SECOND-SOUND WAVES IN CRYOCRYSTALS

V.D. KHODUSOV, A.A. BLINKINA

Karazin Kharkiv National University
(4, Svobody Sq., Kharkiv 61077, Ukraine; e-mail: blinkina@gmail.com, vkhodusov@ukr.net)

PACS 66.70.-f, 61.72.S

The self-coordinated values of parameters of cryocrystals of orthodeuterium, parahydrogen, and neon (temperature, isotope concentrations, and the sizes of samples), which define the range of second-sound waves existence, are found. The limiting isotope concentrations, below which the propagation of second-sound waves is possible, are established. The sizes of samples, starting from which their increase does not essentially influence the damping of second-sound waves are found. The results are presented in three-dimensional plots.

1. Introduction

For the first time, second-sound waves (SSW) in solid bodies were observed experimentally in ^{4}He cryocrystals [1]. The question on an opportunity of the SSW propagation in other cryocrystals was discussed in a number of works [2–5]. For the experimental detection of SSW, it is necessary to know the compatible areas of values for the most essential parameters such as the temperature, impurity and isotope concentrations, and sizes of samples. This compatibility of parameters can be obtained (if the second sound damping factor is known) from a condition of weakness of the damping of these waves. It is possible to write down the damping coefficient using the diffusion times for various dissipative processes running in a phonon gas [6] which are caused by umklapp processes, scattering on impurities and isotopes, and scattering on the sample boundaries. The direct calculation of these times is rather difficult. It is known that the SSW propagation is possible in a vicinity of the maximum of the thermal conductivity coefficient of a cryocrystal. According to this, it is possible to use phenomenological values of collision frequencies connected with these times which are used in the Callaway model to describe experimentally observable dependences of the thermal conductivity coefficient on the temperature in a low-temperature range.

The main contribution to the SSW damping factor is given by the phonon scattering on impurities and isotopes and by the scattering on the sample boundaries. In case of orthodeuterium ($o-\mathrm{D}_2$), parahydrogen ($p-\mathrm{H}_2$), and neon (Ne) cryocrystals, it is possible to produce pure enough samples, in which the role of impurities will be insignificant; however, there remains an essential role of isotopes. It is necessary to consider the phonon scattering on boundaries in an absolutely different way. SSW propagate in crystals in the phonon gas, when the hydrodynamic regime in it is realized due to the fast normal processes of phonon interaction. In this case, before reaching the boundary, a phonon will experience a set of normal collisions. Therefore, the free path will increase due to the boundary scattering, and the contribution of these processes to the damping coefficient will decrease.

In the present work, we determined the coordinated areas of concentrations of hydrogen and ^{22}Ne isotope, sizes of samples, and temperatures, where SSW exist. The obtained results can be used for carrying out the experiments on the SSW registration in orthodeuterium, parahydrogen, and neon.

2. Existence of Second-Sound Waves in Solid Bodies

In work [6], the general theory of secondary waves in a gas of Bose quasiparticles which are, in particular, second-sound waves in a phonon gas has been constructed for solid bodies. Meanwhile, the description of SSW was established in the reduced isotropic crystal model, by using the modulus of elasticity. The SSW dispersion equation has been found in the same work [6] and has the following form:

$$\Omega\left(\Omega^2 - W_{\mathrm{II}}^2\tilde{k}^2\right) - 2iW_{\mathrm{II}}^2\tilde{k}^2\Gamma_{\mathrm{II}} = 0. \quad (1)$$

Here, W_{II} is the SSW phase velocity in an isotropic phonon gas, and Γ_{II} is the SSW damping factor. The expressions for W_{II} and Γ_{II} are as follows [6]:

$$W_{\mathrm{II}} = \left(TS^2/C\tilde{\rho}\right),$$

$$\Gamma_{\mathrm{II}} = \frac{r}{2\tilde{\rho}} + \left[\frac{1}{2\tilde{\rho}}\left(\frac{4}{3}\tilde{\eta} + \tilde{\zeta}\right) + \frac{1}{2C}\tilde{\kappa}\right]k^2, \quad (2)$$

where $\tilde{\rho}$ – phonon density, r – coefficient of external friction caused by the phonon interaction without the quasimomentum conservation (umklapp processes, scattering on impurities, isotopes, and boundaries of the sample), $\tilde{\eta}$ and $\tilde{\zeta}$ – coefficients of first and second viscosities, respectively,, and $\tilde{\kappa}$ – coefficient of hydrodynamical thermal conductivity.

For a phonon gas in the low-temperature range $\Theta_D/T \gg 1$ in the reduced isotropic crystal model, the following expressions for the densities of thermal capacity and entropy, phonon density, Debye temperature, and average sound velocity are obtained: $C = 3S = \frac{2\pi^2}{15}\frac{k_B^4 T^3}{\hbar^3 V_t^3}\left(2+\beta^3\right)$, $\tilde{\rho} = \frac{2\pi^2}{45}\frac{k_B^4 T^4}{\hbar^3 V_t^5}\left(2+\beta^5\right)$, $\Theta_D = 2\pi\hbar V_s/k_B a$, $V_S = V_t\left(\frac{3}{2+\beta^3}\right)^{1/3}$, where V_l, V_t – velocities of longitudinal and transversal sounds, $\beta = V_t/V_l$, k_B – Boltzmann constant, and a – lattice parameter. The second sound velocity

$$W_{II} = \frac{V_t^2}{3}\left(\frac{2+\beta^3}{2+\beta^5}\right). \tag{3}$$

We now introduce the diffusion times by the relations

$$\tau_{\tilde{\eta}} = \frac{\tilde{\eta}}{\tilde{\rho}W_{II}^2};\quad \tau_{\tilde{\zeta}} = \frac{\tilde{\zeta}}{\tilde{\rho}W_{II}^2};\quad \tau_{\tilde{\kappa}} = \frac{\tilde{\kappa}}{CW_{II}^2};\quad \tau_R = \frac{\tilde{\rho}}{r} \tag{4}$$

and the notation

$$\tau_N = \frac{4}{3}\tau_{\tilde{\eta}} + \tau_{\tilde{\zeta}} + \tau_{\tilde{\kappa}}. \tag{5}$$

Then the introduction of the collision frequencies ν_i connected with the diffusion times τ_i by the relation $\nu_i = 1/\tau_i$ allows us to write Γ_{II} in the form

$$\Gamma_{II} = \frac{1}{2}\left[\nu_R + \frac{1}{\nu_N}\Omega^2\right]. \tag{6}$$

The condition of existence of a weakly decaying SSW is $\Gamma_{II} \ll \Omega$, which leads to a condition imposed on the frequency Ω known as a "window" of SSW existence. The given condition is written down in a general form. Therefore, for a more detailed study of the range of SSW existence and for finding the coordinated boundary values of parameters, at which the existence of a weakly decaying SSW is possible, it is necessary to specify it. We demand the next condition to be satisfied:

$$\Gamma_{II} \leq 10\Omega. \tag{7}$$

This leads to the following frequency "window" of SSW existence:

$$0.1\nu_N - \sqrt{\frac{\nu_N^2}{10^2} - \nu_N\nu_R} \leq \Omega_{II} \leq 0.1\nu_N + \sqrt{\frac{\nu_N^2}{10^2} - \nu_N\nu_R}. \tag{8}$$

Equation (8) yields the desired relation between the parameters

$$\nu_N \geq 100\nu_R. \tag{9}$$

3. Definition of the Coordinated Values of Parameters, at Which SSW Exist in Cryocrystals

The expression for the scattering frequency due to resistive processes is as follows:

$$\nu_R = \nu_{\rm iso} + \nu_{\rm Beff} + \nu_{\rm U}. \tag{10}$$

Here, $\nu_{\rm iso}$ – frequency of phonon scattering on isotopes, $\nu_{\rm Beff}$ – frequency of scattering on boundaries of a sample, $\nu_{\rm U}$ – scattering frequency due to umklapp processes.

The collision frequency on isotopes is taken into account for all crystals with the use of the formula [7,8]

$$\nu_{\rm iso}(x) = \frac{a^3}{4\pi V_S^3}\left(\frac{\Delta M}{M}\right)^2 x^4 T^4 C_d, \tag{11}$$

where $C_d = N_d/N$ – concentration of isotopes, ΔM – a difference between nuclear weights of isotopes and that of the basic substance, M – weight of basic atoms, and x – dimensionless variable.

We considered the phonon scattering on boundaries which depends on a relation between the free path length of phonons due to normal processes $l_N = \tau_N V_S$ and that in the sample bulk D. SSW exist in dielectrics if the hydrodynamical mode in the description of phonons is realized. Thus, the inequality $l_N \ll D$ is valid. Phonons that are in the sample bulk will experience a set of normal collisions before reaching the boundary. As a result, the way which is passed between two collisions with boundaries essentially increases. Using the known formulas for the Brown movement, it is easy to show that the trajectory between two collisions with a boundary will be of the order of D^2/l_N [9]. This implies that the effective frequency of phonon collisions with the boundary is

$$\nu_{\rm Beff} = \frac{V_S^2}{D^2\nu_N}. \tag{12}$$

For the numerical estimations and the calculations of conditions for the existence of weakly decaying SSW in cryocrystalls of orthodeuterium and parahydrogen and in crystals of neon, we used data given in works [3,4,10,11] and presented in a tabular form in work [12]. We give here only Table for the averaged frequencies of phonon collisions.

Expressions for the average collision frequencies in the Callaway model for various crystals

Crystal	$o-\mathrm{D}_2$	$p-\mathrm{H}_2$	Ne
ν_N, s^{-1}	$1.4\times10^6T^5$	$1.9\times10^6T^5$	$1.3\times10^7T^4$
ν_{U}, s^{-1}	$1.3\times10^9T^3e^{\left[\frac{-37}{T}\right]}$	$1.4\times10^9T^3e^{\left[\frac{-40}{T}\right]}$	$4\times10^6T^5e^{\left[\frac{-13}{T}\right]}$
ν_{iso}, s^{-1}	$1.3\times10^9T^4C_d$	$2.3\times10^8T^4C_d$	$9.5\times10^7T^4C_d$
ν_{Beff}, s^{-1}	$9\times10^3T^{-5}/D^2$	$4.3\times10^3T^{-5}/D^2$	$0.2\times10^3T^{-4}/D^2$.

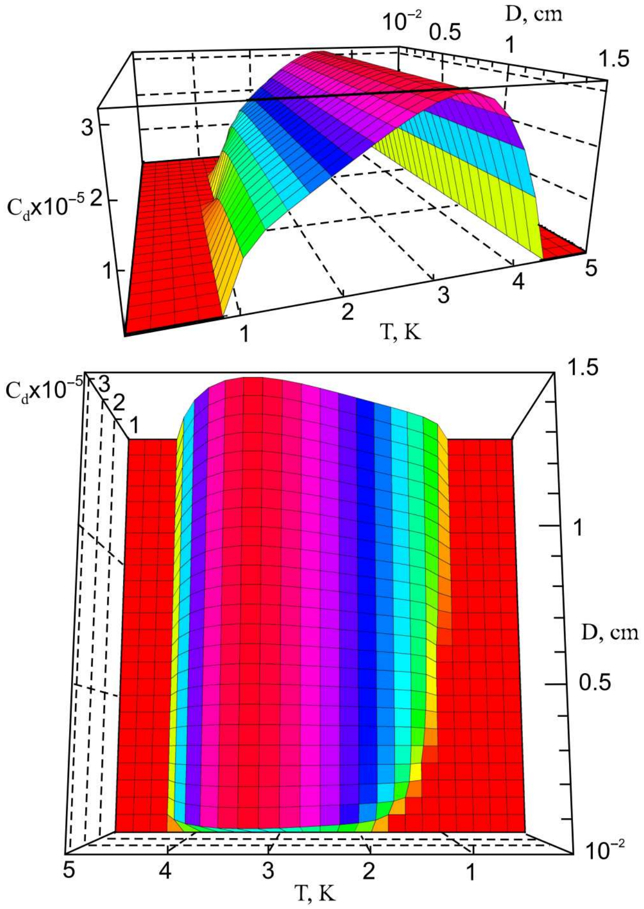

Fig. 1. Range of the self-coordinated parameters for crystals of orthodeuterium

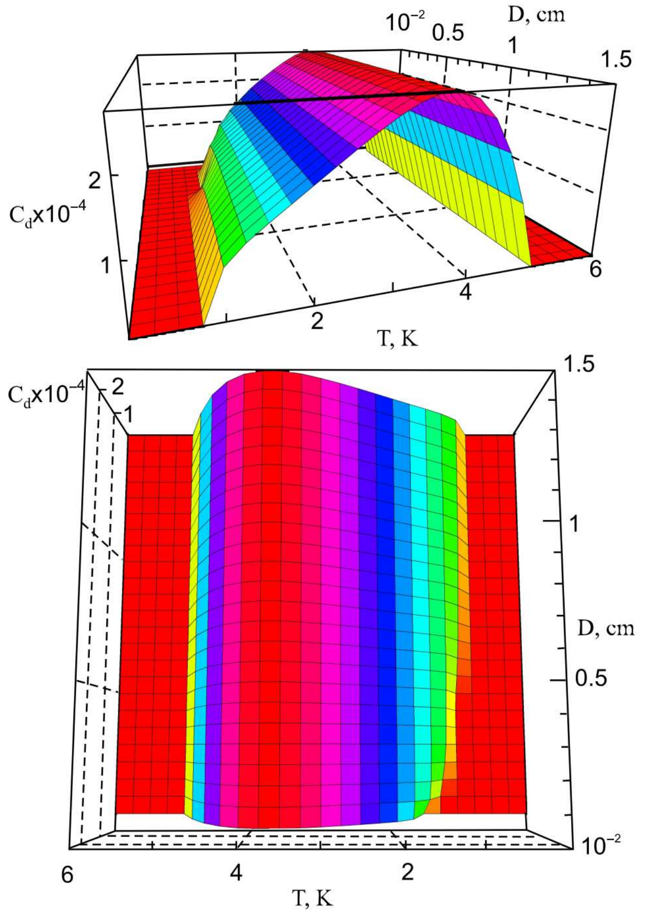

Fig. 2. Range of the self-coordinated parameters for crystals of parahydrogen

Using these average values of frequencies of phonon collisions and inequality (9), we build the plots of self-coordinated values of the parameters (T, C_d, D) determining the range of SSW existence in the considered cryocrystals. These plots are presented in Figs. 1–3.

4. Conclusions

The values of parameters lying on surfaces are limitind values of the self-coordinated parameters, at which the SSW propagation is possible. The range below these surfaces corresponds to the SSW existence in cryocrystals.

The analysis of the resulted plots enables us to establish the limiting values of isotope concentrations, below which the SSW propagation is possible. For orthodeuterium, parahydrogen, and neon, C_{dlim} are, respectively, 3.1×10^{-5}, 2.7×10^{-4}, and 1.3×10^{-3}. In addition, it is seen from plots that, starting from the sizes of the order of 0.5 cm, an increase in the sample size weakly influences the range, where SSW exist.

Choosing the certain parameters in the range of SSW existence from inequality (8), it is possible to find a frequency spectrum of weakly decaying SSW. For example,

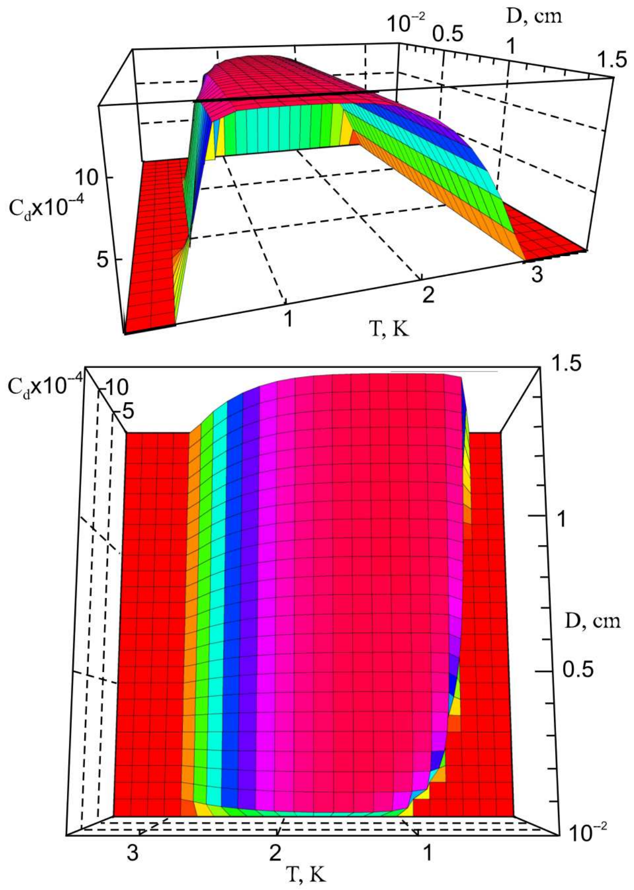

Fig. 3. Range of the self-coordinated parameters for crystals of neon

let us consider parahydrogen: if we take a crystal with $D = 0.7$ cm, the isotope concentration $C_d = 3 \times 10^{-5}$ at $T = 2$ K, the frequency range is $0 \leq \Omega_{\rm II} \leq 4.9 \times 10^6$ s^{-1}. The knowledge of this spectrum is very essential for attempts to register SSW in experiments on the observation of the thermal pulse evolution.

1. C.C. Ackerman and W.C. Overton, Phys. Rev. Letters. **22**, 764 (1969).
2. N.N. Zholonko, B.Ya. Gorodilov, and A.I. Krivchikov, JETP Lett. **55**, 167 (1992).
3. R.M. Kimber and S.J. Rogers, J. Phys. C: Solid State Phys. **6**, 2279 (1973).
4. O.A. Koroluk, B.Ya. Gorodilov, A.I. Krivchikov, and V.V. Dudkin, Fiz. Nizk. Temp. **26**, 323 (2000).
5. T.N. Antsigina, B.Ya. Gorodilov, N.N. Zholonko, A.I. Krivchikov, V.G. Manzhelii, and V.A. Slusarev, Fiz. Nizk. Temp. **18**, 417 (1992).
6. A.I. Akhiezer, V.F. Aleksin, and V.D. Khodusov, Low Temp. Phys. **20**, 939 (1994); Low Temp. Phys. **21**, 1 (1995).
7. R. Berman, *Thermal Conductivity in Solids* (Oxford University Press, Oxford, 1976).
8. A.P. Zhernov and A.V. Inyushkin, Uspekhi. Fiz. Nauk **45**, 573 (2002).
9. R.N. Gurzhi, Uspekhi. Fiz. Nauk **94**, 689 (1968).
10. *Properties of the Condensed Phases of Hydrogen and Oxygen (Reference Book)* (Naukova Dumka, Kiev, 1984).
11. M. M. Tarasenko and V.A. Bezuglyi, *Thermophysical Properties of Substances and Materials* (Standards Publ. House, Moscow, 1976).
12. V.D. Khodusov and A.A. Blinkina, Fiz. Nizk. Temp. **35**, 451 (2009).

Received 22.12.09

ХВИЛІ ДРУГОГО ЗВУКУ У КРІОКРИСТАЛАХ

В.Д. Ходусов, А.А. Блінкіна

Р е з ю м е

Знайдено узгоджені значення параметрів кріокристалів ортодейтерію, параводню та неону (температури, концентрації ізотопів, домішок та розмірів зразків), що визначають область існування хвиль другого звуку. Встановлено граничні концентрації ізотопів, нижче за які можливе поширення хвиль другого звуку, а також визначено розміри зразків, починаючи з яких їх збільшення не впливає на загасання хвиль другого звуку. Результати наведено у вигляді тривимірних графіків.