https://doi.org/10.46813/2023-145-099

GENERAL SOLUTION OF THE EXCITATION PROBLEM OF A SYMMETRIC FLAT DIELECTRIC STRUCTURE BY PLANE ELECTROMAGNETIC WAVES

A.V. Vasiliev, A.I. Povrozin, G.V. Sotnikov*
NSC "Kharkov Institute of Physics and Technology", Kharkiv, Ukraine
****E-mail: sotnikov@kipt.kharkov.ua***

General solution of the excitation problem of a symmetric flat dielectric structure by two laser pulses is obtained. The symmetrical geometry consists of two dielectric prisms separated by a vacuum channel for electron acceleration (so called "sandwich"). Each prism can be illuminated with a separate laser pulse; electric filed amplitudes of pulses can differ. The general solution consist from a symmetric distribution and asymmetric one of a longitudinal electric field across the vacuum channel. In the case of a general solution, the effect of the asymmetric part on the total amplitude of the accelerating and defocusing fields is also analyzed. We determined also conditions when a symmetric or asymmetric solution only is realized. For these cases, the obtained analytical solutions are compared with results of full time-domain numerical simulations of bilateral excitation of dielectric prisms with laser pulses.

PACS: 41.75.Lx, 41.60.-m, 41.75.Ht, 41.85.Ar

INTRODUCTION

To use microscale dielectric structures for high gradient acceleration by waves arising from their illumination with intense laser beams had been proposes in the last century [1, 2]. Further development of dielectric laser accelerators (DLA) took place in several directions: using grating structures [3, 4] (acceleration mechanism is the inverse Smith-Purcell radiation), photonic gap structures [3, 5] and flat структур [6, 7] (acceleration mechanism is the inverse Cherenkov radiation).

However, even A. Lohman [1] drew attention to the fact that for ultra-relativistic electrons the acceleration rate tends to zero when using both single grating structures and single flat structures. The dependence of the accelerating gradient on particle energy $E_{ac} \sim \sqrt{1-\beta^2} = 1/\gamma$ (β is the velocity of a particle synchronous with the excited wave, γ is the relativistic factor) then was confirmed in analytical studies [6, 8]. For the DLA with single flat dielectric structure the exact dependence of the acceleration rate on the beam energy is studied in the paper [9].

The vanishing of the longitudinal electric field E_{ac} when $\beta \to 1$ can be eliminated by using double flat dielectric structures [1, 10] or double grating dielectric structures [1, 8]. At this authors [1, 8, 10] considered the bilateral illumination of double dielectric structures by laser pulse (so called "two-side interaction" [1]).

In a recent study [11], a more general case is considered in comparison [1, 8], when a double flat dielectric structure (double-prism model) is illuminated both from one side and from both sides (unilateral illumination and bilateral illumination by driving laser pulses). It is shown that even when using a unilateral illumination of a double flat dielectric structure the longitudinal electric field E_{ac} does not vanish when $\beta \to 1$.

The analytical solution given in [11] for bilateral illumination of a double prism has a transverse distribution of the longitudinal electric field that is symmetric with respect to the axis of the accelerating channel. At the same time, it is known [12-14] that symmetric dielectric structures have both symmetric and antisymmetric solution of the excitation problem by an external source. Below, we will obtain a general solution to the excitation problem of a two-sided dielectric structure by two laser pulses having the same frequency but differing in the amplitude of the electric field. We will show analytically when the case with a symmetric or the case with an antisymmetric transverse distribution of the longitudinal electric field can be realized. These conclusions then will be confirmed by numerical simulations of excitation by laser pulses of a double dielectric structure.

1. STATEMENT OF THE PROBLEM

We will describe the electromagnetic fields of laser pulses in the plane wave approximation. Let p-polarized plane waves fall from a optically transparent medium with refractive index $n = \sqrt{\varepsilon}$ under an angle θ onto the boundary between the vacuum and this mediums. The frequencies of these waves are equal, but the amplitudes in the general case have different values. If $\sin\theta > 1/n$ the incident electromagnetic wave undergoes total internal reflection. Geometry of the problem and the coordinate system is given in Fig. 1. The boundary surfaces are planes $x = \pm d/2$, the incident plane is xz-plane, and the z-axis is directed along the propagation of accelerated bunch. In such a frame the components of the electric vector of the incident waves will be of the form

$$E_{1z}^i = E_1^i \cos(\theta)\exp(i\Psi_1),\ E_{1x}^i = E_1^i \sin(\theta)\exp(i\Psi_1),$$
$$E_{3z}^i = -E_3^i \cos(\theta)\exp(i\Psi_3),\ E_{3x}^i = E_3^i \sin(\theta)\exp(i\Psi_3), \quad (1)$$
$$\Psi_{1,3} = k_0 n \sin\theta \cdot z \mp k_0 n \cos\theta \cdot (x \mp d/2) - \omega t,$$

where $k_0 = \omega/c$, ω is the frequency of plane wave, c is the speed of light in the vacuum, E_1 and E_3 are amplitudes of incident waves in region 1($x \ge d/2$) and in region 3 ($x \le -d/2$) correspondently.

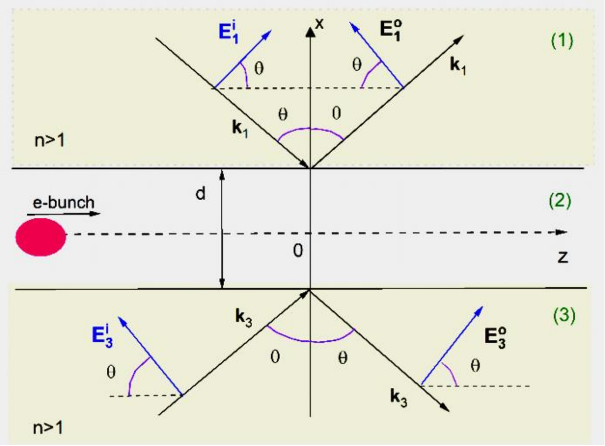

Fig. 1. Geometry of the problem. E_1^i *and* E_3^i *are amplitudes of incident waves in the region (1) and (3),* E_1^o *and* E_3^o *are unknown amplitudes of reflected waves. Region (2) is accelerating channel*

2. ANALYTICAL SOLUTION OF THE PROBLEM

Let us designate an unknown amplitudes of the reflected waves in regions (1) s and (3) as E_1^o and E_3^o respectively. Then the components of the electric field of the reflected waves have the form

$$E_{1z}^o = -E_1^o\cos(\theta)\exp(i\Psi_1),\ E_{1x}^o = E_1^o\sin(\theta)\exp(i\Psi_1),$$
$$E_{3z}^3 = E_3^o\cos(\theta)\exp(i\Psi_3),\ E_{3x}^o = E_3^o\sin(\theta)\exp(i\Psi_3). \quad (2)$$

We will seek the solution of the Maxwell equations for the amplitude of longitudinal electric field E_{2z} of the evanescent wave in region (2) in the form

$$E_{2z} = [C_1\sinh(px) + C_2\cosh(px)]\exp(i\Psi_2), \quad (3)$$

where $\Psi_2 = k_0 n\sin\theta\cdot z - \omega t$, $p = k_0\sqrt{n^2\sin^2\theta - 1}$, C_1 and C_2 are unknown constants.

Taking into account the equation (1), for tte amplitude of transverse electric field E_{2x} from the Maxwell equations follows the expression

$$E_{2x} = -\frac{k_0 n\sin\theta}{p}[C_1\cosh(px) + C_2\sinh(px)]\exp(i\Psi_2)\ .(4)$$

To uniquely find fields in all regionss, it is necessary to determine four unknown constants C_1, C_2, E_1^o, E_3^o. They can be found using the continuity conditions for the longitudinal electric field E_z and the transverse component of the electric field induction vector D_x at the boundary $x = d/2$

$$E_1^i\cos\theta - E_1^o\cos\theta = C_1\sinh(pd/2) + C_2\cosh(pd/2),$$
$$\varepsilon E_1^i\sin\theta + \varepsilon E_1^o\sin\theta = -\frac{ik_0 n\cos\theta}{p}[C_1\cosh(pd/2) \quad (5)$$
$$+ C_2\sinh(pd/2)]$$

and at the boundary $x = d/2$

$$E_3^i\cos\theta - E_3^o\cos\theta = C_1\sinh(pd/2) - C_2\cosh(pd/2),$$
$$\varepsilon E_3^i\sin\theta + \varepsilon E_3^o\sin\theta = -\frac{ik_0 n\cos\theta}{p}[C_1\cosh(pd/2) \quad (6)$$
$$-C_2\sinh(pd/2)]\,.$$

Having solved equations (5) and (6) with respect to the required values, we find the following expressions for the variables C_1, C_2

$$C_1 = (E_1^i + E_3^i)\frac{\sin\theta}{\sinh\left(\frac{pd}{2}\right)\tan\theta - \frac{ik_0 n}{\varepsilon p}\cosh\left(\frac{pd}{2}\right)}, \quad (7)$$

$$C_2 = (E_1^i - E_3^i)\frac{\sin\theta}{\cosh\left(\frac{pd}{2}\right)\tan\theta - \frac{ik_0 n}{\varepsilon p}\sinh\left(\frac{pd}{2}\right)}. \quad (8)$$

The constant C_1 describes antisymmetric part and the constant C_2 describes symmetric part of the transverse distribution of the longitudinal electric field in the accelerating channel (3). From the equation (8) follows if $E_3^i = E_1^i$, i.e. amplitude of incident wave are equal, then the symmetric solution is absent (3). Similarly, from the equation (7) follows if $E_3^i = -E_1^i$, i.e. amplitude of incident wave are equal but opposite in sign, then the antisymmetric solution is absent.

If the amplitude of one of incident wave is equal zero, for example $E_3^i = 0$, we obtain the solution for unilateral illumination of the double flat dielectric structure, given in the ref. [11]. For the ultrarelativic electrons $\beta \to 1$ the value $p \approx k_0/\beta\gamma \to 0$, the constant $C_1 \to 0$ and the antisymmetric part in the equation (3) disappears. Therefore, the qualitative dependences of the longitudinal electric field in the accelerating channel on the electron energy for unilateral and bilateral (if $E_3^i = -E_1^i$) illumination coincide.

3. NUMERICAL SIMULATION OF THE EXCITATION PROBLEM

To verify the analytical results presented in above section we simulate the field excitation by laser pulses in double flat dielectric structure. The double flat dielectric structure consists of two oppositely located rectangular prisms, the hypotenuses of which form a vacuum channel for the accelerated bunch. The angle of the prism is equal to θ, this angle is equal to the angle of incidence of the laser pulse on the dielectric-vacuum channel interface. The field simulations are performed by using an electromagnetic code based on the finite-difference time-domain (FDTD) algorithm [15]. The parameters of the laser pulses and prisms used in the simulations are given in Table.

In Figs. 2–6 are presented the results of the simulations. In Fig. 2 are shown 2D-maps of the distribution of the longitudinal electric field in the simulated region - inside of prisms, acceleration channel and surrounded space (top picture). In the bottom picture is shown the zoomed part of top picture in the acceleration channel.

Simulation parameters

Wavelength of laser pulse………………	790…810 nm
Form of laser pulse ……………………	Gaussian
Electric field amplitude……………....	$3 \cdot 10^9$ V/m
Waist ………………………………..	20 μm
Focus distance…………………………	5 μm
Refractive index (fused silica), n ……..	1.453
Incident angle, θ ……………………….	43.482°
Channel width, d ……………………….	400 nm
Channel length ………………………...	22 μm
Phase shift between laser pulses ………	0°, 180°

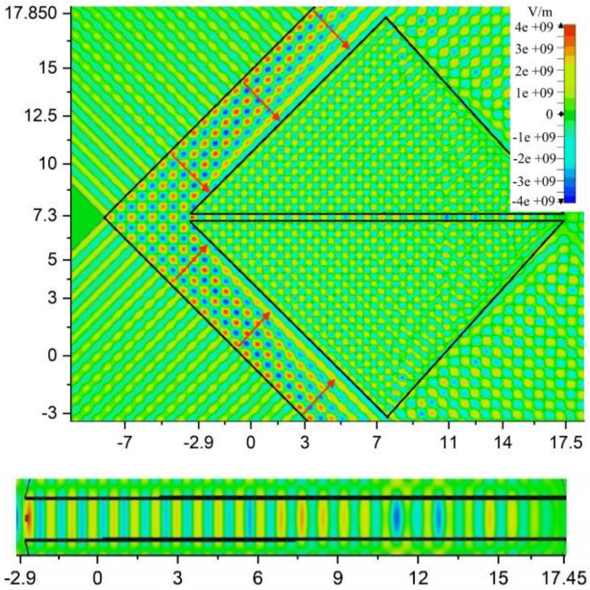

Fig. 2. 2D-distribution of the longitudinal electric field in the x-z plane (x-axis is directed upward). Red arrows show direction and origin of laser pulse injection. Zoomed part of acceleration channel is in the bottom. Coordinates measured in microns. Phase shift between pulses is 180°

The simulation results shown in Fig. 2 are obtained for the case symmetric excitation of the dielectric structure, when phase shift between laser pulses is 0. This case corresponds the case when $E_3^i = -E_1^i$ in the equations (7), (8), (3), (4). As can be seen from the bottom figure, inside the vacuum channel, the amplitude of the longitudinal electric field is almost constant.

In Fig. 3 are shown the axial profile of the longitudinal (top) and transverse profile electric field in accelerating channel in the case of symmetric excitation of the dielectric structure. Maximal amplitude is $4 \cdot 10^9$ V/m in comparison with the value $3 \cdot 10^9$ V/m of amplitude of electric field of each laser. Transverse profile the longitudinal electric field is about the constant.

In Fig. 4 are shown the similar profiles as in Fig. 3, only in the case of antisymmetric excitation of the dielectric structure. This case corresponds the case when $E_3^i = E_1^i$ in the equations (7), (8), (3), (4).

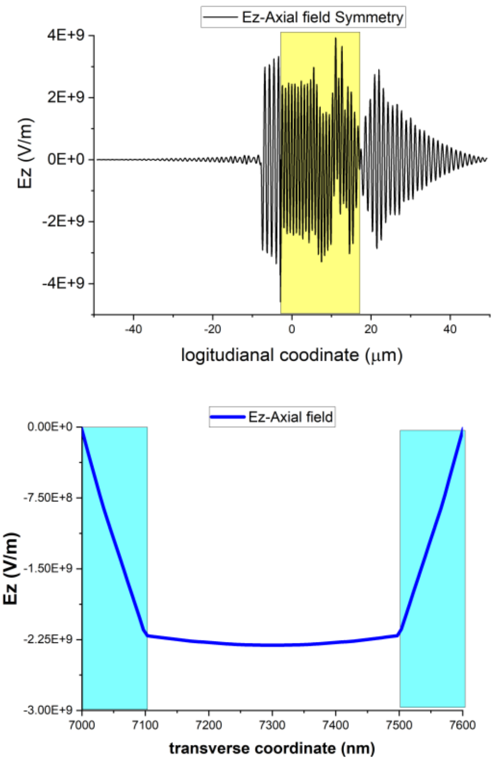

Fig. 3. The distribution of the longitudinal electric field along the center of the accelerating channel (top) and across the accelerating channel (channel) in the section z=5 μm. Symmetric excitation of dielectric structure, phase shift between pulses is 0.Yellow and cyan rectangles show location of dielectric prisms

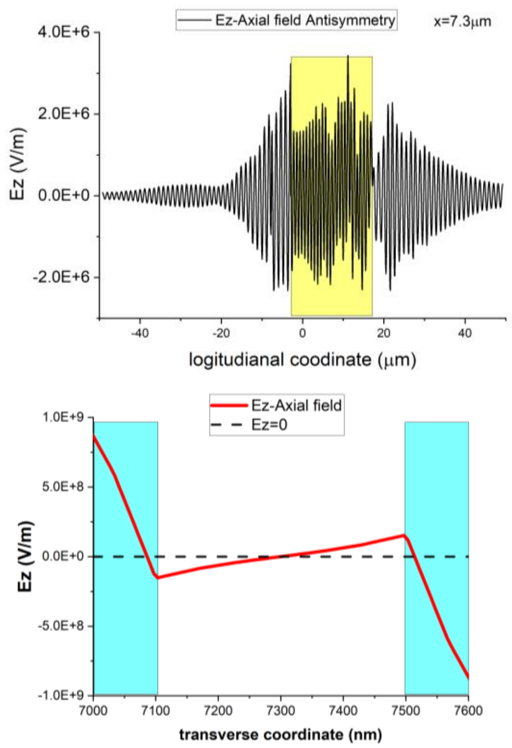

Fig. 4. The same in Fig. 3 for antisymmetric excitation of dielectric structure, phase shift between pulses is 180°.Yellow and cyan rectangles show location of dielectric prisms

The longitudinal electric field near the channel axis is about zero, three orders of magnitude less than in the symmetrical case, not exactly zero due to grid effects. Transverse profile of the longitudinal electric field has antisymmetric distribution with opposite signs of the field at opposite sides of the accelerating channel. Maximal amplitude is the value of $0.15 \cdot 10^9$ V/m, which is significantly less than the amplitude of the field of the incident lase wave.

CONCLUSIONS

Analytical solution of the excitation problem of a symmetric flat dielectric structure by two laser pulses is obtained in the case of arbitrary values of field amplitudes of incident lasers. The general solution consist from a symmetric part and asymmetric one of a longitudinal electric field distribution across the vacuum channel.

If the vectors of the electric field of the incident waves are oriented so that their longitudinal components are directed in one direction, then the case of a symmetrical distribution of the accelerating field across the vacuum channel is realized. In opposite case when electric fields of incident waves are shifted by 180° is realized the antisymmetric distribution of the accelerating field across the vacuum channel.

The obtained analytical results are confirmed with full finite-difference time-domain simulation of the bilateral excitation of double flat dielectric structure.

The presented analytical solutions and numerical simulations are useful in planning experiments on dielectric laser acceleration.

ACKNOWLEDGEMENTS

The National Research Foundation of Ukraine under the program “Leading and Young Scientists Research Support” supports the study (project # 2020.02/0299).

REFERENCES

1. A. Lohmann. Electron Acceleration by Light Waves // *IBM Technical Note TN5.* 1962, p. 169-182.
2. Koichi Shimoda. Proposal for an Electron Accelerator Using an Optical Maser // *Applied Optics.* 1962, v. 1, p. 33-35.
3. J. Breuer J and P. Hommelhoff. Laser-based acceleration of nonrelativistic electrons at a dielectric structure // *Phys. Rev. Lett.* 2013, v. 111, p. 134803.
4. R.J. England et al. Dielectric laser accelerators // *Rev. Mod. Phys.* 2014, v. 86, p. 1337-1389.
5. X. E Lin. Photonic band gap fiber accelerator // *Phys. Rev. ST Accel. Beams*. 2001, v. 4, p. 051301.
6. S.A. Kheifetz. Particle motion a surface on which total internal reflection of electromagnetic wave takes plase // *Proc. 8-th Int. conf. Higt energy ac-cel.* CERN, Geneva. 1971, p. 597-599.
7. R.C. Fernow. Acceleration using total internal reflection // *BNL Report* No.52290. 1991, p. 1-17.
8. J. Breuer, J. McNeur, and P. Hommelhoff. Dielectric
9. Laser acceleration of electrons in the vicinity of single and double grating structures theo ry and simulations // *J. Phys. B: At. Mol. Opt. Phys.* 2014, v. 47(23), p. 234004.
10. O.O. Bolshov, A.V. Vasiliev, A.I. Povrozin, and G.V. Sotnikov. About the acceleration rate of relativistic beams by a surface wave in a dielectric laser accelerator // *Problems of Atomic Science and Technology.* 2021, N 6(136), p. 57-60. https://doi.org/10.46813/2021-136-057
11. B.R. Frandsen, S.A. Glasgow, and J.B. Peatross. Acceleration of free electrons in a symmetric evanescent wave // *Laser Phys.* 2006, v. 16, p. 1311-1314. https://doi.org/10.1134/S1054660X06090040
12. Liwen Zhang, Weihao Liu, Yucheng Liu, et al. Inverse Cherenkov dielectric laser accelerator for ultra-relativistic particles // *J. Phys. D: Appl. Phys.* 2023, v. 56, p. 045103.
13. L. Pincherle. Electromagnetic waves in metal tubes filled longitudinally with two dielectric // *Phys. Rev.* 1944, v. 66, №5-6, p. 118-130.
14. Liling Xiao, Wei Gai, and Xiang Sun. Field analysis of a dielectric-loaded rectangular waveguide accelerating structure // *Phys. Rev. E.* 2001, v. 65, p. 016505(9).
15. Gennadij V. Sotnikov, Ivan N. Onishchenko, and Thomas C. Marshall. 3D Analysis of Wake Field Excitation in a Dielectric Loaded Rectangular Resonator // *AIP Conference Proceedings*. 2006, v. 877, p. 888. https://doi.org/10.1063/1.2409230
16. CST Studio Suite Electromagnetic Field Simulation Software. https://www.3ds.com/products-services/simulia/products/cst-studio-suite/

Article received 19.04.2023

ЗАГАЛЬНЕ РІШЕННЯ ЗАДАЧІ ЗБУДЖЕННЯ СИМЕТРИЧНОЇ ПЛОСКОЇ ДІЕЛЕКТРИЧНОЇ СТРУКТУРИ ПЛОСКИМИ ЕЛЕКТРОМАГНІТНИМИ ХВИЛЯМИ

А.В. Васильєв, А.І. Поврозін, Г.В. Сотніков

Отримано загальне рішення задачі збудження симетричної плоскої діелектричної структури двома лазерними імпульсами. Симетрична геометрія складається з двох діелектричних призм, розділених вакуумним каналом для прискорення електронів (так званий сендвіч). Кожну призму можна опромінювати окремим лазерним імпульсом, амплітуди електричного поля імпульсів можуть відрізнятися. Загальне рішення складається з симетричного та асиметричного розподілів поздовжнього електричного поля впоперек вакуумного каналу. У разі загального рішення аналізується також вплив асиметричної частини на сумарну амплітуду полів, що прискорює і дефокусує. Визначено також умови, коли реалізується лише симетричне чи асиметричне рішення. Для цих випадків одержані аналітичні рішення порівнюються з результатами повного тимчасового чисельного моделювання білатерального збудження.