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FEATURES OF NEW CYCLOTRON RESONANCES, AS WELL AS CONDITIONS FOR RESONANT ACCELERATION OF CHARGED PARTICLES IN A VACUUM WITHOUT A MAGNETIC FIELD

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It is shown that the known conditions for cyclotron resonances are strictly valid only under autoresonance conditions or in the nonrelativistic case. In other cases, it is necessary to use the conditions established in the present work. The main features of charged particle dynamics under new resonant conditions are presented. Conditions are found for infinite acceleration of electrons by a transverse electromagnetic wave in a vacuum without a magnetic field.

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INTRODUCTION

In plasma physics and plasma electronics, two types of fundamental interaction processes play important role. These are the wave-particle and of wave-wave interactions. Below we will explore the wave-particle process in which resonances are of great importance. First of all, this concerns the Cherenkov resonance and cyclotron resonances. These resonances are the most widely used. Obviously, it is very important for applications to increase the intensity of fields interacting with particles. The main parameter that characterizes the level of interacting fields is the wave strength parameter $\varepsilon = eE/mc\omega$ (nonlinear parameter). Usually, it is assumed that this parameter is small ($\varepsilon < 1$). Indeed, this parameter would be of the order of unit in the ten-centimeter range, if the intensity of electromagnetic fields is about 10^5 V/cm. For laser radiation, this intensity should exceed 10^{10} V/cm. So, to describe the interaction with the field of such it is necessary to take into account nonlinear effects. Note that the generally accepted conditions for cyclotron resonances contain only the strength of the external magnetic field. In [1], the nonlinear particle dynamics was taken into account. The conditions for cyclotron resonances were formulated, which explicitly contain the parameter of the wave strength. The papers [1, 2] describe a large number of new features of particle dynamics in high-amplitude fields. The purpose of this work is to generalize the obtained results. In this case, the main attention is paid to identifying the conditions for the possibility of reducing the field strength to have particle acceleration. Physical considerations indicate that this can be achieved in the region of parameters that corresponds to autoresonance. In [1, 2] a wave with only one polarization was considered. In this paper, another polarization for cyclotron resonances is considered, and the results for wave-particle interaction for two polarizations will be presented.

The work consists of Introduction, three sections and Conclusions. In the Section 1, we formulate the statement of the problem and basic equations. In the Section 2, new variables are introduced and conditions for new cyclotron resonances are formulated in more general case than in [1]. It is shown that the main features of resonances are the same for the fields of different polarizations. It is shown that the known conditions of cyclotron resonances are strictly valid only for autoresonances and for the case of nonrelativistic motion of particles ($\gamma \to 1$). In all other cases, the new conditions should be used. The Section 3 describes new variables that made it possible to find the conditions for unlimited acceleration of charged particles (electrons) by a laser field in a vacuum without an external magnetic field. In Conclusions, the most important results of the work are formulated.

1. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Consider a charged particle that moves in the stationary homogeneous external magnetic field and in the field of a plane electromagnetic wave that in the general case has the components

$$\mathbf{E} = \mathrm{Re}(E\boldsymbol{\alpha}\exp(i\omega t - i\mathbf{kr})),$$
$$\mathbf{H} = \mathrm{Re}\left(\frac{c}{\omega}[\mathbf{kE}]\exp(i\omega t - i\mathbf{kr})\right), \tag{1}$$

where $\mathbf{E} = E_0\boldsymbol{\alpha}$, $\boldsymbol{\alpha} = \{\alpha_x, i\alpha_y, \alpha_z\}$ is the wave polarization vector.

We choose a coordinate system in which the wave vector of the wave has only two components k_x and k_z. It is also convenient to use the dimensionless dependent and independent variables

$$\mathbf{p} \to \mathbf{p}/mc, \ \tau \to \omega t, \ \mathbf{r} \to \frac{\omega}{c}\mathbf{r}, \quad \psi = \tau - \mathbf{kr}.$$

The equations of motion in terms of these variables are given by

$$\frac{d\mathbf{p}}{d\tau} = \left(1 - \frac{\mathbf{kp}}{\gamma}\right)\mathrm{Re}\left(\boldsymbol{\varepsilon}e^{i\psi}\right) + \frac{\omega_H}{\gamma}[\mathbf{ph}] + \frac{\mathbf{k}}{\gamma}\mathrm{Re}\left[(\boldsymbol{\varepsilon}\cdot\mathbf{p})e^{i\psi}\right],$$
$$\mathbf{v} = \frac{d\mathbf{r}}{d\tau} = \frac{\mathbf{p}}{\gamma}, \quad \dot{\psi} = \frac{d\psi}{d\tau} = 1 - \frac{\mathbf{kp}}{\gamma}, \tag{2}$$

where $\mathbf{h}=\mathbf{H}/H_0$, $\omega_H=eH_0/mc\omega$, $\boldsymbol{\varepsilon}=\varepsilon_0\boldsymbol{\alpha}$, $\varepsilon_0=(eE_0/mc\omega)$, $\psi=\tau-\mathbf{kr}$, $\mathbf{k}$ is the unit vector in the direction of the wave vector, $\gamma=(1+\mathbf{p}^2)^{1/2}$ is the particle's dimensionless energy (measured in the units of mc^2), $\mathbf{p}$ is the momentum of the particle, $\mathbf{H_0}$ is the external magnetic field directed along the z-axis.

Multiplying the first equation of the set (2) by **p**, we obtain a useful equation that describes the change in particle energy, i.e.

$$\dot{\gamma}=\mathrm{Re}\left(\mathbf{v}\boldsymbol{\mathcal{E}}e^{i\psi}\right). \quad (3)$$

Equations (2) and (3) have integrals given by

$$\begin{aligned}&\mathbf{p}+\mathrm{Re}\left(i\boldsymbol{\varepsilon}e^{i\psi}\right)-\omega_H\left[\mathbf{rh}\right]-\mathbf{k}\gamma=\\&=\mathbf{p_0}-\mathbf{k}\gamma_0+\mathrm{Re}\left(i\boldsymbol{\varepsilon}e^{i\psi_0}\right)\text{-}\omega_H\left[\mathbf{r_0h}\right]=\text{const.}\end{aligned} \quad (4)$$

Here the subscript "0" denotes the values of the initial variables.

2. CYCLOTRON RESONANCES

The set of vector equations (2), (3), even taking into account the integrals (4), can be fully analyzed only by numerical methods. However, many important features of charged particle dynamics can be discovered using new variables

$$\begin{aligned}&p_x=p_\perp\cos\theta,\ p_y=p_\perp\sin\theta,\ p_z=p_\parallel,\\&x=\xi-\frac{p_\perp}{\omega_H\gamma}\sin\theta,\ y=\eta+\frac{p_\perp}{\omega_H\gamma}\cos\theta.\end{aligned} \quad (5)$$

An analysis of the general case leads to the need to analyze very cumbersome and complex systems of equations. In order to simplify expressions obtained for analysis, below we consider the dynamics of particle motion in the field of a polarized wave. Many features of particle dynamics in the field E polarization were described in [1, 2]. Below we will consider the dynamics of particles in the field H polarized wave $\left(E_x,E_z,H_y\right)$. We will assume that the wave vector has the following components (k_x,k_z). In view of the above, the complete system of equations that describe the dynamics of particles in new variables can be reduced to the following

$$\frac{dp_x}{d\tau}=\frac{\omega_H}{\gamma}p_y+\left\{\begin{aligned}&\left(1-\frac{\mathbf{kp}}{\gamma}\right)(\varepsilon_x)+\\&+\frac{k_x}{\gamma}\left[\left(\varepsilon_x\cdot p_x+\varepsilon_z\cdot p_z\right)\right]\end{aligned}\right\}\cos\psi,$$

$$\frac{dp_y}{d\tau}=-\frac{\omega_H}{\gamma}p_xh_0; \quad (6)$$

$$\frac{dp_z}{d\tau}=\left\{\left(1-\frac{\mathbf{kp}}{\gamma}\right)(\varepsilon_z)+\frac{k_z}{\gamma}\left[\left(\varepsilon_x\cdot p_x+\varepsilon_z\cdot p_z\right)\right]\right\}\cos\psi;$$

$$\dot{x}=v_x=\frac{p_x}{\gamma};\ \dot{y}=v_y=\frac{p_y}{\gamma};\ \dot{z}=v_z=\frac{p_z}{\gamma};\ \dot{\psi}=1-\frac{\mathbf{kp}}{\gamma}.$$

To find the conditions for the resonant interaction of waves with particles (the conditions for cyclotron resonances), we use the following expansion of functions into series in terms of Bessel functions [3]:

$$\exp(\pm i\mu\sin\theta)=\sum_{n=-\infty}^{\infty}J_n(\mu)\exp(\pm in\theta). \quad (7)$$

Taking this expansion into account, the system of equations (6) can be rewritten

$$\dot{p}_\perp=\left\{\begin{aligned}&\left(1-\frac{1}{\gamma}\left(k_xp_\perp\cos\theta+k_zp_z\right)\right)\varepsilon_x+\\&+\frac{k_x}{\gamma}\left[\varepsilon_x\cdot p_\perp\cos\theta+\varepsilon_z\cdot p_z\right]\end{aligned}\right\}\cos\theta\cos\psi; \quad (8)$$

$$\dot{p}_z=\left\{\left(1-k_xv_x-k_zv_z\right)\left(\varepsilon_z\right)+\frac{k_z}{\gamma}\left[\left(\varepsilon_x\cdot p_x+\varepsilon_z\cdot p_z\right)\right]\right\}\cos\psi;$$

$$\dot{\xi}=v_\perp(1-\frac{1}{\gamma})\cos\theta-\frac{p_\perp}{\omega_H\gamma}\left[..\varepsilon..\right]_\theta\cos\theta+\frac{\dot{v}_\perp}{\omega_H}\sin\theta;$$

$$\dot{\theta}=-\frac{\omega_H}{\gamma}-\left[..\varepsilon..\right]_\theta,$$

where $\cos\psi=\sum_{n=-\infty}^{\infty}J_n(\mu)\cos(\varphi+n\theta)$;

$$\left[..\varepsilon..\right]_\theta=\frac{1}{p_\perp}\left[\begin{aligned}&\left(1-\frac{1}{\gamma}\left(k_xp_\perp\cos\theta+k_zp_z\right)\right)\varepsilon_x+\\&+\frac{k_x}{\gamma}\left(\varepsilon_x\cdot p_\perp\cos\theta+\varepsilon_z\cdot p_z\right)\end{aligned}\right]\sin\theta\cos\psi.$$

The cyclotron resonance conditions will be as the stationarity conditions for the phase of one term from the sum on the right side of system (8):

$$\theta_n=\tau-k_zz-k_x\xi+\left(n\pm1\right)\theta=const, \quad (9)$$

$$\begin{aligned}&\dot{\theta}_n=0=\left(1-k_zv_z-\left(n\pm1\right)\frac{\omega_H}{\gamma}\right)-\\&-k_xv_\perp\left(1-1/\gamma\right)\cos\theta-\left(n\pm1\right)\left[..\varepsilon..\right]_\theta+\frac{k_x\dot{v}_\perp}{\omega_H}\sin\theta.\end{aligned}$$

It is convenient to rewrite the resulting expression in the form of three terms:

$$\dot{\theta}_n=\Delta_0+\Delta_1+\Delta_2=0, \quad (10)$$

where $\Delta_0=\left(1-k_zv_z-n\frac{\omega_H}{\gamma}\right)$;

$\Delta_1=-k_xv_\perp\left(1-1/\gamma\right)\cos\theta$; $\Delta_1=-k_xv_\perp\left(1-1/\gamma\right)\cos\theta$;

$\Delta_2=-n\left[..\varepsilon..\right]_\theta+\frac{k_x\dot{v}_\perp}{\omega_H}\sin\theta$.

The first term in expression (10) describes the usual condition for cyclotron resonance. The last term describes the role of the electric intensity of the external wave ($\Delta_2\approx\varepsilon$). Note that the second term in Δ_2 can be omitted in almost all cases of interest to us. The middle term (Δ_1), as will be seen below, determines the characteristic features of the particle dynamics on the steps. In addition, the presence of this term significantly changes the cyclotron resonance conditions even when the wave strength parameter can be neglected ($\varepsilon\to0$). The presence of this term indicates the fact that the commonly used widely used cyclotron resonance conditions (the first term in (10) $\Delta_0=0$) are strictly valid only under the conditions of cyclotron autoresonance ($k_x=0$) or in nonrelativistic case ($\gamma\to1$). In all other cases, this term can significantly change the resonance conditions. In particular, this term can lead to a limitation of the energy transferred from the wave to the particles.

2.1. NUMERICAL INVESTIGATION

The analytical results described above point to a significant difference between the dynamics of particles at cyclotron resonances and the usual dynamics of particles at these resonances. However, these results were obtained using a fairly large number of large and small parameters. Therefore, it is of considerable interest to find out how these results look when numerically solving the original equations not transformed to new variables. Such decisions have been made.

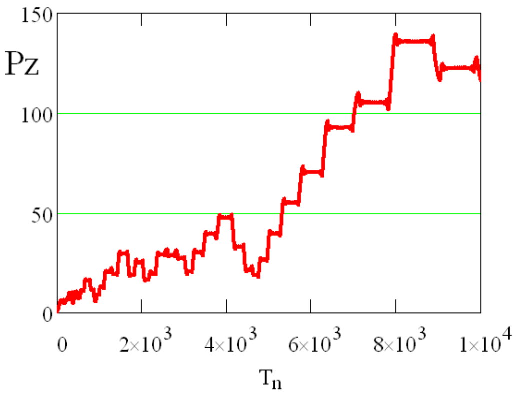

Fig. 1. The dependence of the longitudinal momentum of the particle on time.

Options: $\varepsilon_x = 0.5$, $\varepsilon_z = 0.05$, $\omega_H = 0.99$;

$k_z = 0.8$, $k_x = 0.6$; $p_x(0) = p_y(0) = p_z(0) = 0.01$

Some of the most representative results of these numerical studies are presented below in Figs. 1-4. When selecting pictures for demonstration, preference was given to those values of the parameters at which the new features of particle dynamics were most clearly manifested. In addition, such parameters were chosen at which the wave strength parameter was, if possible, smaller. Fig. 1 shows the dependence of the longitudinal momentum of the particle on time for a sufficiently small value of the wave strength parameter ($\varepsilon = 0.5$). Despite such a small parameter, one can see the most noticeable feature characteristic of particle dynamics at new cyclotron resonances. This feature consists in the appearance of steps in the time dependence of momenta and energy.

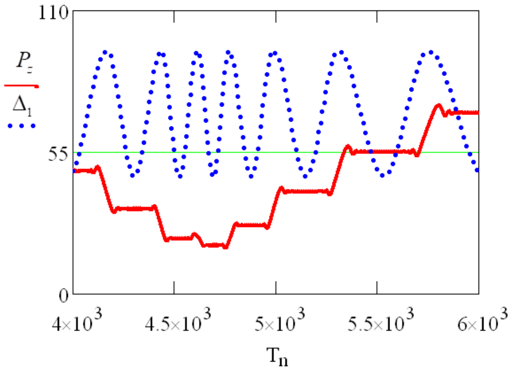

Fig. 2. The same parameters as in Fig. 1

Fig. 2 shows the plot of the dependence of the longitudinal impulse on time, as well as the form of the function Δ_1. One can see the dependence of the characteristics of the steps, as well as the dependence of the moments of jumps on the characteristics of the function Δ_1. This function first appeared in [2]. The presence of this function leads to a significant limitation of the use of familiar conditions for the implementation of cyclotron resonances. It can be seen from this figure that jumps between steps occur in the region of the maximum value of this function. In addition, the time width of the steps is also determined by the period of the function Δ_1. An interesting feature is manifested in the transverse dynamics of particles at resonances. This feature is shown in Fig. 3.

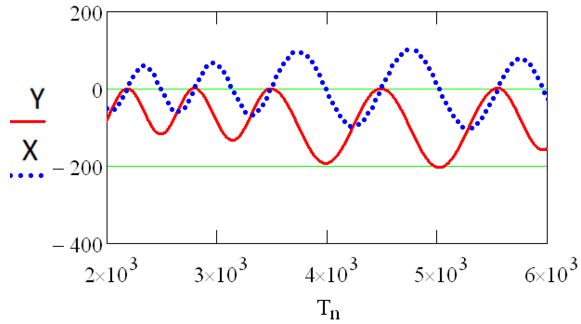

Fig. 3. Time dependence of the transverse coordinates of the particles.

Options: $\varepsilon_x = 0.9$, $\varepsilon_z = 0.09$, $\omega_H = 0.99$;

$k_z = 0.8$, $k_x = 0.6$; $p_x(0) = p_y(0) = p_z(0) = 0.01$

It can be seen that the maxima of the function $y(\tau)$ goes to zero. In addition, it turns out that at the same points vanishes and $x(\tau)$. This feature of the transverse particle trajectory leads to the fact that a point appears on the transverse particle trajectory through which all particle trajectories pass. This feature of the trajectory is shown in Fig. 4.

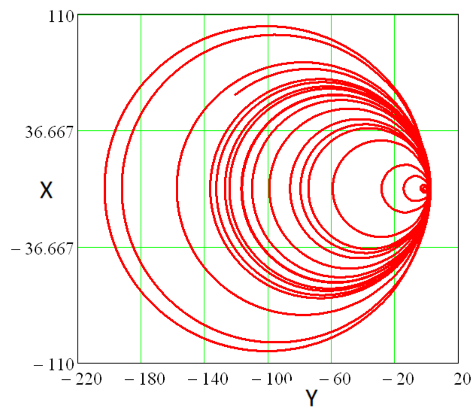

Fig. 4. Trajectory of particles in the transverse plane.

Options: $\varepsilon_x = 0.9$, $\varepsilon_z = 0.09$, $\omega_H = 0.99$;

$p_x(0) = p_y(0) = p_z(0) = 0.01$

It is the appearance of such a common point for all trajectories that leads to jumps of the particle from one step to another. Moreover, this process (these jumps) is random. The characteristics of such randomness were first described in [1]. Similar regimes with dynamic chaos in systems with one degree of freedom are described in [4 - 6]. Note that this randomness is similar to throwing a die with an unlimited number of faces. This feature of the particle dynamics is clearly manifested at sufficiently significant particle energies ($\gamma > 20$). At smaller ones, it is not so noticeable. It is also useful to add that such dynamics (as shown in Fig. 1, for example) is characterized by intermittency. In our case, this means that the particle dynamics on the steps is regular. Randomness occurs only at moments of jumps.

Moreover, the magnitude of the jump is significantly larger than the changes in the magnitudes of the impulses at the steps themselves. The main feature of regimes with intermittency is the appearance of higher moments, which turn out to be larger than the lower moments [7]. Some features and details of this regime are described in [8].

3. RESONANCES AND ACCELERATION OF PARTICLES IN VACUUM WITHOUT EXTERNAL MAGNETIC FIELD

Accelerating charged particles in a vacuum is an attractive option. This is especially true for laser acceleration schemes. There are many attempts to find such acceleration schemes. There is a large number of works that describe various scenarios for such acceleration. One of the last works in this direction is the work [9] (see also the literature cited therein). Below we will show that taking into account the strength of an electromagnetic wave that interacts with particles, as well as the presence in this wave of the transverse component of the wave vector, allows us to formulate the resonant conditions for the interaction of waves with particles, as well as to carry out unlimited acceleration of charged particles in vacuum by transverse electromagnetic waves without an external magnetic field. The initial system of equations is written above (see system of equations (2)). In this system, you need to put $\omega_H = 0$. Formulas (5) in this case must be replaced by other formulas. As the latter, we accept the following transformation formulas:

$$p_x = p_\perp \cos\theta, \quad p_y = p_\perp \sin\theta, \quad p_z = p_\parallel,$$
$$x = \xi - \frac{p_\perp}{\gamma\dot\psi}\sin\theta, \quad y = \eta + \frac{p_\perp}{\gamma\dot\psi}\cos\theta. \qquad (11)$$

To simplify the form of the formulas below, we will analytically present only the expressions for the case when the wave has only the following components (E_y, H_z, H_x). The system of equations (2) for new variables in this case can be rewritten:

$$\dot\theta = \frac{1}{p_\perp}\left[\frac{2}{\gamma}k_x p_\perp \sin\theta + \left(-1+\frac{2}{\gamma}(k_z p_z)\right)\cos\theta\right]\varepsilon\sin\psi,$$
$$\dot p_\perp = -\frac{2}{\gamma}(k_x p_y)\varepsilon\sin\psi\cos\theta +$$
$$+\left[-1+\frac{2}{\gamma}(k_x p_x + k_z p_z)\right]\varepsilon\sin\psi\sin\theta, \qquad (12)$$
$$\dot\xi = v_x + \frac{p_\perp}{\dot\psi\gamma}\dot\theta\cos\theta + \frac{\dot v_\perp\gamma}{\dot\psi\gamma}\sin\theta.$$

Using formula (7) and the considerations that were used above to obtain cyclotron resonant conditions, we can leave only one resonant term in the equation for the new angular variable on the right side:

$$\dot\theta \approx \frac{1}{2p_\perp}\left[-1+\frac{2}{\gamma}(k_z p_z)\right]\varepsilon J_n(\mu)\sin(\psi_n + \theta). \qquad (13)$$

Here $\psi_n = \varphi + n\theta = \tau - k_z z - k_x\xi + n\theta$, $\mu = k_x p_\perp / \gamma\dot\psi$.

At $n = 0$ the resonant conditions will be condition

$$\Phi = (\psi_n + \theta) \approx const; \ \dot\Phi = 0. \qquad (14)$$

These conditions can be conveniently rewritten

$$\dot\Phi = \Delta_0 + \beta\sin\Phi, \qquad (15)$$

where $\Delta_0 = (1 - k_z v_z - k_x v_x)$, $\beta = \frac{\varepsilon}{p_\perp}\left[\frac{k_z p_z}{\gamma} - \frac{1}{2}\right]J_0(\mu)$.

Equation (15) is the Adler equation [10]. This equation has been studied in synchronization theory and is widely used (see, for example [11 - 13]).

3.1. NUMERICAL RESULTS

The particle dynamics is very sensitive to even small changes in parameters. So, a small change in the configuration of an electromagnetic wave can significantly change this dynamics. A typical example is shown in Figs. 5 and 6.

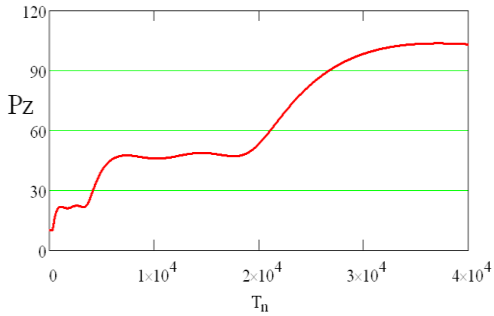

Fig. 5. The dependence of the longitudinal momentum of the particle on time.

Options: $p_z(0) = 10,\ p_x(0) = 0.4,\ p_y(0) = 0.1;$

$\varepsilon_y = 0.5,\ \varepsilon_x = 0,\ \varepsilon_z = 0;$

$k_z = 0.995,\ k_x = 0.099,\ p_{z\max} = 104$

Let us pay attention to the appearance of flat sections in the dependence of momentum on time (see Fig. 5). This feature is characteristic of solutions of the Adler equation.

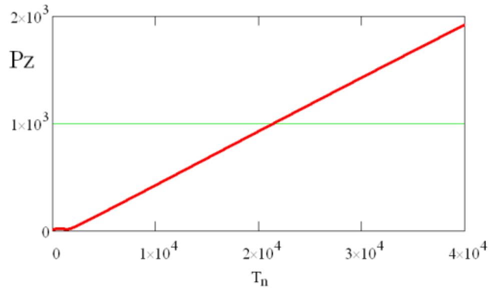

Fig. 6. Time dependence of the longitudinal momentum.

Options: $p_z(0) = 10,\ p_x(0) = 0.1,\ p_y(0) = 0.1;$

$\varepsilon_x = 0.5,\ \varepsilon_y = 0,\ \varepsilon_z = 0.025;$

$k_z = 0.999,\ k_x = 0.051,\ p_{z\max} = 1922$

It is easy to see from Eq. (15) that the capture of particles into unlimited acceleration can occur at sufficiently small ε. Really:

$$\Delta_0 = \left(1 - k_z v_z - k_x v_x\right) \approx 1 - v_z^2\left(1 + k_x^2\right) \approx 1/\gamma^2, \; p_\perp \approx \gamma,$$
$$\varepsilon > 1/\sqrt{\gamma}. \qquad (16)$$

CONCLUSIONS

Let us formulate the most important results of the work.

1. As follows from the new expressions for cyclotron resonances (10), the known conditions for the occurrence of cyclotron resonances ($\Delta_0 = 0$) can be strictly fulfilled only for autoresonance ($k_x = 0$) or in the nonrelativistic case ($\gamma \to 1$). In all other cases, formula (10) should be used.

2. Formula (10) was obtained for an H-polarized wave. However, the resonance structure does not depend on polarization. For the E-wave, only the third term (Δ_2) in formula (10) will change slightly.

3. The most important result of the work is the demonstration of the fact that taking into account the electric strength of the wave, as well as taking into account its transverse component of the wave vector, made it possible to discover the resonant condition for unlimited acceleration of charged particles by the field of transverse electromagnetic waves in vacuum without an external magnetic field. It turned out that rather moderate field strengths can be used to capture particles in unlimited acceleration ($\varepsilon > 1/\sqrt{\gamma}$).

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ОСОБЛИВОСТІ НОВИХ ЦИКЛОТРОННИХ РЕЗОНАНСІВ, А ТАКОЖ УМОВИ РЕЗОНАНСНОГО ПРИСКОРЕННЯ ЗАРЯДЖЕНИХ ЧАСТИНОК У ВАКУУМІ БЕЗ МАГНІТНОГО ПОЛЯ

В.О. Буц, А.Г. Загородній

Показано, що відомі умови для циклотронних резонансів строго справедливі лише в умовах авторезонансу або в нерелятивістському випадку. В інших випадках необхідно використовувати умови, виписані в роботі. Наведено результати дослідження основних особливостей динаміки заряджених частинок за нових резонансних умов. Знайдено умови необмеженого прискорення електронів поперечною електромагнітною хвилею у вакуумі без магнітного поля.