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THE TERMAL DEFORMATION OF ISOTROPIC BEAM ELEMENTS OF TECHNOLOGICAL EQUIPMENT IN THE MANUFACTURING OF CARBON-CARBON COMPOSITE MATERIALS IN FURNACES OF DIRECT HEATING

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For the isotropic beam elements of the technological equipment (BETE), which are used in the production of carbon-carbon composite materials (CCCM) in direct heating furnaces, the analytical solution of the temperature stress-strain state, taking into account the stiffness of the spring of the current collector, was obtained for the first time in the work. Formulas for calculating the critical diameters of loss of compressive strength are given. Using the example of graphite and molybdenum rods, it was established in the work that the decisive factor of the load-bearing capacity is the loss of stability of the elements under consideration.

INTRODUCTION

In the production of CCCM by the method of thermogradient gas-phase densification in direct heating furnaces, beam elements are often used as equipment, which are in conditions of high temperatures (up to 1500 °C) and compression, which occurs as a result of thermal expansion. It is advisable to study the stress-strain state of such elements at the stage of densification processes planning.

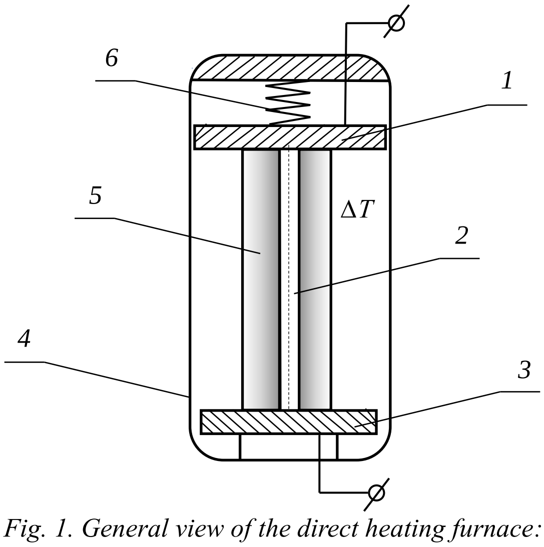

Fig. 1. General view of the direct heating furnace: 1 – upper power supply; 2 – equipment element (heater rod); 3 – lower power supply; 4 – camera body; 5 – product made of CCCM; 6 – spring of the upper current supply

The general view of the direct heating furnace is shown in Fig. 1. It is a chamber 4 that is cooled. During the operation of the furnace, the initial heating of the CCCM frame for densification is often carried out using the rod element of the equipment 2, which is heated to a temperature of 900 °C. For reliable electrical contact, the spring 6 is pressed using a screw mechanism. This spring should also compensate for the thermal expansion of rod 2 and CCCM. At the end of the process, the temperature in the rod and in the center of the product reaches 1300…1500 °C. At the same time, the central areas of the finished product will be compressed between the current leads due to thermal expansion with a temperature difference along the radius of about 600 °C. The equipment element 2 can be a molybdenum rod with a diameter of 6…8 mm or a rod made of carbon material.

1. STRESS-STRAIN STATE (SSS) OF THE FREE HEATER ROD WITHOUT TAKING INTO ACCOUNT THE CURRENT SUPPLY SPRING

First, free expansion of BETE due to heating is considered. The calculation diagram of the deformation of rod element 2 for this case is presented in Fig. 2. It is a rod of constant cross-section fixed from below, which is heated to a temperature of ΔT. Let's determine the displacement of the free end of the rod. If the diameter of the BETE is small, then a hole is made in the upper current lead, which should compensate for the elongation of the rod, provided that electrical contact is maintained. Then moving the free end of the rod will give us information about what the depth of this hole should be.

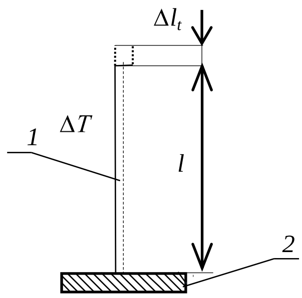

Fig. 2. Calculation diagram the free extension of the heater rod: 1 – BETE; 2 – lower current supply

The elongation of BETE can be found using the expression for temperature deformations ε_t [1]:

$$\varepsilon_t = \alpha \Delta T = \frac{\Delta l_t}{l}, \tag{1}$$

where α is the coefficient of temperature linear expansion of the material (CTLE); Δl_t – elongation of the free end of the rod due to its heating.

Table 1 shows the initial data and calculation results of the movement of the free end of the heater rod (l = 1 m), which is made of MPG-7 graphite at different temperatures [2].

Table 1

Temperature movements of a BETE free end of the MPG-7 graphite

T, °C	$\alpha\cdot10^6$, 1/deg	$\Delta\, l$, mm
1	2	3
200	9.0	1.8
400	13.0	4.8
600	16.0	9.6
800	19.0	15.2

2. SSS OF THE COMPRESSED HEATING ROD WITHOUT TAKING INTO ACCOUNT THE SPRING OF THE CURRENT SUPPLY

Fig. 3 shows a rod of constant cross-section, which is located between two current leads. Its ends cannot move vertically. The temperature has changed by ΔT. Let's determine the axial stress σ [1]:

$$\sigma = E\left(\frac{\Delta l}{l} + \alpha\Delta T\right), \quad (2)$$

where E – modulus of elasticity of the rod material. In (2) in the case of fixed ends $\Delta l = 0$. Thus we have:

$$\sigma = E\alpha\Delta T\,. \quad (3)$$

As can be seen from equation (3), the compressive stresses in the rod do not depend on its diameter and length.

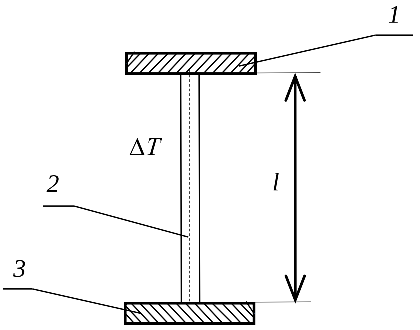

Fig. 3. Calculation diagram of the heating rod deformation without taking into account the spring of the current supply: 1 – upper current lead; 2 – equipment element; 3 – lower current lead

Table 2 shows the initial data and the results of calculations of the axial stress of BETE at different temperatures, from 200 to 800 °C. The length of the rod is 1 m. Data for MPG-7 graphite [2] were taken for CLTE. The compressive strength of MPG-7 graphite of the highest grade is 103.00 MPa [2]. Fig. 4 shows the graphical dependence of the axial stress on the temperature, which is non-linear due to the fact that the CLTE and the modulus of elasticity depend on the temperature.

Table 2

Calculation of the stresses of the BETE due to heating

T, °C	$\alpha\cdot10^6$, 1/deg	E, GPa	$\sigma(T)$, MPa
1	2	3	4
200	9.0	7.10	12.78
400	13.0	7.35	38.22
600	16.0	7.50	72.00
800	19.0	7.84	119.17

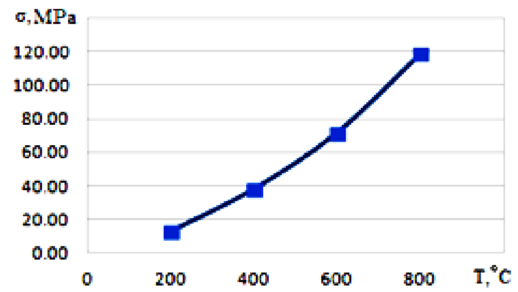

Fig. 4. Dependence of the axial stress of the rod on the temperature without taking into account the stiffness of the current lead spring

3. STRESSED-STRAIN STATE OF BETE, TAKING INTO ACCOUNT THE SPRING OF THE CURRENT SUPPLY

The calculation scheme for this case is shown in Fig. 5. It consists of a heating rod with a diameter d (BETE), length l_1 and a spring with stiffness C. The values characterizing the stressed-strain state of the rod will be denoted by the subscript "*1*".

Thus, we get a static nondeterministic system consisting of two parts. The first section is a heating rod, and the second is a spring with stiffness C. This system deforms in the axial direction due to the heating of the first section.

The generalized view of the calculation scheme in this case is shown in Fig. 6,a.

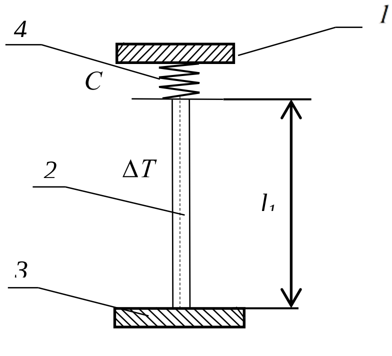

Fig. 5. Calculation scheme of the heating rod taking into account the spring of the current supply: 1 – upper current supply; 2 – BETE; 3 – lower current supply; 4 – spring of the upper current supply

As can be seen from Figs. 6,b and c, the thermal elongation of the first section will consist of the sum of the elastic elongations of the first and second sections:

$$\Delta l_t = \Delta l_1 + \Delta l_2. \quad (4)$$

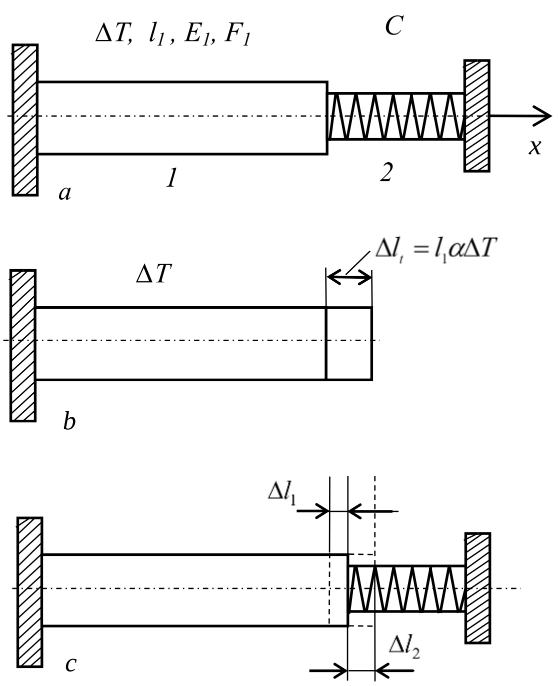

Fig. 6. Deformation of the heater rod together with the spring: a – the system consists of two sections; b – the first section is heated and lengthens by the amount Δl_t; c – thermoelastic deformation of the entire system

In other words, the spring prevents the first section from elongating, (4) is the geometric side of the problem.

The static side of the problem is that the longitudinal force N_x on all sections of the system will be the same and will be equal to the reactions in the resistances (in the current leads). If there is a previous longitudinal compression of the system, then it will be added to the chart of longitudinal forces according to the principle of superpositions [1].

The physical side of the problem [1]:

$$\Delta l_t = l_1 \alpha \Delta T ,$$

$$\Delta l_1 = l_1 \varepsilon_1 = l_1 \frac{\sigma_1}{E_1} = \frac{l_1 N_x^T}{E_1 F_1} , \quad (5)$$

$$\Delta l_2 = \frac{N_x^T}{C} .$$

where F_1 – cross-sectional area of BETE.

After substituting equations (5) into (4), we get the expression for calculating the longitudinal force N_x^T:

$$\frac{l_1 N_x^T}{E_1 F_1} + \frac{N_x^T}{C} = l_1 \alpha \Delta T,$$

$$N_x^T = \frac{\alpha \Delta T l_1}{\dfrac{l_1}{E_1 F_1} + \dfrac{1}{C}}. \quad (6)$$

In the case when $C \to 0$, $N_x \to 0$ we have a free end. When $C \to \infty$, we have a rigid support, $N_x = \alpha \Delta T E_1 F_1$, the formula for the longitudinal force turns into the one resulting from (2).

To obtain the stress in the rod, it is enough to divide the value of the axial force by the cross-sectional area [1]:

$$\sigma_x = \frac{N_x^{\Sigma}}{F_1} . \quad (7)$$

In the case of preliminary compression by force N_x^{Π}, formula (6) takes the form:

$$N_x^{\Sigma} = \frac{\alpha \Delta T l_1}{\dfrac{l_1}{E_1 F_1} + \dfrac{1}{C}} + N_x^{\Pi} . \quad (8)$$

In order to verify that at $C \to \infty$, $N_x \to \alpha \Delta T E_1 F_1$ calculations of axial stresses were carried out at $E = 7.84$ GPa; $\alpha = 19 \cdot 10^{-6}$ 1/deg; $\Delta T = 800$ °C for different values of the stiffness of the spring C and the diameter of rod. The results are given in Table 3.

Table 3

The value of BETE stresses σ_x, MPa at different diameters and stiffness of the spring

d, mm	C, kN/m				
	2	20	200	2000	20000
25	0.03	0.31	3.02	24.58	86.05
15	0.09	0.85	8.02	49.96	104.6
5	0.77	7.27	46.9	103.3	117.4

As can be seen from the data shown in Table 3, with a spring stiffness of 20000 kN/m and a rod diameter of 5 mm, the axial stress of the BETE is close to that obtained earlier with absolutely rigid resistances $\sigma_x = 119{,}17$ MPa (see Table 2).

Table 4 contains the initial data for calculations according to formula (8).

The results of calculations for BETE, which is made of MPG-7 graphite according to formula (8), are presented in Table 5 at different values of the diameter of the graphite rod. The following notations are accepted: $\frac{l_1}{E_1 F_1}$ – axial ductility of the graphite rod, ductility of the spring $1/C = 5 \cdot 10^{-5}$ m/N, η – coefficient of strength reserve for axial stresses due to temperature expansion; taking into account the preliminary compression of 500 N at the compressive strength limit of MPG-7 graphite $[\sigma] = 103.00$ MPa.

Table 4

Input data for temperature SSS calculations

Physical quantity	Value	
	MPG-7 graphite	Molybdenum
Temperature drop ΔT, °C	800	800
CLTE $\alpha \cdot 10^6$, 1/grad	19	6,2
Modulus of elasticity E, GPa	7,84	160
Rod length l_1, m	1	1
Spring stiffness, $C \cdot 10^{-4}$ N/m	2	2
Pre-compression N_x^{Π}, N	500	500

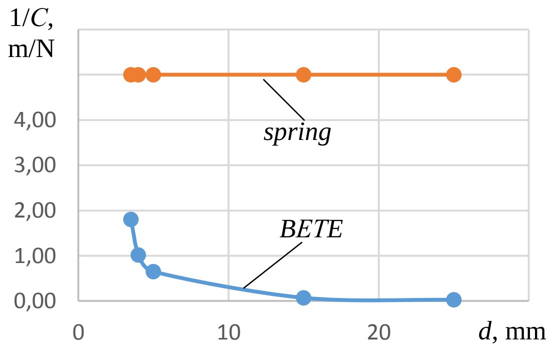

Fig. 7. Comparison of the compliance of the BETE and the current supply spring for different values of its diameter

On the basis of the data entered in Table 5, it can be concluded that the destruction of the BETE due to reaching the strength limit during compression occurs at a diameter of 3 mm.

Fig. 7 shows a graph for comparing the axial compliance of the BETE for different values of its diameter with the compliance of the current supply spring. As can be seen from the graph, the component corresponding to the compliance of BETE becomes significant at diameters that are close to the critical ones for the loss of rod strength.

In order to avoid selecting the critical diameter of the BETE, we obtain the cross-sectional area using formula (7). Let's write down the strength condition:

$$\sigma_x = \frac{N_x^{\Sigma}}{F_1} \le [\sigma_x]. \quad (9)$$

Instead of N_x^{Σ} let's substitute its expression (8) in (9). After bringing similar ones, we get a quadratic equation with respect to F_1 of the form:

$$aF_1^2 + bF_1 + c = 0, \quad (10)$$

where $a = E_1[\sigma]$; $b = [\sigma]l_1C - \alpha\Delta T l_1 E_1 C - N_x^{\Pi} E_1$; $c = -N_x^{\Pi} l_1 C$.

One positive root of the equation (10) was obtained $F_1 = 7.02 \cdot 10^{-6}\,\text{m}^2$ = 0.0702 cm^2. It corresponds to the value of the diameter $d = 2.99$ mm, which coincides with what was obtained by selection and is given in Table 5. With a length of $l_1 = 0.5$ m, the failure occurs at $d = 2.78$ mm. For molybdenum, the critical diameter of temperature compression at a length $l_1 = 1\,\text{m}$ is 2.72 mm, that is, it is almost no different from that obtained for graphite.

4. BUCKLING OF THE ROD ELEMENT OF THE EQUIPMENT UNDER CONDITIONS OF THERMAL EXPANSION

The analysis of the results regarding the critical diameter of graphite BETE failure due to thermal compression shows that the obtained diameter values are very small. In addition, the dangerous diameter depends slightly on the length of the rod element. At the same time, the elements under consideration have a sufficiently large flexibility, so it is appropriate to consider the buckling problem for them.

According to [1], the critical strength of the loss of stability for the rod element of the structure, which has a rigid clamping at the ends (Fig. 8), will be calculated according to the formula:

$$P_{cr} = \frac{4\pi^2 E J_{min}}{l^2}, \quad (11)$$

where E – modulus of elasticity at a given temperature; J_{min} – moment of inertia in the plane of minimum stiffness of the section, for a rod of round cross-section:

$$J_{min} = \frac{\pi d^4}{64}, \quad (12)$$

l – rod lenghth.

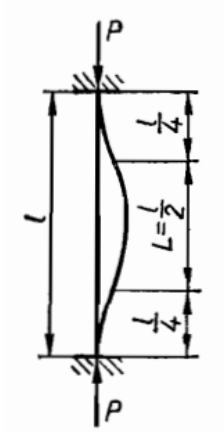

Fig. 8. Calculation scheme for determining the critical strength of the loss of stability of the rod element

The safety margin of the structure under consideration is defined as a fraction of the critical force of buckling and the axial force of temperature expansion. At the same time, we will reduce the diameter of the rod until the safety margin approaches unity. The calculation results for MPG-7 graphite (at $l_1 = 1\,\text{m}$) are given in Table 6, and for molybdenum in Table 7.

The data presented in Tables 6 and 7 confirm the general information about the structural buckling. For example, for a graphite rod, the critical value of the diameter of the loss of stability is 16 mm (for molybdenum, 7 mm), although this diameter corresponds to a compressive stress of only 3.98 MPa, and the loss of strength due to thermal expansion occurs at a diameter of 3.0 mm (see Table 5), that is, the destruction caused by the loss of stability occurs at stresses much lower than the strength limit of the material.

The following values of the critical diameter of buckling were obtained for the length of the BETE $l_1 = 0{,}5\,\text{m}$: graphite MPG-7 11 mm, molybdenum 5 mm.

CONCLUSIONS

The paper investigates the temperature resistance of beam elements of technological equipment used in the production of CCCM in direct heating furnaces. The calculation scheme of the problem is built taking into account the stiffness of the furnace current supply spring, and the deformation model contains such a type of destruction as loss of stability. In the course of numerical studies, it was proved that the buckling is decisive for this case.

Table 5

Axial stresses of a graphite rod at different diameters

$l_1 = 1\ m$						
d, mm	F, cm^2	$\frac{l_1}{E_1 F_1} \cdot 10^5$, m/N	N_x^T, N	N_x^Σ, N	σ_x, MPa	η
25	4.909	0.026	302	802	1.63	63.01
15	1.77	0.072	300	800	4.53	22.76
5	0.196	0.65	269	769	39.17	2.63
3	0.071	1.8	223	723	102.33	1.006

Table 6

Axial force and critical buckling force of a graphite rod at different diameters (l = 1m)

d, mm	$J_{\min} \cdot 10^9$, cm^4	N_x^Σ, N	σ_x, MPa	P_{cr}, N	$\eta = \frac{P_{cr}}{N_x^\Sigma}$
20	7.85	802	2.55	2430	3.03
18	5.153	801	3.15	1594	1.99
16	3.217	800	3.98	996	1.24
15	2.485	800	4.53	769	0.962

Table 7

Axial force and critical buckling force of a molybdenum rod at different diameters (l = 1m)

d, mm	$J_{\min} \cdot 10^{10}$, cm^4	N_x^Σ, N	σ_x, MPa	P_{cr}, N	$\eta = \frac{P_{cr}}{N_x^\Sigma}$
10	4.91	599	7.63	3100	5.18
7	1.18	599	15.56	744.5	1.24
6	0.636	599	21.18	401.8	0.67

The article provides formulas for calculating the critical diameters of the loss of compressive strength due to heating, as well as calculations for determining the buckling critical diameters of rod elements at a temperature of 800 °C, which are made of MPG-7 graphite and molybdenum, and have a length of 1 and 0.5 m.

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ТЕМПЕРАТУРНІ ДЕФОРМАЦІЇ ІЗОТРОПНИХ СТЕРЖНЕВИХ ЕЛЕМЕНТІВ ТЕХНОЛОГІЧНОГО ОСНАЩЕННЯ ПРИ ВИГОТОВЛЕННІ ВУГЛЕЦЬ-ВУГЛЕЦЕВИХ КОМПОЗИТНИХ МАТЕРІАЛІВ У ПЕЧАХ ПРЯМОГО НАГРІВУ

М.В. Мельтюхов, Я.В. Кравцов

Для ізотропних стержневих елементів технологічного оснащення (СЕТО), яке використовується при виготовленні вуглець-вуглецевих композиційних матеріалів (ВВКМ) у печах прямого нагріву, в роботі вперше отримане аналітичне рішення температурного напружено-деформованого стану з урахуванням жорсткості пружини струмовідводу. Наводяться формули для обчислення критичних діаметрів втрати міцності на стискання. На прикладі графітових і молібденових стержнів у роботі встановлено, що вирішальним фактором несучої здатності є втрата стійкості елементів, що розглядаються.