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# The Effect of Damping and Force Application Point on the Non-Linear Dynamic Behavior of a Cracked Beam at Sub- and Super-Resonance Vibrations*

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# Вплив демпфування і місця прикладення сили на нелінійну динамічну поведінку стрижня з тріщиною при суб- і супер-резонансних коливаннях

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*Представлено скінченноелементну модель пружного тіла з тріщиною, що закривається. Показано, що нелінійні спотворення коливань при супер- і субгармонічному резонансах є високочутливим проявом наявності тріщини. Інтенсивність цього прояву суттєво залежить від рівня демпфування в системі та від місця прикладення вимушеної сили.*

***Ключові слова***: тріщина, що закривається, демпфування, нелінійні резонанси, діагностика пошкодження.

**Introduction**. Fatigue cracks are the most widespread damage sources of dynamically loaded structural elements. Such cracks periodically close and open in the process of cyclic deformation of a body (and, for this reason, are often called “closing” or “breathing” one), leading to the instantaneous change of its stiffness. Usually the change of stiffness is modeled by the piece-wise linear characteristic of the restoring force [1, 2] or by the specific modification of the driving force [3].

A closing crack causes the dynamic behavior of vibrating system to be significantly non-linear, creating a series of fundamental difficulties with regard to determining analytical solutions. Numerical investigations of forced vibrations of beams with a closing crack [4–7] have demonstrated that the main distinctive features of such a vibration system are the manifestation of effects associated with non-linearity, namely the presence of sub- and superharmonic resonances and significant non-linearity of the vibration responses (displacement, acceleration,

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strain) at sub- and super-resonances. The sensitivity of these so-called non-linear effects to the crack presence exceeds many times (one or even two orders of magnitude) the sensitivity of natural frequencies and mode shapes, as well as the sensitivity of non-linear effects at resonance. This opens up the prospect of using sub- and super-resonance regimes for the diagnosis of fatigue cracks.

Furthermore, it has been shown in [8] that the manifestation of non-linear effects depend not only on the crack parameters (size and location), but also on the level of damping in a vibrating system. The data of direct experimental investigations [9–13] attest that the fatigue crack growth is accompanied by a considerable increase of damping characteristic of cracked specimens. Consequently, the determination of the relationship between the crack parameters and non-linear effects must be realized while taking into account the change of damping in a vibrating system, rather than assuming constant damping which has been the case in the past; if the increase in damping is neglected, the prediction of damage magnitude will be in error.

The aim of the present work has been to develop the model of a beam with a closing crack which takes into consideration the change of damping due to a crack growth; based on this model, an investigation has been performed into the relationship of non-linear effects on the crack parameters at sub- and super-resonance vibrations, as well as on the driving force application point.

**Model of a Cracked Beam with Account for Damping in a Crack**. The presented mathematical model of the cantilever beam with an edge transverse closing crack is based on the finite element model proposed in [5]. When the crack is closed and its interfaces are completely in contact with each other, the dynamic response can be determined directly as that one of the intact beam. However, when the crack is opened, the stiffness matrix of the cracked element should be introduced in replacement at the appropriate rows and columns of the general stiffness matrix. Under the action of the excitation force, crack opening and closing alternate in time, making the equations of motion of the cracked beam non-linear. Since exact solution of these equations does not exist, it was suggested that the system has the piecewise-linear characteristic of the restoring force. Then the vibration of a cracked beam was described by two linear differential equations in normalized coordinates:

$$\begin{cases} [I]\{\ddot{q}\} + [\Lambda]\{\dot{q}\} + [\omega^2]\{q\} = \{R\}F(t), \\ [I]\{\ddot{q}\} + [\Lambda_d]\{\dot{q}\} + [\omega_d^2]\{q\} = \{R_d\}F(t), \end{cases} \tag{1}$$

where the matrices of a beam with a closed crack are shown without subscripts and the matrices of a beam with an open crack are shown with subscript $d$ (damaged beam), $[I]$ is the mass matrix (unitary), $[\omega^2]$ is the stiffness matrix, $[\Lambda]$ is the damping matrix, $[R]$ is the external load vector, and $F(t)$ is the external load function. The first equation describes the vibration of a beam with a closed crack and the second one – of a beam with an open crack. These equations were solved with the Runge-Kutta method proceeding step-by-step in time.

The mesh with a local concentration ($L_c$) in an area of crack is used for the increase of the accuracy of a crack location determination (Fig. 1). To calculate the additional strain energy caused by the crack, the Cherepanov formula was used [14]:

$$K_{\mathrm{I}} = \frac{4.2M}{bh^{3/2}}[(1-\gamma)^{-3} - (1-\gamma)^{3}]^{1/2}, \tag{2}$$

where $M$ is the bending moment, $b$ and $h$ are the width and height of the beam cross-section, and $\gamma = a/h$ is the relative crack depth ($a$ is the crack depth).

The results of estimation of the reliability of the beam model presented in Table 1 indicate that the model can reliably predict the change of natural frequencies of the cracked beam.

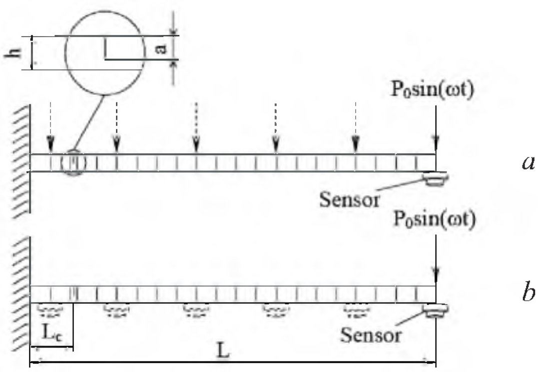


Fig. 1. The finite element mesh for the beam analyzed.

**Calculation Results**. Presented above model of a beam with a closing crack was used for the investigations of non-linear effects arising at forced vibrations of the beam. Geometrical and mechanical properties of the beam are the following: $L = 0.2$ m, $L_c/L = 0.1$ or 0.5, $h = 0.01$ m, $b = 0.01$ m, $E = 206$ GPa (modulus of elasticity), and $\rho = 7850$ kg/m$^3$ (density of the material). All calculations were performed for two levels of damping $\delta = 1\%$ and 10% ($\delta$ is logarithmic decrement of vibration).

As can be seen from Fig. 2, one of the manifestation of the non-linearity of the vibration response of the cantilever beam with a closing crack is the appearance of the set of non-linear resonances, namely superharmonic resonances of order 2/1 and 3/1 and subharmonic resonance of order 1/2. The amplitudes of non-linear resonances are 20–160 times less than the amplitudes of principal resonance at the damping level under study. This is why the reliable revealing of the non-linear resonances by the change of amplitude of vibration may cause a problem. In addition, the possibility of the detection of non-linear resonances depends on the resonant width. The calculated results show that the resonant width at the principal, subharmonic of order 1/2 and superharmonic of the odd orders resonances are comparable by the value, but at superharmonic resonances of even orders the resonant width is about by one order of magnitude less then at principal resonance. Consequently, the excitation of accurate superharmonic resonance of the order 2/1 is much more complex problem then the excitation of principal or subharmonic resonances.

T a b l e 1

**Evaluation of the Model Reliability by the Test Results**

| $L_c/L$ | $a/h$ | $f_c/f$ (experiment) | $f_c/f$ (prediction) | Distinction (%) |
|---|---|---|---|---|
| Steel 15Kh2NMFA (220× 13.8× 4 mm) [15] | | | | |
| 0.077 | 0.101 | 0.999 | 0.9980 | −0.06 |
| | 0.232 | 0.974 | 0.9810 | 0.67 |
| | 0.362 | 0.942 | 0.9450 | 0.34 |
| 0.155 | 0.244 | 0.988 | 0.9810 | −0.70 |
| | 0.347 | 0.969 | 0.9580 | −1.16 |
| | 0.486 | 0.882 | 0.9050 | 2.59 |
| 0.277 | 0.101 | 0.997 | 1.0003 | 0.33 |
| | 0.217 | 0.991 | 0.9910 | 0.03 |
| | 0.312 | 0.983 | 0.9780 | −0.55 |
| Steel ATSTS-1018 (330× 25× 25 mm) [16] | | | | |
| 0 | 0.200 | 0.924 | 0.9720 | 5.18 |
| | 0.400 | 0.871 | 0.8820 | 1.32 |
| | 0.600 | 0.725 | 0.7200 | −0.71 |
| 0.182 | 0.200 | 0.947 | 0.9840 | 3.89 |
| | 0.400 | 0.901 | 0.9280 | 2.99 |
| | 0.600 | 0.830 | 0.8070 | −2.81 |

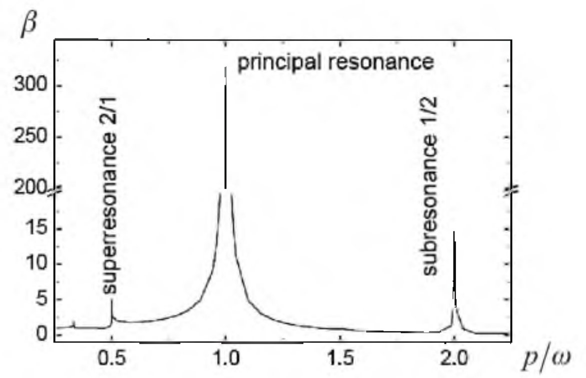


Fig. 2. Frequency-response function of the cracked beam ($a/h = 0.4$, $L_c/L = 0.1$, and $\delta = 1\%$).

The vibration responses at non-linear resonances are substantially non-harmonic. The reason of considerable non-linear distortions of vibration response at non-linear resonances is the fact that at these regimes the vibration range contains the harmonic, the frequency of which coincides with the frequency of the principal resonance. The amplitude of this harmonic is comparable at accurate non-linear resonance with the amplitude of the first harmonic. Therefore, below the ratio of the amplitude of dominating harmonic in the vibration range to the amplitude of the first harmonic is used as a characteristic of damage.

Two original procedures of damage diagnostics are proposed. They are based on the non-linear distortions of vibration response at non-linear resonances. In the first one, the force application point along the beam length is varied and the spectral analysis of the beam end vibrations is executed (Fig. 1a). In the second procedure, the driving force is applied only to the end of the beam and the data from the set of sensors located along the beam length are used for the spectral analysis (Fig. 1b).

As can be seen from Figs. 3 and 4, both procedures significantly respond to the crack appearance by the increase of the amplitude of dominating harmonic. Moreover, the first procedure distinctly reveals the crack location by the sharp local change of the amplitude of dominating harmonic.

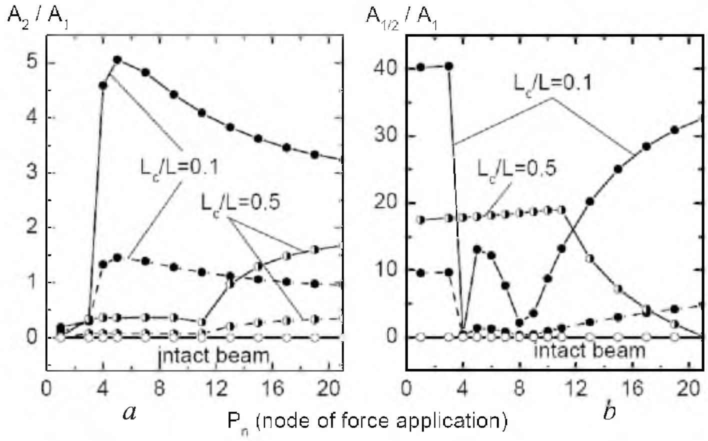


Fig. 3. The amplitudes of dominant harmonics vs. the coordinate of the force application point. Here and in Fig. 4: (*a*) super-resonance (2/1); (*b*) sub-resonance (1/2) ($\delta = 1\%$ – solid lines; $\delta = 10\%$ – dashed lines).

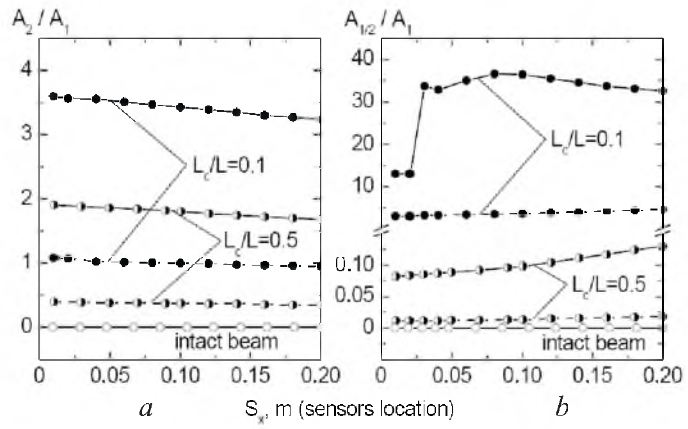


Fig. 4. The amplitudes of dominant harmonics vs. sensors location.

The damping suppresses the sensitivity of the presented damage characteristics. The increase by one order of magnitude of the damping characteristic leads to the drop harmonic at super-resonance up to 5 times and sub-resonance – up to 8 times.

As calculations showed, the qualitative distinction between the sub-resonance and the super-resonance regimes of vibrations takes place (Fig. 5). At sub-resonance the damage characteristic shows up only if the crack reaches a certain definite size. For instance, at damping level $\delta = 1\%$ subharmonic in the vibration range appears only in condition if the relative crack size will be more then $\gamma = 0.07$. Since that value the damage characteristic at subharmonic resonance shows high sensitivity to the crack presence. The increase of damage characteristic at super-resonance with the crack grows is less intensive but it takes place after the minimal crack values.

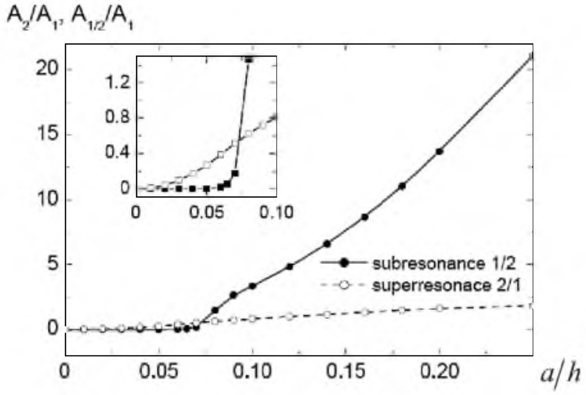


Fig. 5. Crack depth dependence of the dominating harmonics of range of vibrations at non-linear resonances of the beam ($L_c/L = 0.1$ and $\delta = 1\%$).

**Conclusions**. It was shown with the use of the FE model of a cracked beam that the non-linear effects at sub- and super-resonance vibrations of a cantilever beam are very sensitive to the presence of a closing crack. At the same time they are strongly dependent not only on the crack parameters, but on the level of damping in a vibrating system as well. The higher the level of damping, the lower is the manifestation of non-linear effects.

Two procedures of crack detection based on the non-linear effects' utilization are proposed. The first procedure, in which the force application points are varied and the sensor has only one location, makes it possible to determine both crack size and location. The second procedure, in which the set of sensors located along the beam is used and the driving force is applied to the beam end, makes it possible to detect the crack presence, but cannot be used for the estimation of the crack parameters.

## Резюме

Представлена конечноэлементная модель упругого тела с закрывающейся трещиной. Показано, что нелинейные искажения колебаний при супер- и субгармоническом резонансах являются высокочувствительным проявлением наличия трещин. Интенсивность этого проявления существенно зависит от уровня демпфирования в системе и места приложения вынуждающей силы.

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