СДІТ

ПРОБЛЕМИ ПРИЙНЯТТЯ РІШЕНЬ І УПРАВЛІННЯ В ЕКОНОМІЧНИХ, ТЕХНІЧНИХ, ЕКОЛОГІЧНИХ І СОЦІАЛЬНИХ СИСТЕМАХ

UDC 519.5

THEORY OF THE ANALYTIC HIERARCHY AND ANALYTIC NETWORK PROCESS – EXAMPLES. PART 2.2

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In this part we introduce the role of benefits, opportunities, costs and risks (BOCR) in decision-making and how to establish priorities for them. We give an example of a real life application of the US Congress acting on China joining the World Trade Organization (WTO) mailed to the US congressional committee before that decision. We then introduce and apply the Analytic Network Process and its concept of a supermatrix to make decisions with dependence and feedback and illustrate its application for a single “control” criterion of market share. This will be followed in Part 2.3 by a full BOCR application in the context of the ANP.

1. EVALUATING THE BOCR MERITS THROUGH STRATEGIC CRITERIA USING RATINGS

This section was taken from an analysis carried out before the US Congress acted favorably on China joining the WTO and was hand-delivered to many of the members of the committee including its Chairperson [4]. Since 1986, China had been attempting to join the multilateral trade system, the General Agreement on Tariffs and Trade (GATT) and, its successor, the World Trade Organization (WTO)]. According to the rules of the 135-member nation WTO, a candidate member must reach a trade agreement with any existing member country that wishes to trade with it. By the time this analysis was done, China signed bilateral agreements with 30 countries — including the US (November 1999) — out of 37 members that had requested a trade deal with it.

As part of its negotiation deal with the US, China asked the US to remove its annual review of China’s Normal Trade Relations (NTR) status, until 1998 called Most Favored Nation (MFN) status. In March 2000, President Clinton sent a bill to Congress requesting a Permanent Normal Trade Relations (PNTR) status for China. The analysis was done and copies sent to leaders and some members in both houses of Congress before the House of Representatives voted on the bill, May 24, 2000. The decision by the US Congress on China’s trade-relations status will have an influence on US interests, in both direct and indirect ways. Direct impacts will include changes in economic, security and political relations between the two countries as the trade deal is actualized. Indirect impacts will occur when China becomes a WTO member and adheres to WTO rules and principles. China has said that it would join the WTO only if the US gives it Permanent Normal Trade Relations status.

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It is likely that Congress will consider four options, the least likely being that the US will deny China both PNTR and annual extension of NTR status. The other three options are:

1. **Passage of a clean PNTR bill**: Congress grants China Permanent Normal Trade Relations status with no conditions attached. This option would allow implementation of the November 1999 WTO trade deal between China and the Clinton administration. China would also carry out other WTO principles and trade conditions.
2. **Amendment of the current NTR status bill**: This option would give China the same trade position as other countries and disassociate trade from other issues. As a supplement, a separate bill may be enacted to address other matters, such as human rights, labor rights, and environmental issues.
3. **Annual Extension of NTR status**: Congress extends China's Normal Trade Relations status for one more year, and, thus, maintains the status quo.

The conclusion of the study is that the best alternative is granting China PNTR status. China now has that status.

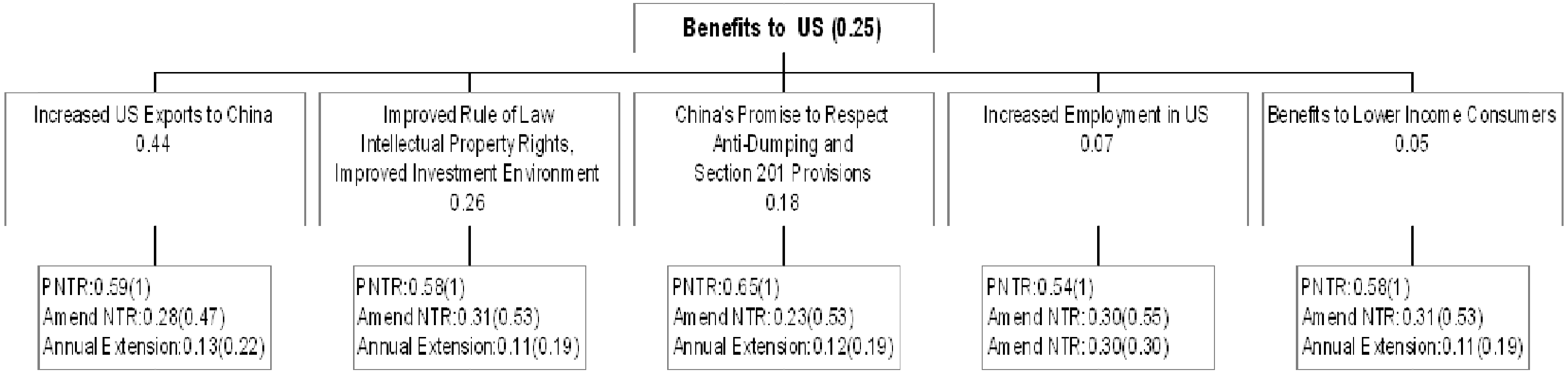
Our analysis involves four steps. First, we prioritize the criteria in each of the benefits, costs, opportunities and risks hierarchies. Fig. 1 shows the resulting prioritization of these criteria. The alternatives and their priorities are shown under each criterion both in the distributive and also in the ideal modes. The ideal priorities of the alternatives were used as appropriate to synthesize their final values beneath each hierarchy.

The priorities shown in Fig. 1 were derived from judgments that compared the elements involved in pairs. For readers to estimate the original pairwise judgments (not shown here), one forms the ratio of the corresponding two priorities shown, leave them as they are, or take the closest whole number, or its reciprocal if it is less than 1.0.

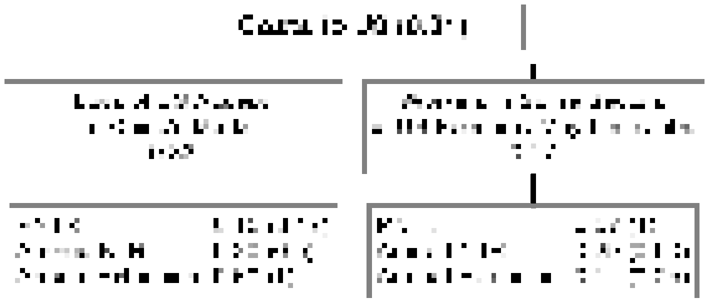
The idealized values are shown in parentheses after the original priorities obtained from the eigenvector. The latter sum to 1 and are called distributive priorities. The ideal values are obtained by dividing each of the distributive priorities by the largest. For the Costs and Risks structures, the question is framed as to which is the *most* costly. That is, the most costly alternative ends up with the highest priority.

It is likely that, in a particular decision, the benefits, costs, opportunities and risks (BOCR) are not equally important, so we must also prioritize them. This is shown in Tabl. 1. The priorities for the economic, security and political factors themselves were established as shown in Fig. 2 and used to rate the importance of the benefits, costs, opportunities and risks in Tabl. 1. Finally, we used the priorities of the latter to combine the synthesized priorities of the alternatives in the four hierarchies, using the normalized reciprocal - priorities of the alternatives under costs and risks, to obtain their final ranking, as shown in Tabl. 2.

How to derive the priority shown next to the goal of each of the four hierarchies shown in Fig. 1 is outlined in Tabl. 1. We rated each of the four merits: benefits, costs, opportunities and risks of the dominant PNTR alternative, as it happens to be in this case, in terms of intensities for each assessment criterion. The intensities, Very High, High, Medium, Low, and Very Low were themselves prioritized in the usual pairwise comparison matrix to determine their priorities. We then assigned the appropriate intensity for each merit on all assessment criteria. The outcome is as found in the bottom row of Tabl. 2.



Benefits Synthesis (Ideal): PNTR 1.00 Amend NTR 0.48 Annual Extension 0.21



Costs Synthesis (which is more costly): PNTR 0.31 Amend NTR 0.50 Annual Extension 0.87
Costs Synthesis (less costly, and idealizing): PNTR 1 Amend NTR 0.61 Annual Extension 0.35

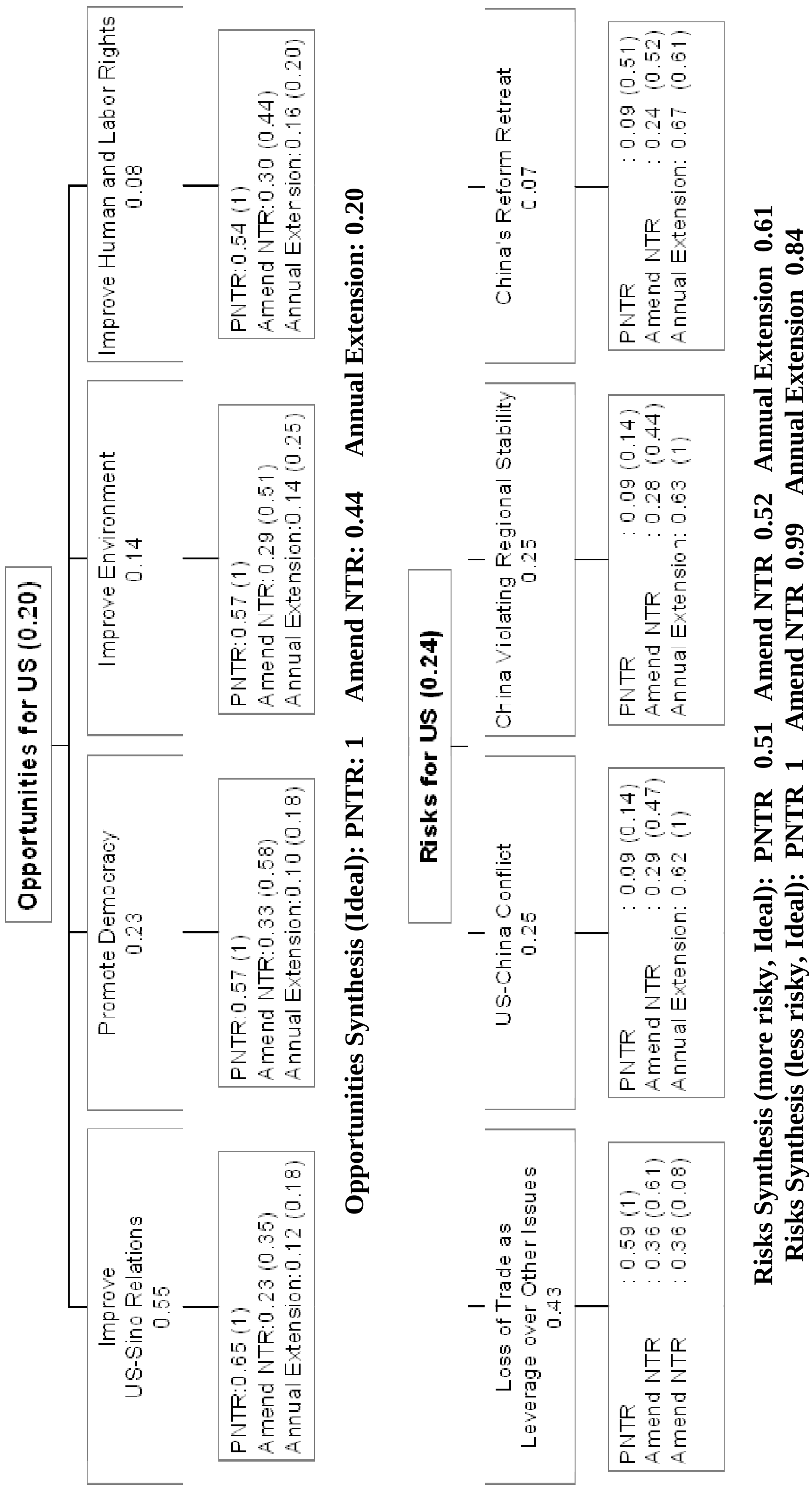


Fig. 1. Hierarchy for Rating Benefits, Costs, Opportunities, and Risks

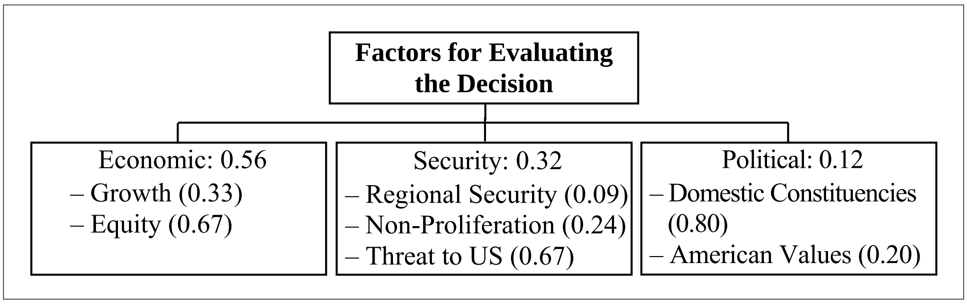


Fig. 2. Prioritizing the Strategic Criteria to be used in Rating the BOCR

Table 1. Priority Ratings for the Merits: Benefits, Costs, Opportunities, and Risks

Intensities: Very High (0.42), High (0.26), Medium (0.16), Low (0.1), Very Low (0.06)

		Benefits	Costs	Opportunities	Risks
Economic (0.56)	Growth (0.19)	High	Very Low	Medium	Very Low
	Equity (0.37)	Medium	High	Low	Low
Security (0.32)	Regional (0.03)	Low	Medium	Medium	High
	Non-Proliferation (0.08)	Medium	Medium	High	High
	Threat to US (0.21)	High	Very High	High	Very High
Political (0.12)	Constituencies (0.1)	High	Very High	Medium	High
	American Values (0.02)	Very Low	Low	Low	Medium
Priorities		0.25	0.31	0.20	0.24

We are now able to obtain the overall priorities of the three major decision alternatives listed earlier, given as columns in Tabl. 2 which gives three ways of synthesize for the ideal mode, we see in bold that PNTR is the dominant alternative any way we synthesize as in the last four columns.

Table 2. Four Methods of Synthesizing BOCR Using the Ideal Mode

Alternatives	Benefsts	Opportunities	Costs	Reciprocals of Costs	Costs (divided by largest reciprocal)	Risks	Reciprocals of Risks	Risks (divided by largest reciprocal)	BO/CR	$bB + oO + c(1/C) + r(11\text{-}R)$	$bB + oO + c(1{-}C) + \mathrm{r}\,(1{-}R)$	$bB + oO$ - $cC - rR$
	(0.25)	(0.20)	(0.31)			(0.24)						
PNTR	1	1	0.31	3.23	1	0.51	1.96	1	1.65	1.01	0.78	0.23
Amend NTR	0.48	0.44	0.50	2.00	0.62	0.52	1.92	0.98	0.22	0.64	0.51	-0.07
Annual Exten.	0.21	0.20	0.87	1.15	0.36	0.61	1.64	0.84	0.03	0.41	0.28	-0.32

2. THE ANALYTIC NETWORK PROCESS (ANP)

At present, in their effort to simplify and deal with complexity, people who work in decision-making use mostly very simple hierarchic structures consisting of a goal, criteria, and alternatives. Yet, not only are decisions obtained from a simple hierarchy of three levels different from those obtained from a multilevel hierarchy, but also decisions obtained from a network can be significantly different from those obtained from a more complex hierarchy. We cannot collapse complexity artificially into a simplistic structure of two levels, criteria and alternatives, and hope to capture the outcome of interactions in the form of highly condensed judgments that correctly reflect all that goes on in the world. We must learn to decompose these judgments through more elaborate structures and organize our reasoning and calculations in sophisticated but simple ways to serve our understanding of the complexity around us. Experience indicates that it is not very difficult to do this although it takes more time and effort. Indeed, we must use feedback networks to arrive at the kind of decisions needed to cope with the future.

The Analytic Network Process is a generalization of the Analytic Hierarchy Process. The basic structure is an influence network of clusters and nodes. Priorities are established in the same way they are in the AHP using pairwise comparisons and judgment. Many decision problems cannot be structured hierarchically because they involve the interaction and dependence of higher-level elements in a hierarchy on lower-level elements. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria. Two bridges, both strong, but the stronger is also uglier, would lead one to choose the strong but ugly one unless the criteria themselves are evaluated in terms of the bridges, and strength receives a smaller value and appearance a larger value because both bridges are strong. Feedback enables us to factor the future into the present to determine what we have to do to attain a desired future.

The feedback structure does not have the top-to-bottom form of a hierarchy but looks more like a network, with cycles connecting its components of elements, which we can no longer call levels, and with loops that connect a component to itself (Fig. 3). It also has sources and sinks. A **source** node is an origin of paths of influence (importance) and never a destination of such paths. A **sink** node is a destination of paths of influence and never an origin of such paths. A full network can include source nodes; intermediate nodes that fall on paths from source nodes, lie on cycles, or fall on paths to sink nodes; and finally sink nodes. Some networks can contain only source and sink nodes. Still others can include only source and cycle nodes or cycle and sink nodes or only cycle nodes. A decision problem involving feedback arises often in practice. It can take on the form of any of the networks just described. The challenge is to determine the priorities of the elements in the network and in particular the alternatives of the decision and even more to justify the validity of the outcome. Because feedback involves cycles, and cycling is an infinite process, the operations needed to derive the priorities become more demanding than has been familiar with hierarchies.

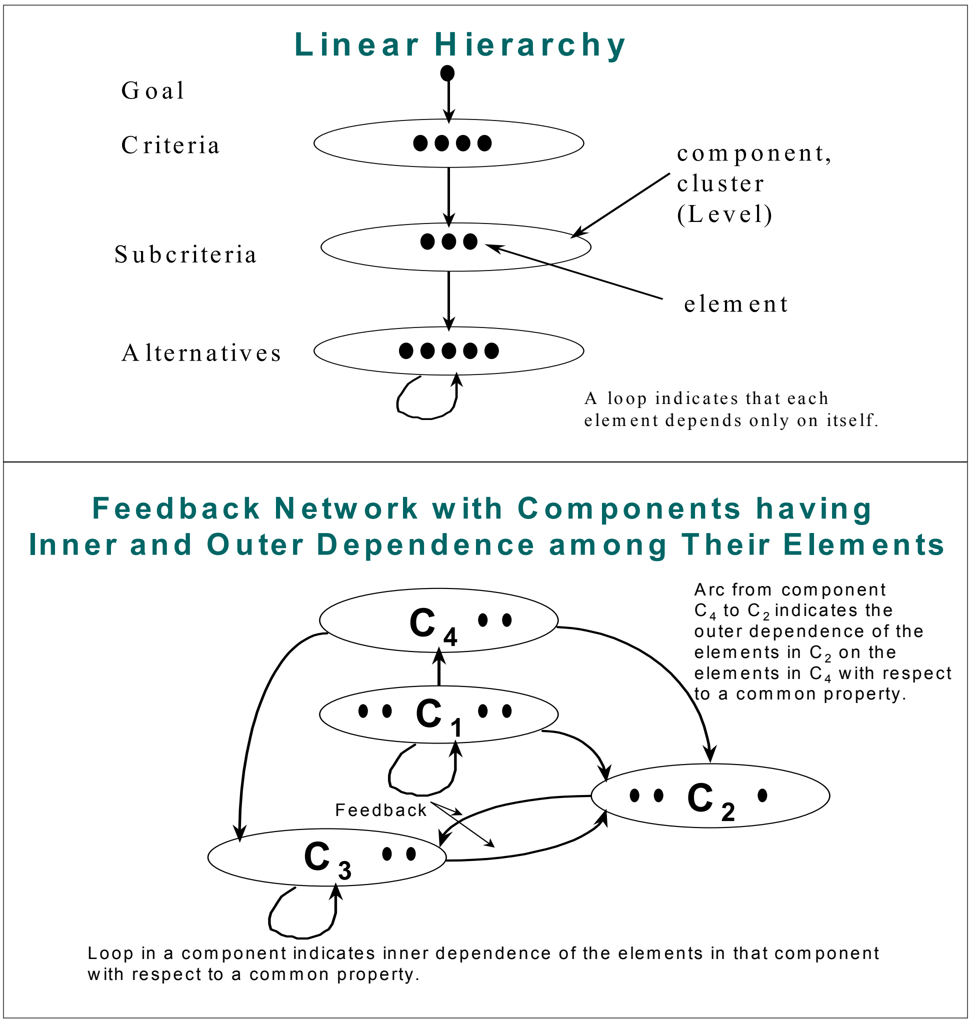


Fig. 3. How a Hierarchy Compares to a Network

To test for the mutual independence of elements such as the criteria, one proceeds as follows: Construct a zero-one matrix of criteria against criteria using the number one to signify dependence of one criterion on another, and zero otherwise. A criterion need not depend on itself as an industry, for example, may not use its own output. For each column of this matrix, construct a pairwise comparison matrix only for the dependent criteria, derive an eigenvector, and augment it with zeros for the excluded criteria. If a column is all zeros, then assign a zero vector to represent the priorities. The question in the comparison would be: For a given criterion, which of two criteria depends more on that criterion with respect to the goal or with respect to a higher-order controlling criterion?

In Fig. 3, a view is shown of a hierarchy and a network. A hierarchy is comprised of a goal, levels of elements and connections between the elements. These connections go only to elements in lower levels. A network has clusters of elements, with the elements being connected to elements in another cluster (outer dependence) or the same cluster (inner dependence). A hierarchy is a special case of a network with connections going only in one direction. In a view of a hierarchy, such as that shown in Fig. 3, the levels in the hierarchy correspond to clusters in a network. One example of inner dependence in a component consisting of a father mother and baby is whom does the baby depend on more for its survival, its mother or itself. The baby depends more on its mother than on itself. Again suppose one makes advertising by newspaper and by television. It is clear that the two influence each other because the newspaper writers watch television and need to make their message unique in some way, and vice versa. If we think about it carefully everything can be seen to influence everything including itself according to many criteria. The world is far more interdependent than we know how to deal with using our existing ways of thinking and acting. We know it but how to deal with it. The ANP appears to be a plausible logical way to deal with dependence.

The Supermatrix of a Network

$$
W = \begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{11}\, e_{12} \cdots e_{1n_1} & e_{21}\, e_{22} \cdots e_{2n_2} & & e_{N1}\, e_{N2} \cdots e_{Nn_N} \end{array} \\
\begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \\ C_N & \begin{array}{c} e_{N1} \\ e_{N2} \\ \vdots \\ e_{Nn_N} \end{array} \end{array} & \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix} \end{array}
$$

W_{ij} **Component of Supermatrix**

$$
w_{ij} = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_2)} & \cdots & W_{i1}^{(jn_j)} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_2)} & \cdots & W_{i2}^{(jn_j)} \\ \vdots & \vdots & \vdots & \vdots \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_2)} & \cdots & W_{in_i}^{(jn_j)} \end{bmatrix}
$$

Fig. 4. The Supermatrix of a Network and Detail of a Component in It

The priorities derived from pairwise comparison matrices are entered as parts of the columns of a supermatrix. The supermatrix represents the influence priority of an element on the left of the matrix on an element at the top of the matrix. A supermatrix along with an example of one of its general entry matrices is shown in Fig. 4. The component C_i in the supermatrix includes all the priority vectors derived for nodes that are "parent" nodes in the C_i cluster. Fig. 5 gives the supermatrix of a hierarchy along with the kth power that yields the principle of hierarchic composition in its $(k,1)$ position.

Supermatrix of a Hierarchy

$$\begin{array}{cc} & \begin{array}{cccccc} C_1 & C_2 & \cdots & C_{N-2} & C_{N-1} & C_N \\ e_{11}\cdots e_{1n_1} & e_{21}\cdots e_{2n_2} & & e_{(N-2)1}\cdots e_{(N-2)n_{N-2}} & e_{(N-1)1}\cdots e_{(N-1)n_{N-1}} & e_{N1}\cdots e_{Nn_N} \end{array} \\ W = \begin{array}{c} C_1 \; \begin{array}{c} e_{11} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 \; \begin{array}{c} e_{21} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots \\ C_N \; \begin{array}{c} e_{N1} \\ \vdots \\ e_{Nn_N} \end{array} \end{array} & \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ W_{21} & 0 & \cdots & 0 & 0 & 0 \\ 0 & W_{32} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & W_{n-1,n-2} & 0 & 0 \\ 0 & 0 & \cdots & 0 & W_{n,n-1} & I \end{bmatrix} \end{array}$$

Supermatrix to n^{th} Power Gives Hierarchical Synthesis

$$W^k = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ W_{n,n-1}W_{n-1,n-2}\ldots W_{32}W_{21} & W_{n,n-1}W_{n-1,n-2}\ldots W_{32} & \cdots & W_{n,n-1}W_{n-1,n-2} & W_{n,n-1} & I \end{bmatrix}$$

for $k > n-1$

Fig. 5. The Supermatrix of a Hierarchy with the Resulting Limit Matrix Corresponding to Hierarchical Composition

Hierarchic composition yields multilinear forms which are of course nonlinear and have the form

$$\sum_{i_1,\cdots,i_p} x_1^{i_1} x_2^{i_2} \cdots x_p^{i_p} ,$$

where i_j indicates the *j*th level of the hierarchy and the x_j is the priority of an element in that level. The richer the structure of a hierarchy in breadth and depth, the more elaborate are the derived multilinear forms from it. There seems to be a good opportunity to investigate the relationship obtained by composition to covariant tensors and their algebraic properties.

More concretely we have the covariant tensor

$$w_i^h = \sum_{i_2,\cdots,i_{h-1}=1}^{N_{h-1},\cdots,N_1} w_{i_1 i_2}^{h-1} \cdots w_{i_{h-2}\, i_{h-1}}^{2}\, w_{i_{h-1}}^{1} \quad i_1 \equiv i$$

for the priority of the *i*th element in the *h*th level of the hierarchy. The composite vector W^h for the entire *h*th level is represented by the vector with covariant tensorial components. Similarly, the left eigenvector approach to a hierarchy gives rise to a vector with contravariant tensor components.

The classical problem of relating space (geometry) and time to subjective thought can perhaps be examined by showing that the functions of mathematical analysis (and hence also the laws of physics) are derivable as truncated series from the above tensors by composition in an appropriate hierarchy. The foregoing is reminiscent of the theorem in dimensional analysis that any physical variable is proportional to the product of powers of primary variables.

Priority means dominance. If we know how capture dominance we would know how to obtain priorities. In the ANP we look for steady state priorities from a limit super matrix. To obtain the limit we must raise the matrix to powers. The outcome of the ANP is nonlinear and rather complex. The limit may not converge unless the matrix is column stochastic that is each of its columns sums to one. If the columns sum to one then from the fact that the principal eigenvalue of a matrix lies between its largest and smallest column sums, we know that the principal eigenvalue of a stochastic matrix is equal to one. Now we know, from a theorem due to J.J. Sylvester that when the multiplicity of each eigenvalue of a matrix *W* is equal to one that an entire function $f(x)$ (power series expansion of $f(x)$ converges for all finite values of (*x*) with *x* replaced by W, is given by

$$f(W) = \sum_{i=1}^{n} f(\lambda_i) Z(\lambda_i), \quad Z(\lambda_i) = \frac{\prod_{j \neq i} (\lambda_j I - A)}{\prod_{j \neq i} (\lambda_j - \lambda_i)},$$

$$\sum_{i=1}^{n} Z(\lambda_i) = I, \quad Z(\lambda_i) Z(\lambda_j) = 0, \quad Z^2(\lambda_i) = Z(\lambda_i),$$

where I and 0 are the i dentity and null matrices respectively.

A similar expression is also available when some or all of the eigenvalues have multiplicities greater than one. The matrix A itself gives the direct dominance of an element on the left over another element on top. But an element can dominate another via a third element. Dominance of an element over another through two step transitivities is obtained by squaring the matrix. Similarly all Nth order transitivities are obtained by raising the matrix to the Nth power which gives the dominance of one element over another in N steps. From each matrix we obtain the relative overall dominance of an element in steps equal to that power of the matrix by adding the coefficients in the row of the matrix corresponding to that element and dividing by the total. According to Cesaro summability, the limit of the Cesaro sum $\lim_{N\to\infty} 1/N\left(\sum_{k=0}^{N} A^k e^T \Big/ \sum_{k=0}^{N} e A^k e^T\right)$, $e=(1,1,...,1)$ that represents the average of all order dominance up to *N*, is the same as the limit of the sequence of the powers of the matrix i.e. $\lim_{N\to\infty} A^N e^T / e A^N e^T$ and thus we need to calculate the limiting powers of A.

How do we capture the priorities in the limit as the steady state priorities? We see that if, as we need in our case, $f(W)=W^N$, then $f(\lambda_i)=\lambda_i^N$ and as $N\to\infty$ the only terms that give a finite nonzero value are those for which the modulus of λ_i is equal to one. The fact that W is stochastic ensures this. We have:

$$\max\sum_{j=1}^{n} a_{ij} \ge \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i} = \lambda_{\max} \text{ for } \max w_i,$$

$$\min\sum_{j=1}^{n} a_{ij} \le \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i} = \lambda_{\max} \text{ for } \min w_i.$$

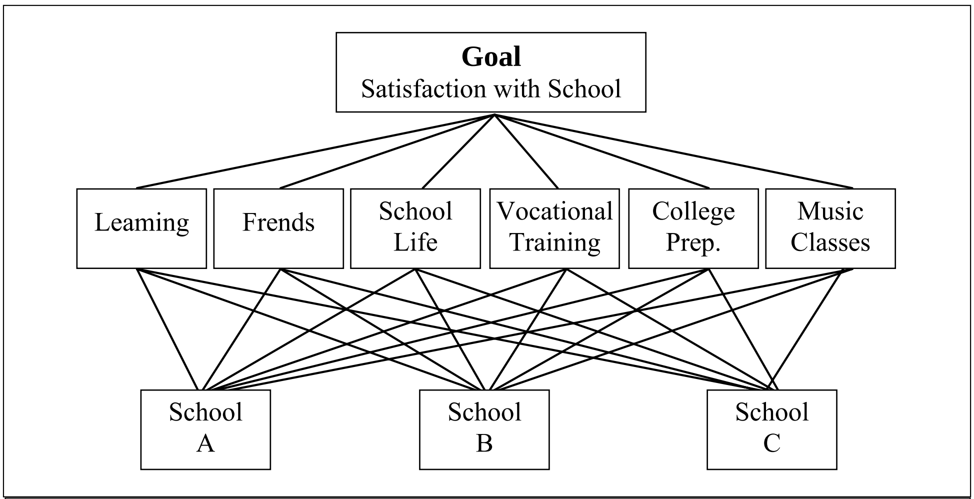
Thus for a row stochastic matrix we have $1=\min\sum_{j=1}^{n} a_{ij} \le \lambda_{\max} \le$

$\le \max\sum_{j=1}^{n} a_{ij} = 1$, thus $\lambda_{\max} = 1$.

The same type of argument applies when a matrix that is column stochastic. For complete treatment, see this author's 2001 book on the ANP [1], and also the manual for the ANP software [2].

The ANP Formulation of the Classic AHP School Example

We show in Fig. 6 below the hierarchy, the corresponding supermatrix, and its limit supermatrix to obtain the priorities of three schools involved in a decision to choose one for the author's son. They are precisely what one obtains by hierarchic composition using the AHP. The priorities of the criteria with respect to the goal and those of the alternatives with respect to each criterion are clearly discernible in the supermatrix itself. Note that there is an identity submatrix for the alternatives with respect to the alternatives in the lower right hand part of the matrix. The level of alternatives in a hierarchy is a sink cluster of nodes that absorbs priorities but does not pass them on. This calls for using an identity submatrix for them in the supermatrix.



The School Hierarchy as Supermatrix

	Goal	Learning	Friends	School life	Vocational training	College preparation	Music classes	A	B	C
Goal	0	0	0	0	0	0	0	0	0	0
Learning	0.32	0	0	0	0	0	0	0	0	0
Friends	0.14	0	0	0	0	0	0	0	0	0
School life	0.03	0	0	0	0	0	0	0	0	0
Vocational training	0.13	0	0	0	0	0	0	0	0	0
College preparation	0.24	0	0	0	0	0	0	0	0	0
Music classes	0.14	0	0	0	0	0	0	0	0	0
Alternative A	0	0.16	0.33	0.45	0.77	0.25	0.69	1	0	0
Alternative B	0	0.59	0.33	0.09	0.06	0.5	0.09	0	1	0
Alternative C	0	0.25	0.34	0.46	0.17	0.25	0.22	0	0	1

Limiting Supermatrix & Hierarchic Composition

	Goal	Learning	Friends	School life	Vocational training	College preparation	Music classes	A	B	C
Goal	0	0	0	0	0	0	0	0	0	0
Learning	0	0	0	0	0	0	0	0	0	0
Friends	0	0	0	0	0	0	0	0	0	0
School life	0	0	0	0	0	0	0	0	0	0
Vocational training	0	0	0	0	0	0	0	0	0	0
College preparation	0	0	0	0	0	0	0	0	0	0
Music classes	0	0	0	0	0	0	0	0	0	0
Alternative A	0.3676	0.16	0.33	0.45	0.77	0.25	0.69	1	0	0
Alternative B	0.3781	0.59	0.33	0.09	0.06	0.5	0.09	0	1	0
Alternative C	0.2543	0.25	0.34	0.46	0.17	0.25	0.22	0	0	1

Fig. 6. Supermatrix of School Choice Hierarchy gives same Result as Hierarchic Composition

The Investment Example with Criterion Weights Automatically Derived from the Supermatrix [3]

Let us revisit the investment example that appeared in my earlier exposition in Part 2.1 on the theory of the AHP/ANP. An individual has three alternate ways, A_1, A_2, and A_3, of investing a sum of money for the same period of time. There are two types of returns, C_1 and C_2 (for example, capital appreciation and interest), as shown in Tabl. 3. The question is, which is the best investment to make in terms of actual dollars earned?

It is easy to calculate the actual total cost for each alternative by simply adding the two numbers; the relative cost is then obtained by normalizing as shown in the table.

Table 3. Calculating Returns Arithmetically

Alternatives	Criterion C_1 Unnormalized weight = 1.0	Criterion C_2 Unnormalized weight = 1.0	Weighted Sum Unnormalized	Normalized or Relative values
A_1	200	150	350	350/1300=0.269
A_2	300	50	350	350/1300=0.269
A_3	500	100	600	600/1300=0.462
Column Totals	1000	300	1300	1

Since we are dealing with tangibles we normalize each column to obtain the priorities for the alternatives under each criterion. We also normalize each row to obtain the priorities of the criteria with respect to each alternative. We enter these in a supermatrix as shown in Tabl. 4; there is no need to weight the supermatrix because it is already column stochastic, so we can raise it to limiting powers and obtain the limit supermatrix in Tabl. 5 in which all the columns are identical. Because the supermatrix is column stochastic the priorities for the alternatives and the criteria each add to 50% of the value in a column. We see that the supermatrix saves us the arithmetic of determining criteria weights based on the values of the alternatives under each criterion.

Table 4. The Unweighted Supermatrix of the Investment Example

		Alternatives			Criteria	
		A_1	A_2	A_3	C_1	C_2
Alternatives	A_1	0.000	0.000	0.000	0.200	0.500
	A_2	0.000	0.000	0.000	0.300	0.167
	A_3	0.000	0.000	0.000	0.500	0.333
Criteria	C_1	0.571	0.857	0.833	0.000	0.000
	C_2	0.429	0.143	0.167	0.000	0.000

Table 5. The Limit Supermatrix of the Investment Example

		Alternatives			Criteria	
		A_1	A_2	A_3	C_1	C_2
Alternatives	A_1	0.135	0.135	0.135	0.135	0.135
	A_2	0.135	0.135	0.135	0.135	0.135
	A_3	0.231	0.231	0.231	0.231	0.231
Criteria	C_1	0.385	0.385	0.385	0.385	0.385
	C_2	0.115	0.115	0.115	0.115	0.115

Normalizing the results for the alternatives, that is, dividing by the sum of their values in Tabl. 5, which is .5, gives the same ratios we obtained for the overall return for each investment in Tabl. 3.

3. TWO EXAMPLES OF ESTIMATING MARKET SHARE — THE ANP WITH A SINGLE BENEFITS CONTROL CRITERION

A market share estimation model is structured as a network of clusters and nodes. The object is to try to determine the relative market share of competitors in a particular business, or endeavor, by considering what affects market share in that business and introducing them as clusters, nodes and influence links in a network. The decision alternatives are the competitors and the synthesized results are their relative dominance. The relative dominance results can then be compared against some outside measure such as dollars. If dollar income is the measure being used, the incomes of the competitors must be normalized to get it in terms of relative market share.

The clusters might include customers, service, economics, advertising, and quality of goods. The customers cluster might then include nodes for the age groups of the people that buy from the business: teenagers, 20–33 year olds, 34–55 year olds, 55–70 year olds, and over 70. The advertising cluster might include newspapers, TV, Radio, and Fliers. After all the nodes are created start by picking a node and linking it to the other nodes in the model that influence it. The "children" nodes will then be pairwise compared with respect to that node as a "parent" node. An arrow will automatically appear going from the cluster the parent node is in to the cluster with its children nodes. When a node is linked to nodes in its own cluster, the arrow becomes a loop on that cluster and we say there is interdependence.

The linked nodes in a given cluster are pairwise compared for their influence on the node they are linked from (the parent node) to determine the priority of their influence on the parent node. Comparisons are made as to which is more important to the parent node in capturing "market share". These priorities are then entered in the supermatrix for the network.

The clusters are also pairwise compared to establish their importance with respect to each cluster they are linked from, and the resulting matrix of numbers is used to weight the components of the original unweighted supermatrix to give the weighted supermatrix. This matrix is then raised to powers until it converges to give the limit supermatrix. The relative values for the companies are obtained from the columns of the limit supermatrix that are all the same. Normalizing these numbers yields the relative market share.

If comparison data in terms of sales in dollars, or number of members, or some other known measures are available, one can use these relative values to validate the outcome. The AHP/ANP has a compatibility metric to determine how close the ANP result is to the known measure. It involves taking the Hadamard product of the matrix of ratios of the ANP outcome and the transform of the matrix of ratios of the actual outcome summing all the coefficients and dividing by n^2. The requirement is that the value should be close to 1 and certainly not much more than 1.1.

We will give three examples of market share estimation showing details of the process in the first example and showing only the models and results in the second and third examples.

Example 1. Estimating the relative market share of Walmart, Kmart and Target

The network for the ANP model shown in Fig. 7 well describes the influences that determine the market share of these companies. We will not use space in this paper to describe the clusters and their nodes in greater detail.

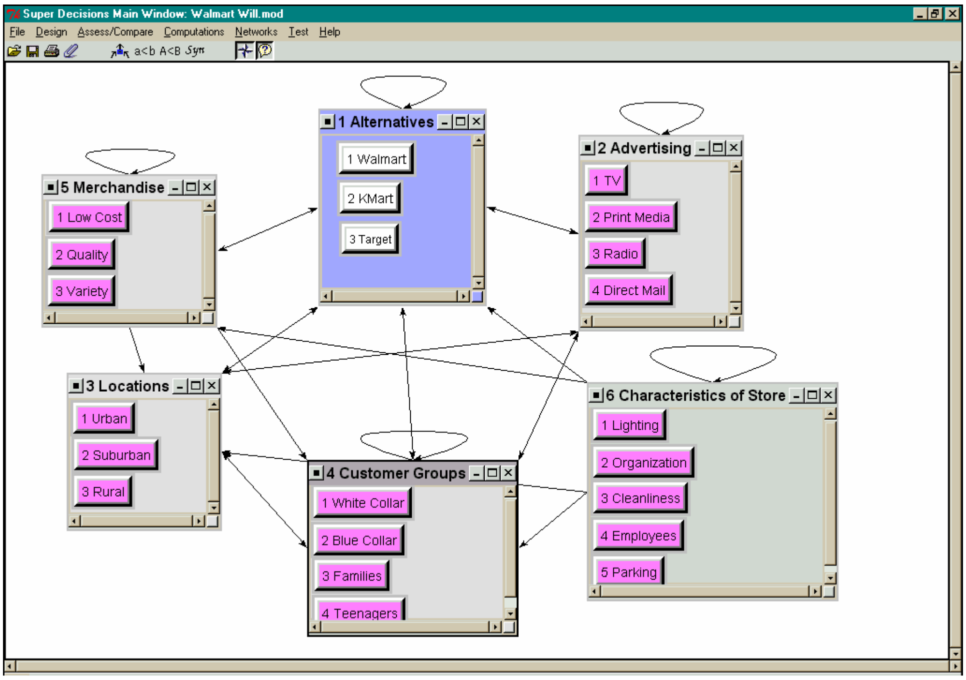


Fig. 7. The Clusters and Nodes of a Model to Estimate the Relative Market Share of Walmart, Kmart and Target

The Unweighted Supermatrix

The unweighted supermatrix is constructed from the priorities derived from the different pairwise comparisons. The nodes, grouped by the clusters they belong to, are the labels of the rows and columns of the supermatrix. The column for a node *a* contains the priorities of the nodes that have been pairwise compared with respect to *a*. The supermatrix for the network in Fig. 7 is shown in Tabl. 6.

The Cluster Matrix

The cluster themselves must be compared to establish their relative importance and use it to weight the supermatrix to make it column stochastic. A cluster impacts another cluster when it is linked from it, that is, when at least one node in the source cluster is linked to nodes in the target cluster. The clusters linked from the source cluster are pairwise compared for the importance of their impact on it with respect to market share, resulting in the column of priorities for that cluster in the cluster matrix. The process is repeated for each cluster in the network to

Table 6. The Unweighted Supermatrix, Displayed in Two Parts

		1. Alternatives			2. Advertising				3. Locations		
		1. Walmart	2. KMart	3. Target	1. TV	2. Print Media	3. Radio	4. Direct Mail	1. Urban	2. Suburban	3. Rural
1. Alternatives	1. Walmart	0.000	0.833	0.833	0.687	0.540	0.634	0.661	0.614	0.652	0.683
	2. KMart	0.750	0.000	0.167	0.186	0.297	0.174	0.208	0.268	0.235	0.200
	3. Target	0.250	0.167	0.000	0.127	0.163	0.192	0.131	0.117	0.113	0.117
2. Advertising	1. TV	0.553	0.176	0.188	0.000	0.000	0.000	0.000	0.288	0.543	0.558
	2. Print Media	0.202	0.349	0.428	0.750	0.000	0.800	0.000	0.381	0.231	0.175
	3. Radio	0.062	0.056	0.055	0.000	0.000	0.000	0.000	0.059	0.053	0.048
	4. Direct Mail	0.183	0.420	0.330	0.250	0.000	0.200	0.000	0.273	0.173	0.219
3. Locations	1. Urban	0.114	0.084	0.086	0.443	0.126	0.080	0.099	0.000	0.000	0.000
	2. Suburban	0.405	0.444	0.628	0.387	0.416	0.609	0.537	0.000	0.000	0.000
	3. Rural	0.481	0.472	0.285	0.169	0.458	0.311	0.364	0.000	0.000	0.000
4. Cust.Groups	1. White Collar	0.141	0.114	0.208	0.165	0.155	0.116	0.120	0.078	0.198	0.092
	2. Blue Collar	0.217	0.214	0.117	0.165	0.155	0.198	0.203	0.223	0.116	0.224
	3. Families	0.579	0.623	0.620	0.621	0.646	0.641	0.635	0.656	0.641	0.645
	4. Teenagers	0.063	0.049	0.055	0.048	0.043	0.045	0.041	0.043	0.045	0.038
5. Merchandise	1. Low Cost	0.362	0.333	0.168	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Quality	0.261	0.140	0.484	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Variety	0.377	0.528	0.349	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6. Characteristic	1. Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Organization	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Cleanliness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4. Employees	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5. Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Termination

		4 Custom. Groups				5 Merchan-dise			6 Charac-teristics of Store				
		1. White Collar	2. Blue Collar	3. Familie	4. Teens	1. Low Cost	2. Quality	3. Variety	1. Light'ng	2. Organiz.	3. Clean	4. Emp-loyees	5. Park
1. Alternat..	1. Walmart	0.637	0.661	0.630	0.691	0.661	0.614	0.648	0.667	0.655	0.570	0.644	0.558
	2. KMart	0.105	0.208	0.218	0.149	0.208	0.117	0.122	0.111	0.095	0.097	0.085	0.122
	3. Target	0.258	0.131	0.151	0.160	0.131	0.268	0.230	0.222	0.250	0.333	0.271	0.320
2. Advertis.	1. TV	0.323	0.510	0.508	0.634	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Print Med.	0.214	0.221	0.270	0.170	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Radio	0.059	0.063	0.049	0.096	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4. Direct Mail	0.404	0.206	0.173	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3. Locations	1. Urban	0.167	0.094	0.096	0.109	0.268	0.105	0.094	0.100	0.091	0.091	0.111	0.067
	2. Suburban	0.833	0.280	0.308	0.309	0.117	0.605	0.627	0.433	0.455	0.455	0.444	0.293
	3. Rural	0.000	0.627	0.596	0.582	0.614	0.291	0.280	0.466	0.455	0.455	0.444	0.641
4.Cust. Grps	1. White Col.	0.000	0.000	0.279	0.085	0.051	0.222	0.165	0.383	0.187	0.242	0.165	0.000
	2. Blue Collar	0.000	0.000	0.649	0.177	0.112	0.159	0.165	0.383	0.187	0.208	0.165	0.000
	3. Families	0.857	0.857	0.000	0.737	0.618	0.566	0.621	0.185	0.583	0.494	0.621	0.000
	4. Teenagers	0.143	0.143	0.072	0.000	0.219	0.053	0.048	0.048	0.043	0.056	0.048	0.000
5.Merchand.	1. Low Cost	0.000	0.000	0.000	0.000	0.000	0.800	0.800	0.000	0.000	0.000	0.000	0.000
	2. Quality	0.000	0.000	0.000	0.000	0.750	0.000	0.200	0.000	0.000	0.000	0.000	0.000
	3. Variety	0.000	0.000	0.000	0.000	0.250	0.200	0.000	0.000	1.000	0.000	0.000	0.000
6. Character.	1. Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.169	0.121	0.000	0.250
	2. Organiz.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.251	0.000	0.575	0.200	0.750
	3. Cleanli.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.673	0.469	0.000	0.800	0.000
	4. Employee	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.308	0.304	0.000	0.000
	5. Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.075	0.055	0.000	0.000	0.000

obtain the matrix shown in Tabl. 7. An interpretation of the priorities in the first column is that Merchandise (0.442) and Locations (0.276) have the most impact on Alternatives, the three competitors.

Table 7. The Cluster Matrix

	1. Alter-natives	2. Adver-tising	3. Loca-tions	4. Customer Groups	5. Mer-chandise	6. Characteris-tics of Store
1. Alternatives	0.137	0.174	0.094	0.057	0.049	0.037
2. Advertising	0.091	0.220	0.280	0.234	0.000	0.000
3. Locations	0.276	0.176	0.000	0.169	0.102	0.112
4. Customer Groups	0.054	0.429	0.627	0.540	0.252	0.441
5. Merchandise	0.442	0.000	0.000	0.000	0.596	0.316
6. Characteristics of Store	0.000	0.000	0.000	0.000	0.000	0.094

Weighted Supermatrix

The weighted supermatrix shown in Tabl. 8 is obtained by multiplying each entry in a block of the component at the top of the supermatrix by the priority of influence of the component on the left from the cluster matrix in Tabl. 7. For example, the first entry, 0.137, in Tabl. 7 is used to multiply each of the nine entries in the block (Alternatives, Alternatives) in the unweighted supermatrix shown in Tabl. 6. This gives the entries for the (Alternatives, Alternatives) component in the weighted supermatrix of Tabl. 8. Each column in the weighted supermatrix has a sum of 1, and thus the matrix is stochastic.

The limit supermatrix shown in Tabl. 9 is obtained from the weighted supermatrix by raising it to powers until it converges so that all columns are identical.

Synthesized Results

The relative market shares of the alternatives, 0.599, 0.248 and 0.154 are displayed as synthesized results in the Super Decisions Program, shown in the middle column of Tabl. 10. They are obtained by normalizing the values for Walmart, Kmart and Target: 0.057, 0.024 and 0.015, taken from the limit supermatrix. The idealized values are obtained from the normalized values by dividing each value by the largest value in that column.

Actual Relative Market Share Based on Sales

The object was to estimate the market share of Walmart, Kmart, and Target. The normalized results from the model were compared with sales as reported in the Discount Store News of July 13, 1998, p.77, of \$58, \$27.5 and \$20.3 billions of dollars respectively. Normalizing the dollar amounts shows their actual relative market shares to be 54.8, 25.9 and 19.2. The relative market share from the model was compared with the sales values by computing the compatibility index using the Hadamard multiplication method below; it was equal to 1.016. Since that value is less than 1.1 it is acceptable.

Table 8. The Weighted Supermatrix, Displayed in Two Parts

		1. Alternatives			2. Advertising				3. Locations		
		1. Walmart	2. KMart	3. Target	1. TV	2. Print Media	3. Radio	4. Direct Mail	1. Urban	2.Suburban	3. Rural
. Alternatives	1. Walmart	0.000	0.114	0.114	0.120	0.121	0.110	0.148	0.058	0.061	0.064
	2. KMart	0.103	0.000	0.023	0.033	0.066	0.030	0.047	0.025	0.022	0.019
	3. Target	0.034	0.023	0.000	0.022	0.037	0.033	0.029	0.011	0.011	0.011
. Advertising	1. TV	0.050	0.016	0.017	0.000	0.000	0.000	0.000	0.080	0.152	0.156
	2. Print Media	0.018	0.032	0.039	0.165	0.000	0.176	0.000	0.106	0.064	0.049
	3. Radio	0.006	0.005	0.005	0.000	0.000	0.000	0.000	0.016	0.015	0.014
	4. Direct Mail	0.017	0.038	0.030	0.055	0.000	0.044	0.000	0.076	0.048	0.061
. Locations	1. Urban	0.031	0.023	0.024	0.078	0.028	0.014	0.022	0.000	0.000	0.000
	2. Suburban	0.112	0.123	0.174	0.068	0.094	0.107	0.121	0.000	0.000	0.000
	3. Rural	0.133	0.130	0.079	0.030	0.103	0.055	0.082	0.000	0.000	0.000
. Cust.Groups	1. White Collar	0.008	0.006	0.011	0.071	0.086	0.050	0.066	0.049	0.124	0.058
	2. Blue Collar	0.012	0.011	0.006	0.071	0.086	0.085	0.112	0.140	0.073	0.141
	3. Families	0.031	0.033	0.033	0.267	0.356	0.275	0.350	0.411	0.402	0.404
	4. Teenagers	0.003	0.003	0.003	0.021	0.024	0.019	0.023	0.027	0.028	0.024
. Merchandise	1. Low Cost	0.160	0.147	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Quality	0.115	0.062	0.214	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Variety	0.166	0.233	0.154	0.000	0.000	0.000	0.000	0.000	0.000	0.000
. Characteristic	1. Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Organization	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Cleanliness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4. Employees	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5. Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Termination

		4. Custom Groups				5. Merchandise			6. Charactof Store				
		1 White Collar	2 Blue Collar	3 Familie	4 Teens	1 Low Cost	2 Quality	3 Variety	1 Light'ng	2 Organ.	3 Clean	4 Emp-loyees	5 Pkg
.Alternat.	1. Walmart	0.036	0.038	0.036	0.040	0.033	0.030	0.032	0.036	0.024	0.031	0.035	0.086
	2. KMart	0.006	0.012	0.012	0.009	0.010	0.006	0.006	0.006	0.004	0.005	0.005	0.019
	3. Target	0.015	0.007	0.009	0.009	0.006	0.013	0.011	0.012	0.009	0.018	0.015	0.049
. Advertising	1. TV	0.076	0.119	0.119	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Print Med.	0.050	0.052	0.063	0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Radio	0.014	0.015	0.012	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4. Direct Mail	0.095	0.048	0.040	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
. Locations	1. Urban	0.028	0.016	0.016	0.018	0.027	0.011	0.010	0.016	0.010	0.015	0.018	0.031
	2. Suburban	0.141	0.047	0.052	0.052	0.012	0.062	0.064	0.071	0.051	0.074	0.073	0.135
	3. Rural	0.000	0.106	0.101	0.098	0.063	0.030	0.029	0.076	0.051	0.074	0.073	0.295
. Cust Grps	1. White Col.	0.000	0.000	0.151	0.046	0.013	0.056	0.042	0.247	0.082	0.156	0.107	0.000
	2. Blue Collar	0.000	0.000	0.350	0.096	0.028	0.040	0.042	0.247	0.082	0.134	0.107	0.000
	3. Families	0.463	0.463	0.000	0.398	0.156	0.143	0.157	0.119	0.257	0.318	0.400	0.000
	4. Teenagers	0.077	0.077	0.039	0.000	0.055	0.013	0.012	0.031	0.019	0.036	0.031	0.000
. Merchand	1. Low Cost	0.000	0.000	0.000	0.000	0.000	0.477	0.477	0.000	0.000	0.000	0.000	0.000
	2. Quality	0.000	0.000	0.000	0.000	0.447	0.000	0.119	0.000	0.000	0.000	0.000	0.000
	3. Variety	0.000	0.000	0.000	0.000	0.149	0.119	0.000	0.000	0.316	0.000	0.000	0.000
. Charact.	1. Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.017	0.000	0.097
	2. Organiz.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.035	0.000	0.079	0.027	0.290
	3. Cleanli.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.092	0.044	0.000	0.110	0.000
	4. Employee	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.042	0.000	0.000
	5. Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.005	0.000	0.000	0.000

Table 9. Limit Supermatrix

		1. Alternatives			2. Advertising				3. Locations		
		1. Walmart	2. KMart	3. Target	1. TV	2. Print Media	3. Radio	4. Direct Mail	1. Urban	2. Suburban	3. Rural
. Alternatives	1. Walmart	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057
	2. KMart	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
	3. Target	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
. Advertising	1. TV	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079
	2. Print Media	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
	3. Radio	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
	4. Direct Mail	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039
. Locations	1. Urban	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022
	2. Suburban	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
	3. Rural	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069
. Cust.Groups	1. White Collar	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068
	2. Blue Collar	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
	3. Families	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240
	4. Teenagers	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036
. Merchandise	1. Low Cost	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043
	2. Quality	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
	3. Variety	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
. Characteristic	1. Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Organization	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Cleanliness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4. Employees	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5. Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Termination

		4. Customer Groups				5. Merchan-dise			6. Charact of Store				
		1. White Collar	2. Blue Collar	3. Familie	4. Teens	1. Low Cost	2. Quality	3. Variety	1.Light'ng	2. Organiz.	3. Clean	4. Emp-loyees	5. Parking
. Alternatives	1. Walmart	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057
	2. KMart	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
	3. Target	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
. Advertising	1. TV	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079
	2. Print Med.	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
	3. Radio	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
	4. Direct Mail	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039
. Locations	1. Urban	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022
	2. Suburban	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
	3. Rural	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069
. Cust.Groups	1. White Col.	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068
	2. Blue Collar	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
	3. Families	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240
	4. Teenagers	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036
. Merchandise	1. Low Cost	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043
	2. Quality	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
	3. Variety	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
. Characteristi.	1. Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2. Organiz.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3. Cleanli.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4. Employee	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5. Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Competitor	ANP Results	Dollar Sales	Actual Market Share as Dollar Sales Normalized
Walmart	59.8	\$58.0 billion	54.8
Kmart	24.8	\$27.5 billion	35.9
Target	15.4	\$20.3 billion	19.2
		Compatibility Index	1.016

Table 10. The Synthesized Results for the Alternative

Alternatives	Ideal Values	Normalized Values	Values from Limit Super-matrix
Walmart	1.000	0.594	0.057
KMart	0.414	0.250	0.024
Target	0.271	0.156	0.015

Example 2. Estimating Relative Market Share of Airlines

An ANP model to estimate the relative market share of 8 American Airlines is shown in Fig. 8. The results from the model are shown in Tabl. 10 below and the comparison with the relative actual market share is shown in Tabl. 11.

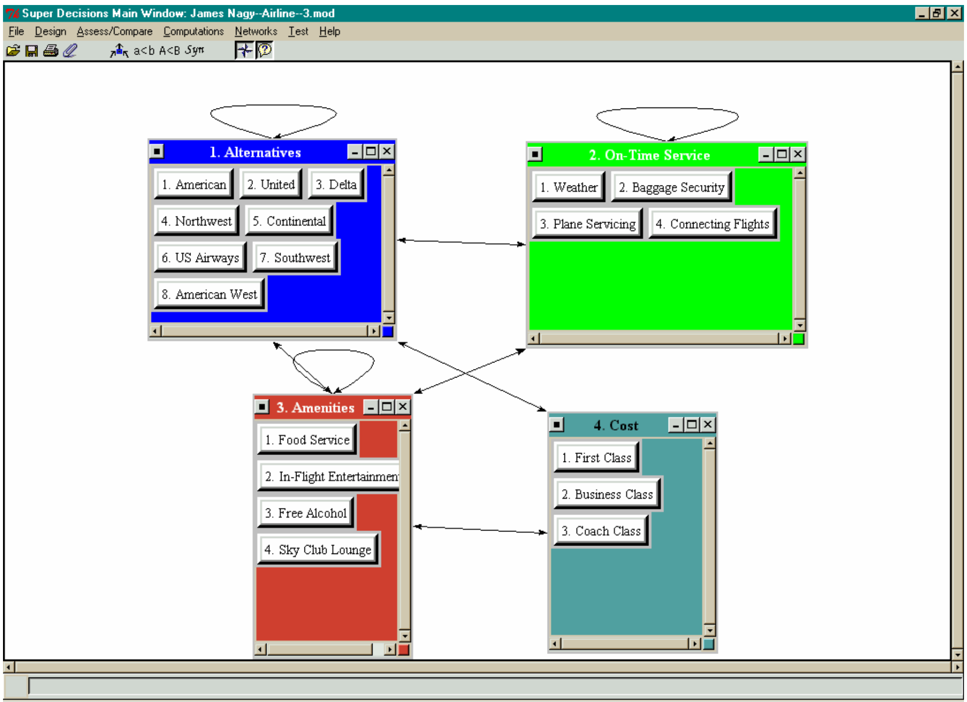


Fig. 8. ANP Network to Estimate Relative Market Share of 8 US Airlines

T a b l e 11. Comparing Model Results with Actual Market Share Data

	Model Results	Actual Market Share (yr 2000)
American	23.9	24.0
United	18.7	19.7
Delta	18.0	18.0
Northwest	11.4	12.4
Continental	9.3	10.0
US Airways	7.5	7.1
Southwest	5.9	6.4
American West	4.4	2.9
	Compatibility Index	1.0247

We summarize by giving the reader a list of the steps we have followed in applying the ANP.

4. OUTLINE OF THE STEPS OF THE ANP

1. Describe the decision problem in detail including its objectives, criteria and subcriteria, actors and their objectives and the possible outcomes of that decision. Give details of influences that determine how that decision may come out.
2. Determine the control criteria and subcriteria in the four control hierarchies one each for the benefits, opportunities, costs and risks of that decision and obtain their priorities from paired comparisons matrices. If a control criterion or subcriterion has a global priority of 3% or less, you may consider carefully eliminating it from further consideration. The software automatically deals only with those criteria or subcriteria that have subnets under them. For benefits and opportunities, ask what gives the most benefits or presents the greatest opportunity to influence fulfillment of that control criterion. For costs and risks, ask what incurs the most cost or faces the greatest risk. Sometimes (very rarely), the comparisons are made simply in terms of benefits, opportunities, costs, and risks in the aggregate without using control criteria and subcriteria.
3. Determine the most general network of clusters (or components) and their elements that applies to all the control criteria. To better organize the development of the model as well as you can, number and arrange the clusters and their elements in a convenient way (perhaps in a column). Use the identical label to represent the same cluster and the same elements for all the control criteria.
4. For each control criterion or subcriterion, determine the clusters of the general feedback system with their elements and connect them according to their outer and inner dependence influences. An arrow is drawn from a cluster to any cluster whose elements influence it.
5. Determine the approach you want to follow in the analysis of each cluster or element, influencing (the preferred approach) other clusters and elements with respect to a criterion, or being influenced by other clusters and elements. The sense (being influenced or influencing) must apply to all the criteria for the four control hierarchies for the entire decision.

6. For each control criterion, construct the supermatrix by laying out the clusters in the order they are numbered and all the elements in each cluster both vertically on the left and horizontally at the top. Enter in the appropriate position the priorities derived from the paired comparisons as subcolumns of the corresponding column of the supermatrix.

7. Perform paired comparisons on the elements within the clusters themselves according to their influence on each element in another cluster they are connected to (outer dependence) or on elements in their own cluster (inner dependence). In making comparisons, you must always have a criterion in mind. Comparisons of elements according to which element influences a given element more and how strongly more than another element it is compared with are made with a control criterion or subcriterion of the control hierarchy in mind.

8. Perform paired comparisons on the clusters as they influence each cluster to which they are connected with respect to the given control criterion. The derived weights are used to weight the elements of the corresponding column blocks of the supermatrix. Assign a zero when there is no influence. Thus obtain the weighted column stochastic supermatrix.

9. Compute the limit priorities of the stochastic supermatrix according to whether it is irreducible (primitive or imprimitive [cyclic]) or it is reducible with one being a simple or a multiple root and whether the system is cyclic or not. Two kinds of outcomes are possible. In the first all the columns of the matrix are identical and each gives the relative priorities of the elements from which the priorities of the elements in each cluster are normalized to one. In the second the limit cycles in blocks and the different limits are summed and averaged and again normalized to one for each cluster. Although the priority vectors are entered in the supermatrix in normalized form, the limit priorities are put in idealized form because the control criteria do not depend on the alternatives.

10. Synthesize the limiting priorities by weighting each idealized limit vector by the weight of its control criterion and adding the resulting vectors for each of the four merits: Benefits (B), Opportunities (O), Costs (C) and Risks (R). There are now four vectors, one for each of the four merits. An answer involving marginal values of the merits is obtained by forming the ratio BO/CR for each alternative from the four vectors. The alternative with the largest ratio is chosen for some decisions. Companies and individuals with limited resources often prefer this type of synthesis.

11. Governments prefer this type of outcome. Determine strategic criteria and their priorities to rate the four merits one at a time. Normalize the four ratings thus obtained and use them to calculate the overall synthesis of the four vectors. For each alternative, subtract the costs and risks from the sum of the benefits and opportunities. At other times one may add the weighted reciprocals of the costs and risks. Still at other times one may subtract the costs from one and risks from one and then weight and add them to the weighted benefits and opportunities. In all, we have four different formulas for synthesis.

12. Perform sensitivity analysis on the final outcome and interpret the results of sensitivity observing how large or small these ratios are. Can another outcome that is close also serve as a best outcome? Why? By noting how stable this outcome is. Compare it with the other outcomes by taking ratios. Can another outcome that is close also serve as a best outcome? Why?

5. CONCLUSIONS

Complete examples applying the AHP to a decision involving BOCR can be found in references 1 and 2. Numerous other examples along with the software Super Decisions for the ANP can be obtained from rozann@creativedecisions.net. The reader now should have a good idea as to how to use the process in a complex decision. The AHP and ANP have found application in practice by many companies and governments. My book *Decision Making for Leaders* is in more than a half a dozen languages. What is happening now is the wide interest shown in the ANP and its applicability to the long discussed project of building a National Missile Defense This application was done in September 2000 and presented at the National Defense University in Washington, DC, in February 2002. Its conclusions were affirmed by President Bush's decision of late December 2002 to construct such a system. Another recent policy study was done regarding whether the US should challenge Iraq directly or go through the UN. The administration decided to go through the UN. There is also the hopeless Middle East conflict. An ANP analysis showed that the best option is for Israel and the US to help the Palestinians both set up a state and in particular achieve a viable economy. There are two things to tell the reader about it in this regard. The ANP book is now translated to Russian and will soon appear and the manual for the Super Decisions sofware can be obtained by sending an email to rozann@creativedecisions.net. My forthcoming book *The Encyclicon* will have nearly 100 examples of such applications and will be out in the near future.

As a final word, the AHP has developed a group of critics who think it cannot be used for multicriteria methods because it is simple. They say that it is linear. I say it is non-linear. Hierarchic composition yields nonlinear forms that are dense in the space of polynomials (multinomials). According to the theorem of Weierstrasse the latter can be used to approximate a continuous function arbitrarily close. Multilinear forms and polynomials are intimately related, particularly when we see that we can identify all the variables into a single or into several variables raised to powers. Thus depending on how rich a hierarchic structure is, one can use hierarchic composition to come close to the real answer underlying a decision. This author, a mathematician, with a Ph.D. in mathematics from Yale University, and postdoctoral work at the Sorbonne in Paris, has written many first works in the field of Operations Research. Among them is Mathematical Methods of Operations Research, translated to many languages. Also, he has spent the first half of his career working at the Pentagon, the Navy Department, and the Department of State, all the time searching for and applying, when feasible, mathematical models of operations research. The biggest weakness of these methods is that they could only be understood by the experts and occasionally used by the practicing layman, with doubt and hesitation. As an adult I took it upon myself since my first involvement in research on negotiations between the US and the Soviet Union in Geneva in the 1960's and after having written a book on the Mathematical Models of Arms Control and Disarmament in 1968, translated to Russian in 1977, to always try to simplify the AHP so that even a child can use it. And children have used it without the need to explain the mathematics but even if one has to, it is possible to do that with some patience but without a great deal of prolixity and confusion. This is why the AHP and ANP are intentionally simple. More full blown decision examples will be given in Part 2.3.

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