequences are the complementary sequences. Their comparison is shown in **Table 3.**

Thus, by the criterion of value of ACF sidelobes after signal processing, the proposed systems of sequences are slightly inferior to known complementary sequences and equal to Barker sequences. Another important factor in the application of the proposed systems of sequences is noise stability of a telecommunication or radar system. In the case of application of such systems of sequences, after signal processing non-stationary noise tends to appear if stationary noise is at the input (unlike complementary systems, where after signal processing the stationary noise appears). However, multiplication of useful signals during signal processing (used for Barker-like systems of sequences) may give better noise stability than adding of useful signals (used for complementary systems). The issue of noise stability when using the suggested Barker-like systems of sequences will be addressed in further research.

Conclusions

The proposed systems of binary sequences and signal processing using such systems make it possible to obtain ACF-signals with the same maximum normalized absolute value of sidelobes as the one for the Barker sequences. It has been established using computer modeling, that the maximum length of binary sequences of such systems is at least 320.

Comparison of the suggested sequences with the known complementary sequences shows that both this types are complementary, but matched filtering results in the case of the proposed sequences are multiplied, while for the known sequences the results are added.

Due to their properties after signal processing, the suggested systems of sequences can be used in the pulse-compression radar technology, in synchronizing system, and in DSSS technology for wideband signal forming and data transfer.

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БАРКЕРОПОДІБНІ СИСТЕМИ ПОСЛІДОВНОСТЕЙ ТА ЇХ ОБРОБКА

Запропоновано та проаналізовано нові системи бінарних послідовностей, які дають такі ж властивості функції автокореляції після обробки сигналів, що і послідовності Баркера. Розглянуто принцип їх обробки, шляхи використання в радіосистемах, виконано їх порівняльний аналіз з комплементарними послідовностями.

Ключові слова: послідовності Баркера, комплементарні послідовності, кореляційні властивості, придушення бічних пелюсток, обробка сигналів.

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БАРКЕРОПОДОБНЫЕ СИСТЕМЫ ПОСЛЕДОВАТЕЛЬНОСТЕЙ И ИХ ОБРАБОТКА

Предложены и проанализированы новые системы бинарных последовательностей, которые дают такие же свойства функции автокорреляции после обработки сигналов, что и последовательности Баркера. Рассмотрены принцип их обработки, пути использования в радиосистемах, выполнен их сравнительный анализ с комплементарными последовательностями.

Ключевые слова: последовательности Баркера, комплементарные последовательности, корреляционные свойства, подавление боковых лепестков, обработка сигналов.