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By the method of comparison of decisions, built on the arctic landmark with the two point of interface for the separate system of differential equalizations of Lezhandra, Bessel and Euler by the method of functions Koshi and by the method of the proper hybrid integral transformation, polyparametric family of known integrals is calculated.

Key words: *Not own integrals, functions Koshi, the main decisions, hybrid integrated transformation, the basic identity, condition of unequivocal resolvabitili, the logic scheme.*

Отримано: 05.06.2011

УДК 519.21

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STOCHASTIC APPROXIMATION PROCEDURE WITH IMPULSIVE MARKOV PERTURBATIONS

In this paper we discuss asymptotic behavior of the stochastic approximation procedure in case when the regression function is perturbed by the Markov impulsive process. Also we consider the stochastic approximation procedure stability conditions in the terms of existence of Lyapunov's function for the averaged evolution system.

Key words: *stochastic approximation procedure, Markov process, impulsive perturbation.*

Introduction. The goal of the Robbins-Monro Stochastic Approximation Procedure (SAP) [1] is to find the solution of the equation $C(u) = 0$ in the case when the measurements of regression function $C(u)$ are made with some errors. It is widely used in the mathematical statistics [2], control theory, image, signal and voice recognition theory [3], etc.

Let us consider the situation when estimated function errors are defined by impulsive process. Then the continuous SAP is defined by the differential equation

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$$du^{\varepsilon}(t) = a(t)\left[C\left(u^{\varepsilon}(t), x\left(t/\varepsilon^4\right)\right)dt + \varepsilon d\eta^{\varepsilon}(t) \right], \tag{1}$$

with $C(u,\cdot) \in C^2(R^n)$.

Where a Markov process $x(t), t \ge 0$ in the standard phase space $\left(X, \aleph\right)$ is defined by the generator:

$$Q\varphi(x) = q(x)\int_X P(x,dy)\left[\varphi(y) - \varphi(x)\right], \ \varphi \in B(X),$$

here $B(X)$ is the Banach space of real bounded functions with supremum-norm $\|\varphi\| = \max_{x\in X} |\varphi(x)|$.

A uniformly ergodic embedded Markov chain $x_n = x(\tau_n)$, $n \ge 0$ with stationary distribution $\rho(B), B \in \aleph$ is defined by the stochastic kernel $P\left(x, B\right), x \in X, B \in \aleph$. A stationary distribution $\pi(B), B \in \aleph$ of the Markov process $x(t), t \ge 0$ is defined by the representation:

$$\pi(dx)q(x) = q\rho(dx), \ q = \int_X \pi(dx)q(x).$$

Lets denote by R_0 potential operator of the generator Q: $R_0 = \Pi - \left(\Pi + Q\right)^{-1}$, where $\Pi\varphi(x) = \int_X \pi(dy)\varphi(y)$ is the projector on the zeros subspace $N_Q = \left\{\varphi : Q\varphi = 0\right\}$ of the generator Q.

Impulsive Perturbation Process. An impulsive perturbation process (IPP) $\eta^{\varepsilon}(t), \ t \ge 0$ is defined by the representation [4]:

$$\eta^{\varepsilon}(t) = \int_0^t \eta^{\varepsilon}\left(ds; x\left(s/\varepsilon^4\right)\right); \tag{2}$$

where the family of independent increment processes $\eta^{\varepsilon}(t,x), \ t \ge 0, \ x \in X$ is defined by the generators:

$$\Gamma^{\varepsilon}(x)\varphi(w) = \varepsilon^{-4}\int_R \left[\varphi\left(w + \varepsilon^2 v\right) - \varphi(w)\right]\Gamma\left(dv; x\right), \ x \in X. \tag{3}$$

Generator (3) can be rewritten in the asymptotic form

$$\Gamma^{\varepsilon}(x)\varphi(w) = \varepsilon^{-2}\Gamma_1(x)\varphi(w) + \Gamma_2(x)\varphi(w) + \varepsilon^2\gamma^{\varepsilon}(x)\varphi(w),$$

where

$$\Gamma_1(x)\varphi(w) = b_1(x)\varphi'(w); \ b_1(x) = \int_R v\Gamma\left(dv; x\right) \tag{4}$$

$$\Gamma_2(x)\varphi(w) = \frac{1}{2}b_2(x)\varphi''(w); \ b_2(x) = \int_R v^2\Gamma\left(dv; x\right), \tag{5}$$

and the remaining term satisfies the condition $\left\|\varepsilon^2\gamma^\varepsilon(x)\varphi(w)\right\| \to 0$ while $\varepsilon \to 0$.

Let also the following balance conditions take place:

$$\Pi b_1(x) = \int_X \pi(dx) b_1(x) = 0;\ b_1(x) = \int_R v\Gamma\left(dv; x\right). \tag{6}$$

Stochastic Approximation Procedure behaviour. Let us consider the continuous SAP (1) convergence under the exponential stability conditions of the averaged evolution system:

$$\frac{d\widehat{u}(t)}{dt} = \widehat{C}\left(\widehat{u}(t)\right), \tag{7}$$

where

$$\widehat{C}(u) = \int_X \pi(dx) C(u, x).$$

Balance condition must be satisfied for the averaged evolution system equilibrium point u_0 existence:

$$\Pi C\left(u_0, x\right) = \int_X \pi(dx) C\left(u_0, x\right) = 0. \tag{8}$$

Without limiting the generality, further assume that $u_0 = 0$.

Theorem. Let there exists Lyapunov function $V(u) \in C^3(R)$ that provides the exponential stability of the averaged system (7):

$$C1 : \widehat{C}V(u) \le -cV(u),\ c > 0. \tag{9}$$

Also let the additional conditions hold true:

$$\begin{aligned}
&C2 : \left|\Gamma_2(x)V(u)\right| \le c_2\left(1 + V(u)\right),\ c_2 > 0,\\
&C3 : \left|\gamma^\varepsilon(x)V(u)\right| \le c_3\left(1 + V(u)\right),\ c_3 > 0,\\
&C4 : \left|\Gamma_1(x)R_0\Gamma_1(x)V(u)\right| \le c_4\left(1 + V(u)\right),\ c_4 > 0,\\
&C5 : \left|\Gamma_1(x)R_0\tilde{C}(x)V(u)\right| \le c_5\left(1 + V(u)\right),\ c_5 > 0,\\
&C6 : \left|C(x)R_0\Gamma_1(x)V(u)\right| \le c_6\left(1 + V(u)\right),\ c_6 > 0,\\
&C7 : \left|C(x)R_0\tilde{C}(x)V(u)\right| \le c_7\left(1 + V(u)\right),\ c_7 > 0,\\
&C8 : \left|\Gamma_2(x)R_0\Gamma_1(x)V(u)\right| \le c_8\left(1 + V(u)\right),\ c_8 > 0\\
&C9 : \left|\Gamma_2(x)R_0\tilde{C}(x)V(u)\right| \le c_9\left(1 + V(u)\right),\ c_9 > 0,\\
&C10 : \left|\gamma^\varepsilon(x)R_0\Gamma_1(x)V(u)\right| \le c_{10}\left(1 + V(u)\right),\ c_{10} > 0,\\
&C11 : \left|\gamma^\varepsilon(x)R_0\tilde{C}(x)V(u)\right| \le c_{11}\left(1 + V(u)\right),\ c_{11} > 0,
\end{aligned} \tag{10}$$

with

$$\tilde{\mathrm{C}}(x) = \mathrm{C}(x) - \mathrm{L}.$$

In addition let the function $C(u,x)$ has the third derivative on $u \in R$, and is uniformly bounded on $x \in X$. Furthermore, let the balance condition (6) takes place. Let also pick control function $a(t) > 0$ in such way that it satisfies the conditions [2]:

$$\int_0^\infty a(t)dt = \infty, \ \int_0^\infty a^2(t)dt < \infty.$$

Then the solution of the evolution equation (1) converges weakly (with probability 1) to the equilibrium point $u_0 = 0$ of the averaged system (7) for all initial values $u^\varepsilon(0) = u$ and all $\varepsilon < \varepsilon_0$, where ε_0 is small enough, i.e.:

$$P\left\{\lim_{t\to\infty} u^\varepsilon(t) = 0\right\} = 1.$$

The proof of the theorem is made with the help of the singular perturbation problem solution and martingale characterization of the two-component Markov process.

Let's consider Markov process

$$u^\varepsilon(t), \ x_t^\varepsilon := x\left(t/\varepsilon^4\right), \ t \ge 0. \tag{11}$$

This process is heterogeneous in time because the SAP $u^\varepsilon(t)$ (1) depends on the control function $a(t)$.

Lemma 1. A generator of the two-component Markov process $u^\varepsilon(t)$, $x\left(t/\varepsilon^4\right)$, $t \ge 0$ has the form

$$\mathrm{L}_t^\varepsilon(x)\varphi(u,x) = \varepsilon^{-4}\mathrm{Q}\varphi(u,x) + \varepsilon a(t)\Gamma^\varepsilon(x)\varphi(u,x) + a(t)\mathrm{C}(x)\varphi(u,x), \tag{12}$$

where $\mathrm{C}(x)\varphi(u,w,x) = C(u,x)\varphi'_u(u,x)$,

Proof. Let us denote $u^\varepsilon(t) = u_t$, $x\left(t/\varepsilon^4\right) = x_t$. Then according to the Markov process generator definition [5] we obtain:

$$\mathrm{L}_t^\varepsilon(x)\varphi(u,x) = \lim_{\Delta\to 0}\frac{1}{\Delta}E\Big[\varphi\Big(u^\varepsilon(t+\Delta), x\big((t+\Delta)/\varepsilon^4\big)\Big) -$$

$$-\varphi\Big(u^\varepsilon(t), x\big(t/\varepsilon^4\big)\Big) \mid u^\varepsilon(t) = u, x\big(t/\varepsilon^4\big) = x\Big] =$$

$$= \lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u_{t+\Delta}, x_{t+\Delta}\right) - \varphi(u,x)\right] =$$

$$= \lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u_{t+\Delta}, x_{t+\Delta}\right) - \varphi\left(u, x_{t+\Delta}\right) + \lim_{\Delta\to 0}\frac{1}{\Delta}E[\varphi\left(u, x_{t+\Delta}\right) - \varphi\left(u, x\right)\right]$$

According to (1): $u\left(t+\Delta\right)=u+a(t)C\left(u,x\right)\Delta+\varepsilon a(t)\Delta\eta^{\varepsilon}(t)+o(\Delta)$, therefore

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u_{t+\Delta},x_{t+\Delta}\right)-\varphi\left(u,x_{t+\Delta}\right)\right]=$$

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u+a(t)C(u,x)\Delta+\varepsilon a(t)d\eta^{\varepsilon}(t)+o(\Delta),x_{t+\Delta}\right)-\varphi\left(u,x_{t+\Delta}\right)\right]$$

Let us add and substitute $\varphi\left(u+a(t)C(u,x)\Delta+o(\Delta),x_{t+\Delta}\right)$ in the last limit. Thus we obtain:

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u_{t+\Delta},x_{t+\Delta}\right)-\varphi\left(u,x_{t+\Delta}\right)\right]=$$

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u+a(t)C(u,x)\Delta+\varepsilon a(t)d\eta^{\varepsilon}(t)+o(\Delta),x_{t+\Delta}\right)-\right.$$

$$\left.-\varphi\left(u+a(t)C(u,x)\Delta+o(\Delta),x_{t+\Delta}\right)\right]+$$

$$+\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u+a(t)C(u,x)\Delta+o(\Delta),x_{t+\Delta}\right)-\varphi\left(u,x_{t+\Delta}\right)\right]$$

According to the generator $\Gamma^{\varepsilon}(x)$ definition:

$$\Gamma^{\varepsilon}(x)\varphi(u)=\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u+\Delta\eta^{\varepsilon}(t)\right)-\varphi(u)\right].$$

Let's also denote $v=u+a(t)C(u,x)\Delta+o(\Delta)$ (in this case $v\to u$ while $\Delta\to 0$). As a result we obtain:

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(v+\varepsilon a(t)\Delta\eta^{\varepsilon}(t),x_{t+\Delta}\right)-\varphi\left(v,x_{t+\Delta}\right)\right]=\varepsilon a(t)\Gamma^{\varepsilon}(x)\varphi(u,x)$$

Then we can transform the second limit to the form

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u+a(t)C(u,x)\Delta,x_{t+\Delta}\right)-\varphi\left(u,x_{t+\Delta}\right)\right]=$$

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi_u'\left(u,x_{t+\Delta}\right)\left(a(t)C(u,x)\Delta+o(\Delta)\right)\right]=a(t)C(u,x)\varphi_u'(u,x).$$

So, using the process $x\left(t/\varepsilon^{4}\right)$ generator definition:

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E\left[\varphi\left(u,x_{t+\Delta}\right)-\varphi(u,x)\right]=\varepsilon^{-4}\mathrm{Q}\varphi(u,x),$$

we obtain (12).

Lemma 2. A generator $\mathrm{L}_t^{\varepsilon}(x)$ of the Markov process $u^{\varepsilon}(t)$, $x\left(t/\varepsilon^{4}\right)$, $t\geq 0$ can be rewritten in the asymptotic form

$$\mathrm{L}_{\mathrm{t}}^{\varepsilon}(x)\varphi(u,x) = \varepsilon^{-4}\mathrm{Q}\varphi(u,x) + \varepsilon^{-1}a(t)\Gamma_1(x)\varphi(u,x) + \\ +a(t)\mathrm{C}(x)\varphi(u,x) + \theta_t^{\varepsilon}(x)\varphi(u,x), \quad (13)$$

where

$$\theta_t^{\varepsilon}(x) = \varepsilon^2 a(t)\gamma^{\varepsilon} + \varepsilon a(t)\Gamma_2(x)\varphi(u,x),$$

and generators $\Gamma_1(x), \Gamma_2(x)$ are defined in (4) and (5) respectively.

Also the remaining term is such that $\left\|\theta_t^{\varepsilon}(x)(x)\varphi(u,x)\right\| \to 0$ while $\varepsilon \to 0$.

We can proof this lemma using the representation (3) of the generator $\Gamma^{\varepsilon}(x)$ and lemma 1.

Lemma 3. Under the conditions of the theorem, the singular perturbation problem for the operator (13) on the perturbed Lyapunov function

$$V_t^{\varepsilon}(u,x) = V(u) + \varepsilon^3 a(t)V_1(u,x) + \varepsilon^4 a(t)V_0(u,x)$$

has the solution in the form

$$\mathrm{L}_t^{\varepsilon}(x)V_t^{\varepsilon}(u,x) = a(t)\mathrm{L}V(u) + \varepsilon\theta_t^{\varepsilon}(x)V(u), \quad (14)$$

where the limit generator L is such that

$$\mathrm{L}V(u) \le -ca(t)V(u) \quad (15)$$

and the remaining term $\theta_t^{\varepsilon}(x)$ satisfies the inequality:

$$\left|\theta_t^{\varepsilon}(x)V(u)\right| \le c^* a^2(t)\left(1+V(u)\right).$$

Proof. Let us collect the similar terms with respect to ε in order to prove equality (14):

$$\mathrm{L}_{\mathrm{t}}^{\varepsilon}(x)V_t^{\varepsilon}(u,x) = \Big[\varepsilon^{-4}\mathrm{Q} + \varepsilon^{-1}a(t)\Gamma_1(x) + a(t)\mathrm{C}(x) + \\ +\varepsilon a(t)\Gamma_2(x) + \varepsilon^2 a(t)\gamma^{\varepsilon}(x)\Big] \\ \Big[V(u) + \varepsilon^3 a(t)V_1(u,x) + \varepsilon^4 a(t)V_0(u,x)\Big] = \\ = \varepsilon^{-4}\mathrm{Q}V(u) + \varepsilon^{-1}a(t)\big[\mathrm{Q}V_1(u,x) + \Gamma_1(x)V(u)\big] + \\ +a(t)\big[\mathrm{Q}V_0(u,x) + \mathrm{C}(x)V(u)\big] + \varepsilon a(t)\Gamma_2(x)V(u) + \\ +\varepsilon^2 a(t)\Big[a(t)\Gamma_1(x)V_1(u,x) + \gamma^{\varepsilon}(x)V(u)\Big] + \\ +\varepsilon^3 a(t)^2\big[\Gamma_1(x)V_0(u,x) + \mathrm{C}(x)V_1(u,x)\big] + \\ +\varepsilon^4 a(t)^2\big[\mathrm{C}(x)V_0(u,x) + \Gamma_2(x)V_1(u,x)\big] + \\ +\varepsilon^5 a(t)\Big[a(t)\Gamma_2(x)V_0(u,x) + \gamma^{\varepsilon}(x)V_1(u,x)\Big] + \varepsilon^6 a(t)\gamma^{\varepsilon}(x)V_0(u,x). \quad (16)$$

Since $\varphi(u,w)$ doesn't depend on x, then

$$\mathrm{Q}V(u)=0,\Leftrightarrow V(u)\in N_Q.$$

The following equation can be solved under the balance condition (6)

$$\mathrm{Q}V_1(u,x)+\Gamma_1(x)V(u)=0.$$

That is why

$$V_1(u,x)=R_0\Gamma_1(x)V(u).$$

We can obtain the limit process L using the solution condition of the last equation:

$$a(t)\mathrm{Q}V_0(u,x)+a(t)\mathrm{C}(x)V(u)=a(t)\mathrm{L}V(u).$$

Thus

$$\mathrm{L}V(u)=\Pi\mathrm{C}(x)V(u)=\Pi C(u,x)V'(u)$$

and then

$$V_0(u,x)=\left[a(t)\right]^{-1}R_0\left[a(t)\mathrm{C}(x)-a(t)\mathrm{L}\right]V(u)=R_0\tilde{\mathrm{C}}(x)V(u).$$

We can obtain $\theta_t^\varepsilon(x)$ from the rest of terms in (16):

$$\theta_t^\varepsilon(x)V(u)=a(t)\Gamma_2(x)V(u)+\varepsilon a(t)\left[a(t)\Gamma_1(x)V_1(u,x)+\gamma^\varepsilon(x)V(u)\right]+$$
$$+\varepsilon^2a(t)^2\left[\Gamma_1(x)V_0(u,x)+\mathrm{C}(x)V_1(u,x)\right]+$$
$$+\varepsilon^3a(t)^2\left[\mathrm{C}(x)V_0(u,x)+\Gamma_2(x)V_1(u,x)\right]+$$
$$+\varepsilon^4a(t)\left[a(t)\Gamma_2(x)V_0(u,x)+\gamma^\varepsilon(x)V_1(u,x)\right]+\varepsilon^5a(t)\gamma^\varepsilon(x)V_0(u,x),$$

and then, using the representation of the Lyapunov function perturbations $V_0(u,x)$ and $V_1(u,x)$, we can obtain

$$\theta_t^\varepsilon(x)V(u)=a(t)\Gamma_2(x)V(u)+\varepsilon a(t)\left[a(t)\Gamma_1(x)R_0\Gamma_1(x)V(u)+\gamma^\varepsilon(x)V(u)\right]+$$
$$+\varepsilon^2a(t)^2\left[\Gamma_1(x)R_0\tilde{\mathrm{C}}(x)V(u)+\mathrm{C}(x)R_0\Gamma_1(x)V(u)\right]+$$
$$+\varepsilon^3a(t)^2\left[\mathrm{C}(x)R_0\tilde{\mathrm{C}}(x)V(u)+\Gamma_2(x)R_0\Gamma_1(x)V(u)\right]+$$
$$+\varepsilon^4a(t)\left[a(t)\Gamma_2(x)R_0\tilde{\mathrm{C}}(x)V(u)+\gamma^\varepsilon(x)R_0\Gamma_1(x)V(u)\right]+$$
$$+\varepsilon^5a(t)\gamma^\varepsilon(x)R_0\tilde{\mathrm{C}}(x)V(u).$$

Let's denote by $\theta_0-\theta_5$ corresponding expressions near ε and $a(t)$

Since Lyapunov function $V(u)$ is smooth enough and under the condition $C2$, for $\theta_0=\Gamma_2(x)V(u)$ we obtain

$$|\,\theta_0\,|\le c_2\left(1+V(u)\right).$$

Second term $\theta_1=\Gamma_1(x)R_0\Gamma_1(x)V(u)+\gamma^\varepsilon(x)V(u)$ under the conditions $C3$ and $C4$ satisfies the inequality

$$|\theta_1| \le (c_3 + c_4)(1 + V(u)).$$

Using the theorem conditions $C5$ and $C6$, we can obtain the estimate for the term $\theta_2 = \Gamma_1(x) R_0 \mathrm{C}(x) V(u) + \mathrm{C}(x) R_0 \Gamma_1(x) V(u)$:

$$|\theta_2| \le (c_5 + c_6)(1 + V(u)).$$

Similarly for $\theta_3 = \mathrm{C}(x) R_0 \mathrm{C}(x) V(u) + \Gamma_2(x) R_0 \Gamma_1(x) V(u)$ (according to $C7$ and $C8$) we obtain:

$$|\theta_3| \le (c_7 + c_8)(1 + V(u)),$$

and for $\theta_4 = \Gamma_2(x) R_0 \mathrm{C}(x) V(u) + \gamma^{\varepsilon}(x) R_0 \Gamma_1(x) V(u)$ (according to $C9$ and $C10$) there is a similar estimate:

$$|\theta_4| \le (c_9 + c_{10})(1 + V(u)).$$

And, using the condition $C11$, we can bound $\theta_5 = \gamma^{\varepsilon}(x) R_0 \mathrm{C}(x) V(u)$ with:

$$|\theta_5| \le c_{11}(1 + V(u)).$$

Thus, for the remainder term $\theta_t^{\varepsilon}(x) V(u)$ cumulative estimate holds:

$$\left|\theta_t^{\varepsilon}(x) V(u)\right| \le c^* a^2(t)(1 + V(u)). \qquad (17)$$

Theorem proof. Using the condition $C1$ and inequality (17) we can obtain:

$$\mathrm{L}_{\mathrm{t}}^{\varepsilon}(x) V_t^{\varepsilon}(u, x) \le -ca(t) V(u) + c^* a^2(t)(1 + V(u)). \qquad (18)$$

Then the stochastic approximation procedure convergence follows from the inequality (18) and the theorem 1 in [6].

Conclusions. Stochastic approximation procedure converges weakly to the equilibrium point of the averaged system under the condition of the exponential stability of that system and additional limitations on the regression function smoothness. These results can be used to identify the SAP asymptotic behaviour [7; 8; 9]. Such SAP can also be used as a scratch for the stochastic optimization procedure (Kiefer-Wolfowitz method [10; 11]) in impulsive perturbations case.

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Розглянуто асимптотичну поведінку процедури стохастичної апроксимації для випадку, коли функція регресії збурена марковським імпульсним процесом. Одержано достатні умови збіжності процедури в умовах існування функції Ляпунова для усередненої еволюційної системи.

Ключові слова: *процедура стохастичної апроксимації, марковський процес, імпульсне збурення.*

Отримано: 23.04.2011