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STUDY OF O(6) SYMMETRY IN ^{108,110,112}Ru ISOTOPES BY IBM-1 CALCULATION

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Interacting Boson Model (IBM-1) is very important for the low-lying combined state in even mass nuclei. We have described the O(6) limit in the ^{108,110,112}Ru nuclei by IBM-1 model. The B(E2) strength and energy levels in different bands were established in good agreement theoretically and experimentally. The calculated potential energy surfaces (PES) of those nuclei were suggested O(6) characters.

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INTRODUCTION

Interacting Boson Model (IBM-1) is useful in explaining the collective nuclear states of even-even nuclei. The IBM-1 is established by a fixed number of bosons (N_b). The neutrons and protons have not been differentiated in IBM-1. The nuclear structure is complicated because each nucleon interacts with every other nucleon. Iachello and Arima [1] fruitfully designated the collective nuclear characters in intermediate mass nuclei using a system of bosons. Depending on its angular momentum L, each boson can occupy one of two levels: s-boson or d-boson. Moreover, this model had generated the algebra U(6) group: O(6), SU(3) and U(5). Nevertheless, some scientists proposed the construction which consists of SU(3)–O(6), U(5)–SU(3) and U(5)–O(6) transitional [4, 5].

The electromagnetic strength, yrast levels and potential energy surface of Ru isotopes were discussed by several researchers [6–11]. Hossain et al. [12, 13] studied O(6) symmetry of the nuclear structure of ^{104,106}Ru by IBM-1 calculation. Sharrad et al. [14] studied the U(5) symmetry of ^{96}Ru and ^{98}Ru nuclei using the interacting boson model (IBM-1). Luo Y. X. et al. [15] studied Odd-parity bands of ^{108}Ru, ^{110}Ru and ^{112}Ru isotopes. For this it is interesting to study the even-parity of the ground state band of more neutron-rich nuclei of Ru (A = 108, 110, 112) by IBM-1 calculation. This model signifies a vital stage to comprehend the structure of those nuclei.

The even ^{108,110,112}Ru isotopes consist of Z = 44 protons and neutrons N= 64, 66, 68 respectively which fill the orbits near a double shell closure ^{100}Sn. The configurations of ^{108,110,112}Ru isotopes have $\pi g_{9/2}^{-6}$ (6 holes of protons) and $\nu g_{9/2}^{14,16,18}$ (14, 16, and 18 neutrons particle) near to ^{100}Sn. It is generally believed that the nuclei far from a close shell indicate structure from spherical to deformed symmetry. These neutrons rich deformed nuclei will be an excellent topic of interest to be studied further.

1. METHODOLOGY

The nuclei comprising N nucleons, the IBM model allocates to occupy a truncated model space. It is responsible for a numerical clarification of indistinguishable elements through L = 0 or 2 forming pairs. The equation of Hamiltonian in IBM-1 is shown as follow equation:

$$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_2\hat{Q}\cdot\hat{Q} + a_3\hat{T}_3\cdot\hat{T}_3 + a_4a_3\hat{T}_4\cdot\hat{T}_4. \quad (1)$$

The equations of three types of symmetry are given as follows:

U(5):

$$E(n_d,\upsilon,L) = \varepsilon n_d + \frac{a_1}{12}n_d(n_d+4) + \left(\frac{a_3}{7} - \frac{a_1}{10} - \frac{3a_4}{70}\right)\upsilon(\upsilon+3) + \frac{1}{14}(a_4-a_3)L(L+1). \quad (2)$$

SU(3):

$$E(\lambda,\mu,L)=\frac{a_2}{2}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)) + \left(a_1-\frac{2a_2}{8}\right)L(L+1). \tag{3}$$

O(6):

$$E(\sigma,\tau,L)=\frac{a_0}{4}(N-\sigma)(N+\sigma+4) + \frac{a_3}{2}\tau(\tau+3)+\left(a_1-\frac{a_3}{10}\right)L(L+1). \tag{4}$$

The Hamiltonian and eigen-values for the three limits are [16],

U(5):

$$\hat{H}_{U(5)}=\varepsilon\hat{n}_d+a_1(\hat{L}\cdot\hat{L})+a_3(\hat{T}_3\cdot\hat{T}_3) + a_4(\hat{T}_4\cdot\hat{T}_4),$$

$$E(n_d,\upsilon,L)=\varepsilon n_d+K_1 n_d(n_d+4) + K_4\upsilon(\upsilon+3)+K_5L(L+1), \tag{5}$$

with

$$K_1=\frac{1}{12}a_1,\ K_4=-\frac{1}{10}a_1+\frac{1}{7}a_3-\frac{3}{70}a_4,$$

$$K_5=-\frac{1}{14}a_3+\frac{1}{14}a_4.$$

O(6):

$$\hat{H}_{O(6)}=a_0\hat{P}.\hat{P}+a_1\hat{L}.\hat{L}+a_3\hat{T}_3.\hat{T}_3,$$

$$E(\sigma,\upsilon,L)=K_3[N(N+4)-\sigma(\sigma+4)] + K_4\upsilon(\upsilon+3)+K_5L(L+1), \tag{6}$$

with

$$K_3=\frac{1}{4}a_0,\ K_4=\frac{1}{2}a_3,\ K_5=-\frac{1}{10}a_3+a_4.$$

SU(3):

$$\hat{H}_{U(3)}=a_1(\hat{L}\cdot\hat{L})+a_2(\hat{Q}\cdot\hat{Q}),$$

$$E(\lambda,\mu,L)=K_2(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)) + K_5L(L+1), \tag{7}$$

with $K_2=\frac{1}{2}a_2,\ K_5=a_1-\frac{3}{8}a_2.$

2. RESULTS AND DISCUSSION

Using IBM-1 model the results of different types of energy band, strength of reduced transition probabilities and potential energy surfaces (PES) of $^{108,\,110,\,112}$Ru nuclei are explained as follows:

2.1. ENERGY BAND

The energy band consists of ground state (G.S.), gamma and beta band. These are used for the phenomenological interaction boson model (IBM-1) to describe experiential data of ^{108,110,112}Ru nuclei.

2.1.1. O(6) SYMMETRY AND BOSON NUMBER

The O(6) symmetry is realistic for those nuclei by means of available statistics of energy level ($E_2 : E_4 : E_6 : E_8$ = 1.0 : 2.74 : 5.12 : 8.02) [17–19]. According to double shell closure of ^{100}Sn, the Ru isotopes with N= 64 and 66, have 3 holes of proton boson and 7 and 8 particles of neutron boson respectively. The ^{112}Ru nucleus has 3 hole of proton boson and 7 holes of neutron boson according to shell closure ^{132}Si. Therefore total bosons numbers are 10, 11, and 10 of ^{108}Ru and ^{110}Ru and ^{112}Ru respectively. Usually three symmetry U(5), O(6) and SU(3) depends on R = $E4_1^+/E2_1^+ \approx$ 2.00, 2.50 and 3.33 respectively [17–19]. Here $E2_1^+$ and $E4_1^+$ are at energy level 2_1^+ and 4_1^+ respectively. The experimental $R_{4/2}$ of ^{108}Ru, ^{110}Ru and ^{112}Ru isotopes are 2.75, 2.75 and 2.73 respectively which characterized by a non-spherical spatial distribution of nuclear density and are known as deformed nuclei gamma soft O(6).

Table 1

Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV, excepted N, and CHQ=0, ε=0, a_2=0, a_4=0

A	*N*	a_0	a_1	a_3
^{108}Ru	10	0.088	0.010	0.130
^{110}Ru	11	0.094	0.015	0.109
^{112}Ru	10	0.093	0.018	0.092

The adopted values of parameters were determined by the equation (4) using the investigational eigen values E(n_d, υ, L), where n_d, υ and L are quantum numbers. The finest fit was occupied up to 12^+ of Ru isotopes for the neutron N= 64, 66 and 68. Table 1 show all parameters (ε, a_0, a_1, a_2, a_3, a_4) are given in MeV.

2.1.2. GROUND STATE BAND

The excitation energy up to 12^+ spins of ground state band of ^{108,110,112}Ru [17–19] nuclei are presented in Fig.1. The 0^+, 2^+, 4^+,6^+, 8^+, 10^+ and 12^+ members of the ground state band of ^{108}Ru occur at 0, 0.242, 0.665, 1.241, 1.942, 2.741 and 3.529 MeV respectively and their corresponding value of IBM-1 are 0.0, 0.244, 0.596, 1.057, 1.626, 2.303, and 3.089 MeV. The measured values of g-band (0^+,2^+,4^+,6^+,8^+,10^+,12^+) of ^{110}Ru and ^{112}Ru are 0.0, 0.241, 0.664, 1.239, 1.945, 2.759 and 3.647 MeV and 0.0, 0.237, 0.645, 1.189, 1.839, 2.56 and 3.326 MeV respectively. The corresponding theoretical calculation of IBM-1 in ^{110}Ru and ^{112}Ru are 0.0, 0.241, 0.619, 1.137, 1.792, 2.586 and 3.518 MeV and 0.0 0.236, 0.634, 1.192, 1.911, 2.792 and 3.834 MeV respectively. It is shown that theoretical and empirical data establish good agreement and the ^{108}Ru and ^{110}Ru and ^{112}Ru nuclei are good contenders for O(6) symmetry.

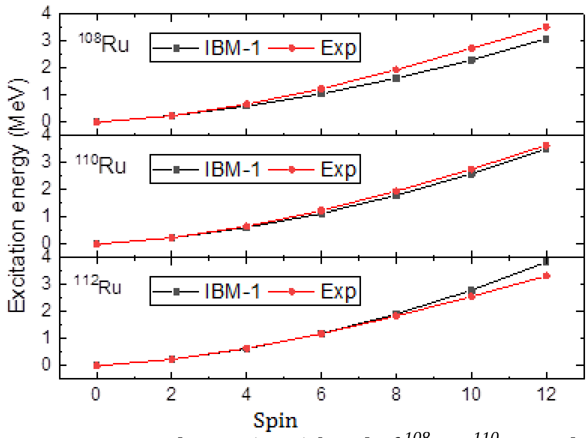

Fig. 1. Ground state (G.S.) band of ^{108}Ru, ^{110}Ru and ^{112}Ru isotopes.IBM-1 and Expt data are indicated by black and red color respectively

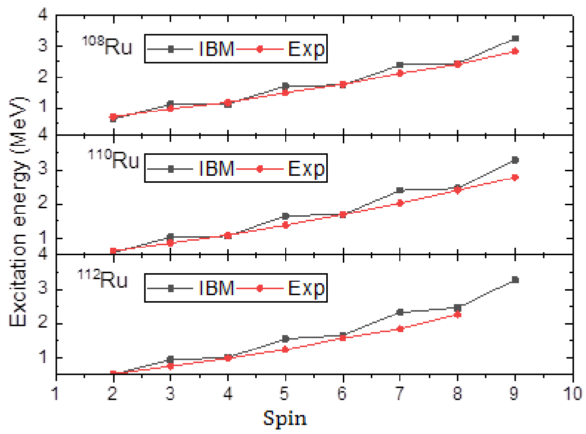

Fig. 2. Gamma band of ^{108}Ru, ^{110}Ru and ^{112}Ru isotopes. IBM-1 and Expt data are indicated by black and red color respectively

2.1.3. GAMMA BANDS

The excitation of γ-bands (2^+, 3^+, 4^+….9^+) is favorable when the potential energy surfaces are found soft for both beta and gamma deformation parameters. The γ bands usually observed at nuclear chart associated with ellipsoidal nuclear shape. The γ bands were observed up to 9^+ (2.844 MeV) in ^{108}Ru [17], 9^+ (2.777 MeV) in ^{110}Ru [18] and 8^+ (2.263 MeV) levels in ^{112}Ru [19]. The calculated excitation energy in MeV from 2^+ to 9^+ for gamma bands is compared with previous experimental data and they are plotted in Fig. 2. From the figures it is shown that gamma band of all IBM calculations of those nuclei are in good agreements with available previous measured data [17–19].

2.1.4. BETA BANDS

The measured members of the beta-vibrational band observed at 0^+ (0.976 MeV), 2^+ (1.249 MeV), 4^+ (1.644 MeV) and 10^+ (3.149 MeV) in ^{108}Ru [17] and 0^+ (1.137 MeV) , 2^+ (1.396 MeV), 4^+(1.618 MeV) in ^{110}Ru [18]. There is no measured member of beta band in ^{112}Ru. The IBM-1 calculation of β bands are presented in Table 2. The excitations energy in MeV for beta bands (0^+, 2^+, 4^+….10^+) is compared with available previous experimental data. It is shown that beta band of all IBM calculations of those nuclei are in good agreements with the available previous measured data [17–19].

Table 2

β-band (in MeV)

J^π	^{108}Ru		^{110}Ru		^{112}Ru	
	IBM	Exp.	IBM	Exp.	IBM	Exp.
0^+	0.976	0.976	0.986	1.137	0.831	–
2^+	1.219	1.249	1.480	1.396	1.261	–
4^+	1.572	1.644	1.605	1.618	1.464	–
6^+	2.032	–	2.341	–	2.207	–
8^+	2.601	–	3.032	–	2.937	–
10^+	3.279	3.150	3.826	–	3.817	–

2.2. B (E2) STRENGTHS

The B(E2) strength provides the additional possessions of the construction of nuclei. It is known that the bosons numbers must be conversed if electric quadruple transition E2 operator should be a Hermitian tensor of rank two. The $B(E2)$ strength of $^{108, 110, 112}$Ru in IBM-1 [17–19] is calculated:

$$T^{E2} = \alpha_2[d^\dagger s + s^\dagger d]^{(2)} + \beta_2[d^\dagger d]^{(2)} = e_B\hat{Q} \quad . \qquad (8)$$

The symbol ($s^\dagger$, $d^\dagger$) is creation and (s, d) is annihilation operators for s and d bosons, respectively, although α_2 and β_2 symbols are two parameters. $\alpha_2 = e_B$ effective charge of boson and $\beta_2 = \chi\alpha_2$

$$B(E2, J_i \to J_f) = \frac{1}{2J_i+1}\left|\left\langle J_f \left\| T^{E2} \right\| J_i \right\rangle\right|^2 \quad . \qquad (9)$$

The parameters, α_2 and β_2 of Eq. (8), accommodated suitably a set to produce the published B(E2; $2_1^+ \to 0_1^+$). The calculated effective charge (e_B) of those nuclei is given in Table 3.

Table 3

Effective charge used to reproduced B(E2) values for even-even $^{108\text{-}112}$Ru nuclei

Isotopes	N	e_B (eb)
^{108}Ru	10	0.082
^{110}Ru	11	0.079
^{112}Ru	10	0.092

Table 4

Experimental and the IBM-1 values of B(E2) for ^{108,110,112}Ru nuclei in e^2b^2

J_i^+ -J_f^+	^{108}Ru		^{110}Ru		^{112}Ru	
	Exp	IBM	Exp	IBM	Exp.	IBM
$2_1^+ \to 0_1^+$	0.237	0.237	0.207	0.207	0.189	0.189
$4_1^+ \to 2_1^+$	–	0.327	0.269	0.286	0.287	0.261
$4_2^+ \to 2_2^+$	–	0.189	–	0.167	–	0.151
$6_1^+ \to 4_1^+$	–	0.362	0.376	0.319	–	0.289
$6_2^+ \to 4_2^+$	–	0.250	–	0.224	–	0.199
$8_1^+ \to 6_1^+$	–	0.367	–	0.328	–	0.293
$10_1^+ \to 8_1^+$	–	0.352	–	0.320	–	0.281
$10_2^+ \to 8_2^+$	–	0.259	–	0.243	–	0.207
$2_2^+ \to 2_1^+$	–	0.327	–	0.286	–	0.261
$4_2^+ \to 4_1^+$	–	0.172	–	0.152	–	0.137
$6_2^+ \to 6_1^+$	–	0.117	–	0.104	–	0.093
$8_2^+ \to 8_1^+$	–	0.085	–	0.077	–	0.067

The values of e_B in unit of eb were estimated to reproduce experimentally (B(E2; $2_1^+ \to 0_1^+$). The ^{108,110}Ru and ^{112}Ru isotopes have O(6) character and the values β_2= 0. The calculated and measured strength of B(E2) in e^2b^2 for those isotopes are shown in Table 4. We have normalized the experimental and theoretical data using B(E2:$2_1^+ \rightarrow 0_1^+$) states in each nuclei and then corresponding value up to B(E2: $8_2^+ \rightarrow 8_1^+$) are given in the Table 4. The present data of IBM-1 is good with the obtainable published measured results.

2.3. POTENTIAL ENERGY SURFACE (PES)

The potential energy surface (PES) stretches the evidence for invention microscopic and geometric shapes of nuclei. The IBM-1 energy surface is created by combining the IBM-1 Hamiltonian's expectation value with the coherent state ($|N,\beta,\gamma\rangle$) [20]. The creation operators (b_c^+) act on a state of boson vacuum $|0\rangle$ to produce the coherent state as follows:

$$\left|N,\beta,\gamma\right\rangle = \frac{1}{\sqrt{N}}(b_c^\dagger)^N\left|0\right\rangle, \tag{10}$$

where

$$b_c^\dagger = \frac{1}{\sqrt{1+\beta^2}}\Big\{s^\dagger + \beta[\cos\gamma(d_0^\dagger) + \sqrt{\frac{1}{2}}\sin\gamma(d_2^\dagger + d_{-2}^\dagger)]\Big\}. \tag{11}$$

The EPS can be written in terms of β and γ as

$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)}[\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4]. \tag{12}$$

The shape of a nucleus could be spherical or distorted depending on whether β=0 or not. The variation in nucleus symmetry is represented by γ term, when γ = 0, the nucleus has a prolate shape; when γ=60, it has an oblate shape. The plot of EPS of even even $^{108-112}$Ru O(6) Symmetry are shown in Fig. 3.

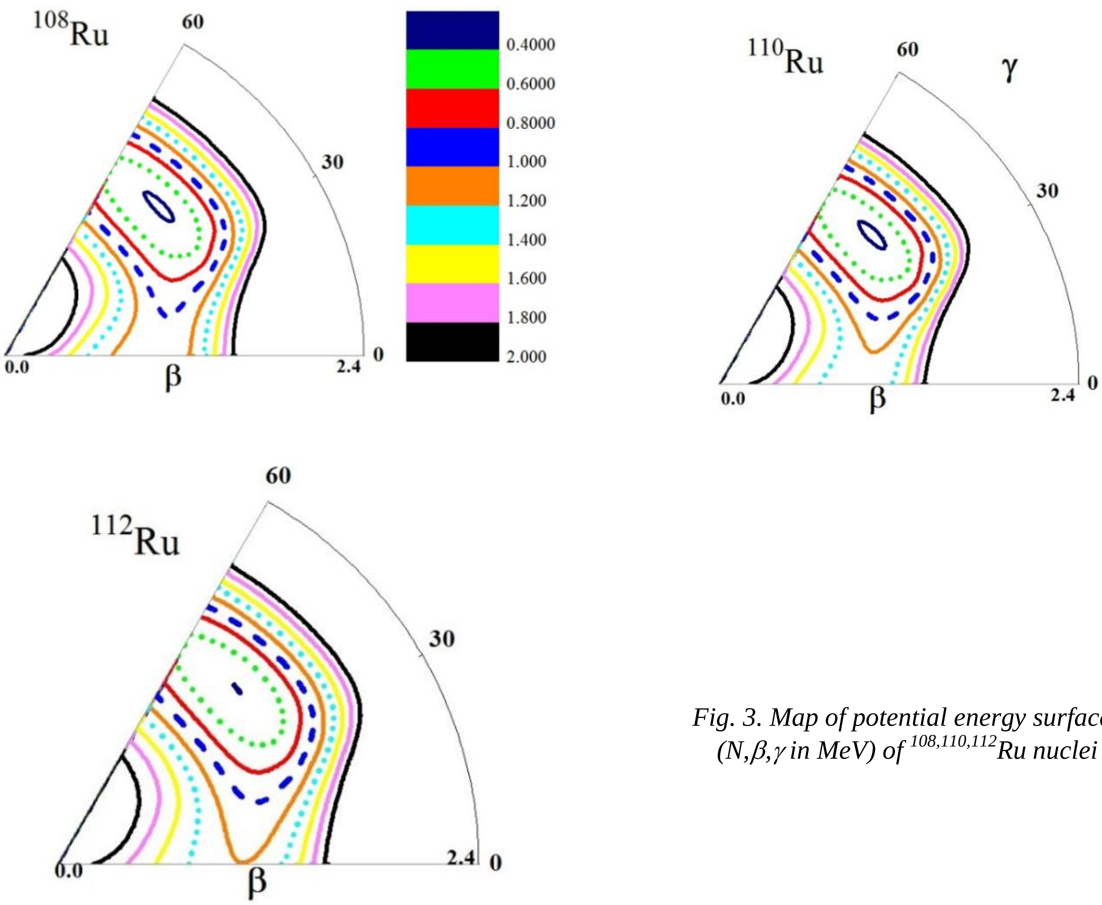

Fig. 3. Map of potential energy surfaces (N,β,γ in MeV) of ^{108,110,112}Ru nuclei

3. CONCLUSIONS

We have used the phenomenological Interaction Boson Model, IBM-1 to describe the experimental data for the energies and transition probabilities B(E2) in the ^{108}Ru, ^{110}Ru and ^{112}Ru nuclei. The yrast band, gamma band, beta band, electromagnetic transition and potential energy surface of those nuclei have been calculated in term of O(6) limit of IBM-1. The parameters for excitation up to energy 3.8 MeV of those isotopes were established using Hamiltonian of IBM-1. The results of calculation were in acceptable arrangement with the investigational data. It is established that interacting boson approximations for those isotopes are suitable to describe the gamma soft O(6) symmetry.

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ДОСЛІДЖЕННЯ СИМЕТРІЇ O(6) В ІЗОТОПАХ ^{108,110,112}Ru ЗА РОЗРАХУНКОМ IBM-1

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Модель взаємодіючих бозонів (IBM-1) дуже важлива для низькорозташованого комбінованого стана в ядрах рівної маси. Ми описали межу O(6) в ядрах ^{108,110,112}Ru за моделлю IBM-1. Сила B(E2) і рівні енергії в різних діапазонах були встановлені в хорошому узгодженні теоретично і експериментально. Розраховані поверхні потенціальної енергії (PES) цих ядер були запропоновані O(6) характеру.