https://doi.org/10.46813/2023-146-061 INVESTIGATION OF PARAMETERS OF ELECTRON AND POSITRON BUNCHES IN A PLASMA-DIELECTRIC WAKEFIELD ACCELERATOR

P.I. Markov, R.R. Kniaziev, G.V. Sotnikov National Science Center "Kharkov Institute of Physics and Technology", Kharkiv, Ukraine E-mail: pmarkov@kipt.kharkov.ua

The results of numerical PIC-simulation of the dynamics of accelerated positron and drive electron bunches under wake acceleration in a dielectric waveguide filled with plasma with a vacuum channel are presented. The wake field was excited by an electron bunch in a quartz dielectric tube inserted into a cylindrical metal waveguide. The inner region of the dielectric tube was filled with plasma with a vacuum channel along the waveguide axis. The difference in the energy and spatial characteristics, acceleration efficiency, emittance, and energy spread for positron and electron bunches is studied for different radii of the vacuum channel and two models of the plasma density dependence on the radius: a homogeneous and an inhomogeneous dependence characteristic of a capillary discharge.

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INTRODUCTION

Recently, to improve the bunch transport in dielectric wakefield accelerators (DWA) it was proposed to use the filling of the vacuum channel with plasma [1].

Subsequent more detailed studies (theoretical and experimental) showed that plasma-filled dielectric wakefield accelerators (PDWAs) make it possible to reduce the transverse size of the accelerated bunches [2 - 5] and suppress beam breakup instability [6].

Compact PDWAs with a higher acceleration gradient than conventional RF accelerators can find application in different areas of industry, technology, medicine, etc. However, if we consider the PDWA in relation to high energy physics as a possible candidate for future electron-positron colliders, it is necessary that the quality of accelerated bunches meet the collider requirements [7 - 12]. The quality of accelerated bunches is characterized by the following characteristics: emittance and energy spread. We have already begun to study these characteristics for positron bunches in the PDWA with an axial vacuum channel inside the plasma column. We started with the study of the transport of positron bunches [13], because this is a more challengeable task than the transport of electron bunches. This problem is similar to that of the PWFA method and a vacuum channel is used to mitigate it [14].

In present paper we continue our research of acceleration of positron bunch in PDWA [13]. We have increased the acceleration length and in more detail investigated dynamic characteristics of accelerated positron and drive electron bunches (emittance, energy spread, efficiency of energy transfer from the drive electron bunch to the witness positron bunch) to see if these stats could saturate with acceleration time. In addition, we investigate the dependences from the size of the vacuum channel.

The article is organized as follows. Section 1 presents the statement of the problem. The results of the 2.5-dimensional PIC-simulation of positron acceleration in PDWA are presented in Section 2. In Section 3 characteristics of the accelerated positron bunch (emittance, energy spread and efficiency) are given. In Conclusions are summarized main results.

1. THE PROBLEM DEFINITION

The statement of the problem is the next. A dielectric tube with inner radius *a* and outer *b*, inserted into cylindrical metal waveguide; the internal area of dielectric tube is filled with annular plasma with inner radius r_{p1} and outer radius *a*. Thus, the paraxial region $r < r_{p1}$ is a vacuum channel. The cylindrically shaped drive electron bunch of radius r_{b1} pass through the slowing-down structure along its axis and excite a wakefield. After the delay time t_{del} following the drive bunch, positron bunch with an absolute value of charge much smaller than that of the drive bunch, is injected in the system along its axis. The radius of positron bunch is r_{b2} . The plasma-filled structure with the drive electron bunch and the witness positron bunch is the plasma dielectric wakefield accelerator of positrons (PDWAP).

For numerical simulation of drive electron and test positron bunch dynamics we used own 2.5D particle-incell (PIC) code [1]. The parameters of the structure and bunches, used in the simulations are specified in Table.

The parameters of the waveguide, drive and witness (test) bunches, employed in PDWAP calculations

Inner radius of dielectric tube, a	0.5 mm
Outer radius of dielectric tube, b	0.6 mm
Inner plasma cylinder radius, r_{p1}	00.5 mm
Waveguide length, L	80 mm
Dielectric permittivity, ɛ	3.75
Bunch energy, E_0	5 GeV
Drive electron bunch charge	-3 nC
Witness (test) positron bunch charge	0.05 nC
Longitudinal rms deviation of drive bunch	0.1 mm
charge, $2\sigma_1$ (Gauss charge distribution)	
Longitudinal rms deviation of positron bunch	0.05 mm
charge, $2\sigma_2$ (Gauss charge distribution)	
Total drive bunch length in PIC simulation	0.2 mm
Total positron bunch length in PIC simulation	0.1 mm
Drive bunch diameter, $2r_{bl}$	0.9 mm
Positron bunch diameter, $2r_{b2}$	0.7 mm
Paraxial plasma density (n_{p0}) when $r_{p1} = 0$	$2 \cdot 10^{14} \text{cm}^{-3}$



Fig. 1. Models of plasma density-radius relationship $n_p(r)$ for two cases: the plasma fills completely the interior of the dielectric tube (dash lines) and the plasma cylinder of internal radius $r_{pl} = 0.2 \text{ mm}$ (solid lines).

Red line (2) corresponds to the inhomogeneous distribution of plasma density [15], cyan line (1) is for the homogeneous one

When simulating plasma two different models of the plasma density-radius relationship $n_p(r)$ are studied: 1) the homogeneous model and 2) the inhomogeneous dependence, characteristic to the capillary discharge [15]. In the general case, there is a vacuum channel inside the plasma. At the vacuum-plasma boundary at $r = r_{p1}$ the stepwise behavior of $n_p(r)$ as a functions of radius r is assumed. Examples of these dependencies for the case $r_{p1} = 0.2$ mm and $r_{p1} = 0$ (no vacuum channel) are shown in Fig. 1. A homogeneous profile of a hollow core plasma channel can easily be obtained by the high order Bessel beams [14, 16]. As for a hollow core plasma from the capillary discharge, a coaxial geometry of the DWA structure with two dielectric tubes, can be suitable. The wall thickness of the inner coaxial has to be significantly smaller than the wall thickness of the outer capillary, so that it does not change the frequency of the main accelerating mode.

2. RESULTS OF 2.5-DIMENSIONAL PIC-SIMULATION

Dependence of test bunch radius on the inner plasma tube radius from 0 to 0.5 mm at time t = 266.9 ps is shown in Fig. 2 (top panel).

As appears from the curves shown in Fig. 2 for the homogeneous plasma distribution as r_{p1} increases from 0 to 0.25 mm, the gradual increase in the test positron bunch focusing is observed. With a further r_{p1} increase to 0.35 mm (a positron bunch radius), the focusing slightly reduces. Further increase of r_{p1} leads to a drastic degradation in the test bunch focusing. If $r_{p1} \ge 0.4$ mm the test bunch focusing is practically absent. In case of inhomogeneous transverse profile of plasma density, a gradual increase in the test positron bunch focusing is observed with an r_{p1} increase from 0 to 0.37 mm (see Fig. 2). It should be noted that the behavior of bunch radius dependencies differ a slightly from presented in paper [13], especially for the case of homogeneous plasma density distribution in drift channel.





Lower panel of Fig. 2 show the energy gain ΔE of the test bunch (blue curve) and the slowed-down drive bunch (red curve) as function of the inner plasma-tube radius r_{p1} . With r_{p1} increase from 0 to 0.25 mm, a small energy gain ΔE reduction of the test bunch is observed. The further r_{p1} increase from 0.25 to 0.5 mm leads to ΔE increase of the test bunch. The dependence of drive bunch energy loss is given for homogeneous plasma density case, for the inhomogeneous plasma density case in drift channel, an obtained dependence is practically coincides with the depicted one.

3. CHARACTERISTICS OF ACCELERATED POSITRON AND DRIVE ELECTRON BUNCHES

Here we study the characteristics of the accelerated positron and drive electron bunches. Figs. 3-5 show the efficiency of energy transfer to the positron bunch from the drive electron bunch, the trace-space emittance of the positron and electron bunches, and their energy spread. For the calculation of the efficiency of energy transfer of drive bunch to the witness bunch, we used the following equation:

$$\eta_{ir} = \frac{\left|\sum_{i=1}^{N_{ac}} m_i^{ac} \left(\gamma_i^{ac} - \gamma_{0i}^{ac}\right)\right|}{\sum_{i=1}^{N_{dr}} m_i^{dr} \left(\gamma_i^{dr} - \gamma_{0i}^{dr}\right)}\right| \cdot 100\% ,$$

where m_i^{ac} and m_i^{dr} are the masses of macroparticles of the witness bunch and the drive bunch; γ_i^{ac} and γ_i^{dr} are the relativistic factors of the macroparticles of the witness bunch and the drive bunch; γ_{0i}^{ac} and γ_{0i}^{dr} their initial values; N_{ac} and N_{dr} are the number of macroparticles in the witness bunch and the drive bunch.

The maximum efficiency was about 3.21% and achieved when the drift channel was completely free from plasma. The minimum of efficiency was achieved when r_{n1} provide the best positron bunch focusing what

can be seen in Fig. 3, c, d: in the inhomogeneous case it is $\approx 1.92\%$; in the homogeneous case it is $\approx 1.90\%$. In the presence of a vacuum channel, the efficiency is smaller, that is associated with a smaller value of the accelerating field.

Fig. 4 shows the change in the trace-space emittance [17] of positron and electron bunches as positron bunch accelerates in the PDWA. For a cylindrically symmetric on-axis beam, it is calculated by the formulas [18]:



Fig. 3. Transfer energy efficiency η_{tr} from the drive electron bunch to the witness positron bunch versus time: a, c correspond the inhomogeneous plasma density; b, d – the homogeneous plasma density in the drift channel; a, b are the 2D charts; c, d – 3D charts with r_{p1} as the depth axis. Time t = 0 corresponds to the beginning of the injection of the drive electron bunch

As follows from Fig. 4 trace-space emittance of the positron beam increases during acceleration in the PDWA, but after travelling ≈ 70 mm it is increased to acceptable values. The smallest value of the emittance $7.9 \cdot 10^{-5}$ mm·mrad is realized for the case when the drift channel is completely free from plasma. The largest value of the emittance is obtained in the case of best positron bunch focusing what can be seen in Fig. 4, d, e: in the inhomogeneous plasma density in the drift channel case it is $4.51 \cdot 10^{-2}$ mm·mrad, in the homogeneous case it is $4.65 \cdot 10^{-2}$ mm·mrad. It can be seen that even for this non-optimal case, there is already a tendency to

reach saturation. Extrapolating the above curves with the parabolic quadratic dependencies, we obtain that in the case of homogeneous plasma the emittance at the maximum is $\approx 4.67 \cdot 10^{-3}$ mm·mrad and is achieved at an acceleration length of ≈ 94 mm, and in the case of inhomogeneous plasma, the emittance at the maximum is $\approx 4.6 \cdot 10^{-2}$ mm·mrad and is achieved at the acceleration length of ≈ 150 mm. In the case of incomplete filling the trace-space emittances turn out to be higher, both in the homogeneous and the inhomogeneous cases of plasma, and their value are approximately the same.

where

$$\left\langle r^{2} \right\rangle = \frac{1}{Q} \sum_{i=1}^{N} q_{i} r_{i}^{2}, \ \left\langle r \cdot r' \right\rangle = \frac{1}{Q} \sum_{i=1}^{N} q_{i} r_{i} \frac{v_{ri}}{v_{zi}},$$
$$\left\langle \left(r'\right)^{2} \right\rangle = \frac{1}{Q} \sum_{i=1}^{N} q_{i} \left(r_{i}'\right)^{2} = \frac{1}{Q} \sum_{i=1}^{N} q_{i} \left(v_{ri}/v_{zi}\right)^{2}$$

 q_i, r_i, v_{ri}, v_{zi} are charge, radius, radial and longitudinal macroparticle velocity, Q is total bunch charge, N is number of macroparticles in the bunch.



Fig. 4. Trace-space emittance of the witness positron bunch (a, b, d, e) and drive electron bunch (c, f) versus time: a, d correspond the inhomogeneous plasma density; b, e - the homogeneous plasma density in the drift channel; a, b, c are the 2D charts; d, e, f - 3D charts with r_{pl} as the depth axis

Trace-space emittance of electron driver is significantly smaller than emittance of positron witness and reach $6.42 \cdot 10^{-3}$ mm·mrad value at $r_{p1} = 0$. In our case the saturation of drive electron bunch emittance occurs when plasma leaves a bunch boundary. However, the deterioration of the emittance of the drive bunch is not yet critical for continuing the process of positron bunch acceleration. In Fig. 4, d, e, f one can see that when plasma leaves a bunch boundary the trace-space emittances sharply decreases to values $\approx 7.87 \cdot 10^{-5}$ mm·mrad for positron witness (see Fig. 4, a, b, d, e) and $\approx 3.36 \cdot 10^{-4}$ mm·mrad for electron driver (see Fig. 4, c, f).

In Fig. 5 is shown the change of the energy spread of the positron and electron bunches when positron bunch accelerates along the PDWA.



Fig. 5. Energy spread of the witness positron bunch (a, b, d, f) and driver electron bunch (c, f) versus time: a, d correspond the inhomogeneous plasma density; b, e - the homogeneous plasma density in the drift channel; a, b, c are the 2D charts; d, e, f - 3D charts with r_{p1} as the depth axis

Energy spread in our simulations for unequally weighted macroparticles is computed using the next equation:

$$\sigma_{\delta}(\% RMS) = (\Delta E_{RMS} / \langle E \rangle) \cdot 100$$

where

$$\begin{split} \Delta E_{RMS} = \sqrt{\left\langle E^2 \right\rangle - \left\langle E \right\rangle^2}, \ \left\langle E^2 \right\rangle = \frac{1}{Q} \sum_{i=1}^N q_i E_i^2 \\ \left\langle E \right\rangle = \frac{1}{Q} \sum_{i=1}^N q_i E_i, \ E_i = \gamma_i - 1, \end{split}$$

 γ_i is the relativistic factor of the macroparticle.

Changes in the energy spread of the positron bunch during its acceleration are shown in the Fig. 5, a, d and Fig. 5, b, e. The smallest energy spread is realized in the case of complete filling of the drift channel of the dielectric structure with plasma and is equal to 0.01% for the case of the capillary discharge and 0.012% for the homogenous plasma case. These values are significantly lower than the requirements for collider applications [7, 18]. The spread has not yet reached saturation, but there is already a tendency for it to saturate. At the maximum using the parabolic extrapolation we obtain the energy spread of $\sim 0.015\%$ for the case of the capillary discharge.

The largest energy spread is realized in the case when $r_{p1} = 0.375$ mm, i.e. when radius of vacuum tube is greater then positron bunch radius and is equal to 0.024% for the case of the capillary discharge and 0.022% for the homogenous plasma case. At further increase in r_{p1} the energy spread sharply falls down to the minimum value.

Changes in the energy spread of the electron bunch at its motion in the drift chamber is shown in the Fig. 5, c, d. Practically, $\sigma_{\delta}(t)$ represents linear relation from time t. Dependence of $\sigma_{\delta}(r_{p1})$ on the vacuum cylinder radius r_{p1} is weak. The largest driver energy spread is realized in the case of complete filling of the drift channel of the dielectric structure with plasma and is equal to 0.13%. The smallest driver energy spread is realized in the case when the drift channel was completely free from plasma and is equal to 0.11%.

CONCLUSIONS

In this article, the main attention is paid to the study of the characteristics of accelerated positron and driver electron bunches by the numerical PIC simulation of wake excitation and self-consistent dynamics of charged particles in a plasma-dielectric slow-wave structure of the THz-band with both a homogeneous model plasma and an inhomogeneous plasma generated by a capillary discharge in waveguide.

We also showed that the presence of plasma in the system leads to focusing of an accelerated positron bunch, and the presence of a paraxial vacuum channel increases focusing by 9.25% in the case of a homogeneous plasma model and by 17.12% in the case of inhomogeneous plasma. In the absence of plasma, there is no focusing of the positron bunch, and the increase in positron energy gain is 36.8 and 34.6% higher than in the presence of homogeneous and inhomogeneous plasma, respectively, and the highest focusing of the bunch.

A study of the energy transfer efficiency from an electron beam to a positron one showed that the maxi-

mum efficiency of 3.21% is achieved in the plasma absence and the minimum is 1.90 and 1.92% in the homogeneous and inhomogeneous plasma presence, respectively, and the highest focusing of the bunch.

An analysis of the emittance showed that the presence of plasma in the system significantly increases the emittance of the beams, and the presence of a vacuum channel increases the emittance of the positron bunch even more. Thus, for a vacuum channel corresponding to maximum focusing, the positron beam emittance values $4.65 \cdot 10^{-2}$ mm·mrad reaches and $4.51 \cdot 10^{-2}$ mm·mrad in the presence of homogeneous and inhomogeneous plasma, respectively, while in the absence of a vacuum channel, the emittance decreases to $3.85 \cdot 10^{-3}$ mm·mrad and $1.29 \cdot 10^{-2}$ mm·mrad for the studied plasma models, respectively. In the absence of plasma, the value of the maximum emittance of the positron beam is $7.87 \cdot 10^{-5}$ mm·mrad.

At the same time, the presence of a vacuum channel in the plasma leads to a decrease in the emittance of the drive electron bunch. When the drift channel is completely filled with plasma, the emittance of the electron bunch is $6.42 \cdot 10^{-3}$ mm·mrad, and in the plasma absence case, the emittance drops to $\approx 3.36 \cdot 10^{-4}$ mm·mrad. The greatest change in the emittance is observed when the vacuum channel radius is close to the radius of the charged particles bunch.

The presence of a vacuum channel also leads to an increase in the energy spread of the positron bunch particles. Thus, when the drift channel is completely filled with plasma, the scatter is 0.01%, while for a vacuum channel corresponding to maximum focusing, the energy spread increases to 0.022 and 0.024% in the cases of homogeneous and inhomogeneous plasma, respectively. The filling of the drift channel with plasma and the presence of a vacuum channel in the plasma have a much smaller effect on the energy spread of the driver electron bunch particles than on the positron bunch spread. If, when the drift channel is completely filled with plasma, the energy spread of electrons is 0.13%, then in the plasma absence case, the spread decreases to 0.11%.

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ДОСЛІДЖЕННЯ ПАРАМЕТРІВ ЕЛЕКТРОННИХ І ПОЗИТРОННИХ ЗГУСТКІВ У ПЛАЗМОВО-ДІЕЛЕКТРИЧНОМУ КІЛЬВАТЕРНОМУ ПРИСКОРЮВАЧІ

П.І. Марков, Р.Р. Князєв, Г.В. Сотніков

Наведено результати чисельного PIC-моделювання динаміки прискореного позитронного та драйверного електронного згустків при кільватерному прискоренні в діелектричному хвилеводі, заповненому плазмою з вакуумним каналом. Кільватерне поле збуджувалося електронним згустком у кварцовій діелектричній трубці, вставленій у циліндричний металевий хвилевід. Внутрішня область діелектричної трубки була заповнена плазмою з вакуумним каналом вздовж осі хвилеводу. Досліджено відмінність в енергетичних та просторових характеристиках, ефективності прискорення, емітансі та енергетичному розкиді для позитронного та електронного згустків при різних радіусах вакуумного каналу та двох моделях залежності щільності плазми від радіуса: однорідної та неоднорідної залежності характерної для капілярного розряду.