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TRIVELPIECE-GOULD MODES OF WAVEGUIDE PARTIALLY FILLED WITH NON-NEUTRAL PLASMA. PART 3

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The possibility of explaining the nature of low-frequency oscillations observed in devices with non-neutral plasma by the instability of the relative azimuth motion of non-neutral plasma components and by anisotropy of the ion distribution function is discussed. The resonance condition for ions with a diocotron mode with a finite value of the longitudinal wave vector k_z is studied. Numerical estimations are made for the plasma parameters and unstable oscillations, that are characteristic for experiments. Frequencies, growth rates, and other characteristics and features of the expected electron-ion instability are estimated. The conclusion is made that the nature of low-frequency oscillations observed in devices with non-neutral plasma can be explained by this instability.

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INTRODUCTION

In the papers [1, 2] the behavior of the frequencies of electron eigenmodes (Trivelpiece-Gould modes) with the azimuth number m=+1 of a waveguide partially filled with non-neutral plasma with an excess of electrons was analyzed. It was shown that the family of lower hybrid modes and the diocotron mode, due to the Doppler shift caused by the rotation of electrons along the azimuth, can turn out to be low-frequency in the laboratory frame of reference. They can have the order of characteristic ion frequencies. Under these conditions, electron-ion instability is possible due to the relative motion along the azimuth of the non-neutral plasma components. In non-neutral plasma, electrons always rotate faster than ions. This is a general property of non-neutral plasma. This situation coincides with the condition for the origin of the Buneman instability, under which electrons move faster than ions [3].

In the MISTRAL device, in which non-neutral plasma is produced, unstable low-frequency oscillations are observed. They run along the azimuth, have an azimuthal numbers m=1, 2 and frequency of the order of the cyclotron frequency of the working gas ions, $\omega \sim \omega_{ci}$ [4-6]. The nature of the fluctuations has remained unexplained reliably for several decades. Low frequency oscillations were also observed in many other experiments (see, for example, [7-9]).

In this paper, we investigate the possibility of explaining the nature of low-frequency oscillations observed in experiments by the instability of the relative motion along the azimuth of electrons and ions of non-neutral plasma. Using the results of papers [1, 2, 10, 11], the characteristics of the expected electron-ion instability are estimated for the values of parameters typical for experiments [4-9]. The frequencies and growth rates of oscillations are estimated depending on the intensities of the radial electric and longitudinal magnetic fields. The peculiarities of this instability are compared with the characteristic features of unstable oscillations observed in experiments.

All results in present paper, as well as in [1, 2, 11], are presented as dependences on the parameter

$$q=2\omega_{pe}^2/\omega_{ce}^2. \tag{1}$$

Parameter q (1), along with the charge neutralization coefficient $f=n_i/n_e$, determines the ratio between the radial electric and longitudinal magnetic fields. It determines the characteristic frequencies of both electrons and ions, and the equilibrium and stability of non-neutral plasma itself. Just the quantities, that determine the parameter q, are controlled and changed in the experiment.

1. RESONANCE OF AN ION WITH A DIOCOTRON MODE

Within the framework of the model of a waveguide partially filled with a non-neutral cold homogeneous plasma, it was found [1, 2], that the electronic modes of the slow lower hybrid family (in [1, 2] they are denoted by "SLH") and diocotron mode (mode "1" in [1, 2]) with an azimuth number m=1 and a finite value of the longitudinal wave vector k_z can be low-frequency for certain values of the parameter q and coefficient of plasma charge neutralization f. In this range of parameters, electron modes can interact resonantly with ions, which should lead to electron-ion instability.

Let us determine at what values q and other parameters the frequency of the diocotron mode coincides with the resonant ion frequencies. We determine also the values of the resonant frequencies themselves. This will make it possible to estimate the frequencies and growth rates of unstable oscillations, which can be expected in such non-neutral plasma, and to evaluate the possibility of explaining by this instability the nature of unstable oscillations observed in experiments with non-neutral plasma [4-9].

The condition for the resonance of an ion with a wave in non-neutral plasma has the form [12]:

$$\omega_{res} \approx m\omega_{rot}^i + n\Omega_i \tag{2}$$

(m=0, +1, +2, … , n=0, ±1, ±2, …), where

$$\omega_{rot}^i = \left(-\omega_{ci} + \Omega_i\right)/2 > 0 \tag{3}$$

is the “slow” ion rotation frequency, $\omega_{ci}=e_iB/(m_ic)>0$,

$$\Omega_i=\mathrm{sgn}(e)\left(\omega_{ci}^2-\frac{4eE_r}{m_ir}\right)^{1/2}=\omega_{ci}\left[1+\frac{m_i}{m_e}q(1-f)\right]^{1/2}$$

$$=|\omega_{ce}|\left[\left(\frac{m_e}{m_i}\right)^2+\left(\frac{m_e}{m_i}\right)q(1-f)\right]^{1/2} \qquad (4)$$

is the “modified” cyclotron frequency of ion. Depending on q value, the frequency Ω_i (4) can take values from ω_{ci} to the maximum value $\omega_{ci}(1+m_i/m_e)^{½} >> \omega_{ci}$. Resonance (2) under condition $n=0$ is the Cherenkov resonance, and under condition $n\neq 0$ it is the cyclotron resonance.

As it is shown in [2], the frequency of the diocotron mode ω_1 is equal to

$$\omega_1=(q/4)|\omega_{ce}|\left\{\left[\left(R_p/R_w\right)^2-f\right]-\left(k_zR_p\right)^2\right\}. \qquad (5)$$

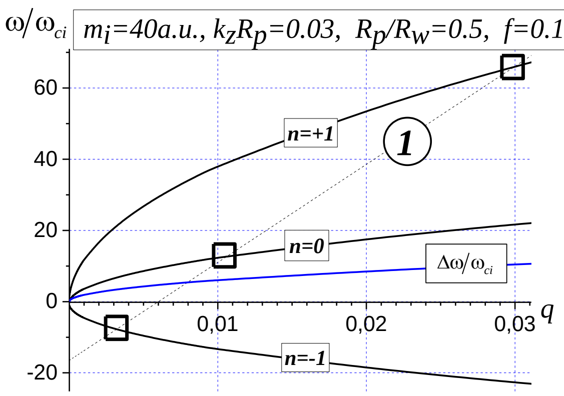

Fig. 1. Resonant frequencies (2) of argon ions at azimuth wave number m=+1 and resonance multiplicities n=+1, 0, –1. The frequency of the diocotron mode (5) is also shown by the dotted line and the number “1”. The intersections of mode “1” and resonant frequencies are indicated by squares. Dependence Δω (15) is also shown. The numerical values of the calculation parameters are indicated at the top of the figure. All frequencies are normalized to ω_{ci}

It reaches the area of low (ion) frequencies and zero frequency under the condition $(R_p/R_w)^2>f$ (see Fig. 2,a in [2]). We consider this condition to be satisfied. The behavior of diocotron mode and resonant frequencies of ions (2) for $m=1$ and lower resonance multiplicities $n=+1$, 0, –1 are shown in Fig. 1. In experiments [4-9], unstable low-frequency oscillations rotating in the positive direction ($\omega/m>0$) are observed. That is why we consider positive resonant frequencies (2). When $m=1$ the lowest positive resonant ion frequency (2) is the frequency with $n=0$. This is the Cherenkov resonance between the electron mode and the ions. The electron-ion instability under the conditions of this resonance was studied in [10] at $k_z=0$. Let us consider this resonance ($\omega_1=\omega_{res}$) at $k_z\neq 0$ and $f\neq 0$.

Substituting in the left hand side of equation (2) the expression for the frequency of diocotron mode (5) and putting in the right hand side of equation (2) $m=1$ and $n=0$ we get the equation for q, at which the frequency of diocotron mode is resonant for ions:

$$\frac{m_i}{2m_e}\left[q\left(\frac{R_p^2}{R_w^2}-f\right)-\left(k_zR_p\right)^2\right]= \qquad (6)$$

$$=-1+\left[1+\frac{m_i}{m_e}q(1-f)\right]^{1/2}.$$

By introducing a variable

$$y\equiv\frac{\Omega_i}{\omega_{ci}}=\left[1+\frac{m_i}{m_e}q(1-f)\right]^{1/2}\geq 1, \qquad (7)$$

we obtain from (6) a simpler equation for y:

$$\eta y^2-2y-\eta-k_z^2R_p^2\frac{m_i}{m_e}+2=0. \qquad (8)$$

In (8), the notation is introduced

$$\eta\equiv\left(R_p^2/R_w^2-f\right)/(1-f). \qquad (9)$$

The behavior of η depending on f is shown in Fig. 2. We are interested in the area where value η is positive: $(R_p/R_w)^2>f$. As follows from (9), always $\eta\leq 1$, and when $R_p/R_w<1/2$, the inequality $\eta << 1$ is satisfied.

From the two roots of equation (8), only one root that is equal to

$$y=\eta^{-1}\left[1+\sqrt{(1-\eta)^2+\eta\left(k_zR_p\right)^2\frac{m_i}{m_e}}\right], \qquad (10)$$

satisfies inequality (7). We study only it. Solution (10) determines the resonant frequency (2) and the q value in the point of intersection with diocotron mode (5):

$$\frac{\omega|_{res}}{\omega_{ci}}=\frac{1}{2}(y-1),\quad q_{res}=\frac{m_e}{m_i}\frac{y^2-1}{1-f}. \qquad (11)$$

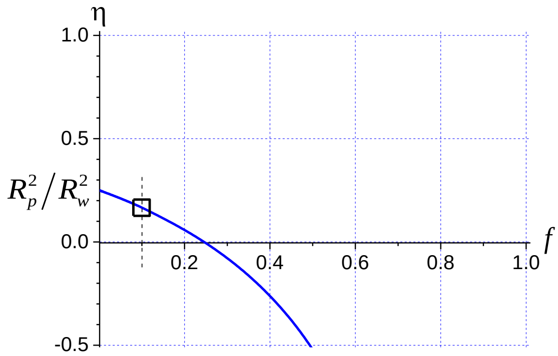

Fig. 2. Dependence η on f (9) for $R_p/R_w=0.5$. The square denotes the value of η at f =0.1, discussed in Section 2

The dependences of the resonance frequency (11) on the parameters f and $(R_p/R_w)^2$ in the area $(R_p/R_w)^2>f$ are shown in Figs 3. For $k_zR_p=0.03$ (upper curve) inequality (12) is satisfied, for $k_zR_p=0.003$ (lower curve) the opposite inequality is satisfied. The values of other calculation parameters are shown in Figs 3.

Expressions for y (10) and ω_{res} (11) are simplified in limiting cases. So, when

$$\eta(\eta-1)^{-2}\left(k_zR_p\right)^2<<m_e/m_i \qquad (12)$$

solution (10) takes the form

$$\frac{\omega_{res}}{\omega_{ci}}\approx\frac{1}{\eta}-1=\frac{1-R_p^2/R_w^2}{R_p^2/R_w^2-f}. \qquad (13)$$

In the case opposite to inequality (12), solution (11) is given by the expression

$$\frac{\omega_{res}}{\omega_{ci}} \approx \frac{1}{2\eta}\left[1+\left(\eta k_z^2 R_p^2 \frac{m_i}{m_e}\right)^{1/2}\right]-\frac{1}{2}. \qquad (14)$$

Expressions (10), (13), (14), as well as (5), are valid when $k_zR_p<<1$, $q<<q_{max}=1/(1-f)$. As follows from (13), (14), the resonance frequency is proportional to magnetic field and inversely proportional to the mass of ion ($\omega_{res}\propto\omega_{ci}$). Expression (14) gives larger values ω_{res} than (13), The latter does not depend on k_zR_p. In Fig. 3 the behavior of resonance frequency ω_{res} is presented depending on f and $(R_p/R_w)^2$. In Fig. 3,a the results of calculations using the exact formulas (10), (11) for argon (m_i=40 *a.u.*). In Fig. 3,b – using the simplified formula (13).

With a decrease in the parameter (k_zR_p), the frequency ω_{res} decreases to the level (13). The value q_{res}, at which the resonance between diocotron mode (5) and an ion (2) ($\omega_1=\omega_{res}$) is reached, is determined by the second equality (11). When $f\to(R_p/R_w)^2$, the value η decreases, and the frequency ω_{res} and q_{res} increase.

As follows from (9)-(14), when $(R_p/R_w)^2<1/2$ the resonance of ions with diocotron mode is realized at frequencies greater and much greater than the cyclotron frequency ω_{ci}. Only when $(R_p/R_w)^2>1/2$ the resonant frequency ω_{res} (13) can become less than ω_{ci}. This is possible when condition (12) is satisfied, i.e., when parameter (k_zR_p) has a sufficiently small value. For illustration in Fig. 4 the area is presented where $\omega_{res}<\omega_{ci}$. It has the form of a triangle (shaded in Fig. 4). At the upper boundary of the triangle, where $(R_p/R_w)^2=1$, we have $\omega_{res}=0$. At the bottom of the triangle, we have $\omega_{res}=\omega_{ci}$.

As we see, the resonance of an ion (2) with the diocotron mode (5) can be achieved at frequencies both lower and higher than the ion cyclotron frequency ω_{ci}.

2. RESONANCE OF AN ION WITH DIOCOTRON MODE IN EXPERIMENTS

At what frequencies the resonance of the ion (2) with the diocotron mode (5) is achieved when the values of the parameters are typical for experiments [4-9] and others similar to them? We estimate these frequencies from formulas (9)-(14) and Figs. 2, 3. We use characteristic values of the parameters in experiments.

We estimate the minimum value k_z for a plasma cylinder of finite length L in the “usual” way: $k_z\sim\pi/L$. For typical dimensions of the plasma cylinder in experiments $R_p\sim1$ cm, $L\sim100$ cm we obtain $(k_zR_p)^2\sim10^{-3}$.

We put the degree of neutralization f equal to f=0.1, and the geometric parameter R_p/R_w is put equal to R_p/R_w=0.5. In this case, the parameter η is equal to η=0.167 (see Fig. 2). The parameter (m_e/m_i), for example, for argon, which is often used in experiments, is equal $m_e/m_i\approx1.36\cdot10^{-5}$. For such parameter values, the inequality opposite to (12) is well satisfied: the left hand side of inequality (12) is 18 times greater than the right hand side. Under these conditions, the resonant frequency is determined by formula (14), which gives the value: $\omega_{res}/\omega_{ci}\approx13$. A approximately the same value ($\omega|_{res}/\omega_{ci}\approx12.8$) is also given by Fig. 3,a. It is indicated by a square in Fig. It’s interesting, the corresponding value of Ω_i is equal, according to (14), $\Omega_i\approx27\omega_{ci}$. For resonance multiplicity n>0, the resonant value Ω_i will be even greater, and for n<0, it will be smaller.

The above estimates indicate that under the chosen values of the parameters typical for experiments [4-9], the coincidence of the frequency of the diocotron mode (5) with the resonant frequency of ions (2) at azimuth wave number m=1 and resonance multiplicity n=0 occurs at a frequency significantly exceeding the ion cyclotron frequency ω_{ci}. For lighter ions, a thinner and longer plasma cylinder, formula (13) will become applicable and the resonant frequency can be comparable to the ion cyclotron frequency ω_{ci}.

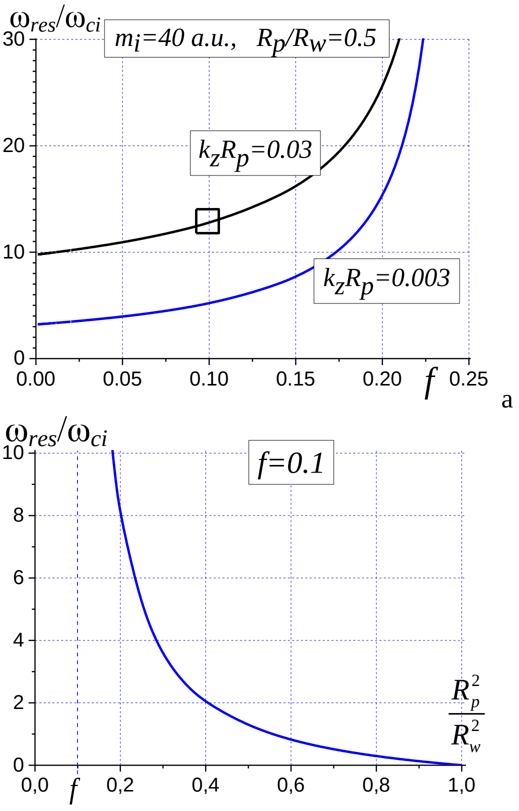

Fig. 3. Values of resonant ion frequencies (2) at the point of intersection with diocotron mode (5): depending on f for two values of the parameter k_zR_p (k_zR_p=0.03; 0.003) (a) and depending on the parameter $(R_p/R_w)^2$ for f=0.1 (b). The square in Fig. 3,a indicates the value ω_{res} for parameter values typical for the experiments, discussed in Section 2

3. FREQUENCIES OF EXPECTED INSTABILITY

It is natural to expect that the electron-ion instability, excited due to the relative motion of electrons and ions, has a maximum growth rate in the vicinity of the resonance $\omega_1=\omega_{res}$. It was shown in [10] that in the hydrodynamic approximation and for k_z=0, instability exists only in the vicinity of this resonance. However, the hydrodynamic approximation is inapplicable for description of ions in a real experiment. Ions produced in crossed fields have a developed transverse motion, which, by the way, was analyzed in detail in the same paper [10], and the kinetic description is necessary.

The form of the distribution function of such ions, which adequately takes into account the features of their production, was determined in paper [13]. It is highly anisotropic and non-Maxwellian in transverse energies. This is an additional reason for plasma instability. According to the analytical and numerical calculations performed in [11] for a waveguide completely filled with non-neutral plasma, taking into account the finite value of the longitudinal wave vector $k_z \neq 0$ and the ion distribution function, electron-ion instability also exists far from the resonance in the region $q<q_{res}$, although with a slower growth rate ($\mathrm{Im}\omega \equiv \gamma \sim \omega_{pi}$). Moreover, instability also takes place at values that are an order of magnitude smaller than q_{res} (11).

We believe that the behavior of the modes does not change radically when the waveguide is partially filled. Let us indicate a number of factors that should lead to a decrease in the frequency of unstable oscillations in comparison with the resonant frequency of ions.

1. According to [11], in the range of values $q<q_{res}$, the frequency ω of the unstable mode generally follows the resonant frequency of ions (2) (in the case under consideration $m=1$, $n=0$ it is equal $\omega_{res} \approx \omega^i_{rot}$ (3)), remaining lower than it by the value

$$\Delta\omega \equiv \left|\mathrm{Re}\,\omega - \omega^i_{rot}\right| \sim \gamma \sim \omega_{pi} = \qquad (15)$$

$$= \omega_{ce}\left[q\frac{f}{2}\frac{m_e}{m_i}\right]^{1/2} = \omega_{ci}\left[q\frac{f}{2}\frac{m_i}{m_e}\right]^{1/2} \simeq \Omega_i\left[\frac{f}{(1-f)}\right]^{1/2}.$$

From the last equality it is seen that the value $\Delta\omega$ can be comparable with Ω_i. (Dependence $\Delta\omega$ (15) on q is shown in Fig. 1.) In this case, the frequency of unstable oscillations ω becomes less or much less than Ω_i and can become of the order of the ion cyclotron frequency ω_{ci} even at those values q for which $\Omega_i >> \omega_{ci}$. Oscillations with such frequencies are observed, for example, in experiments with non-neutral plasma at the MISTRAL plasma device [4-6]. The nature of these oscillations is discussed for decades. In other experiments (for example, in [9]), the frequency of unstable oscillations exceeds the cyclotron frequency ω_{ci} by tens of times and is associated by the authors with the frequency of radial oscillations of the ion Ω_i in regimes with a strong radial electric field, when $\Omega_i >> \omega_{ci}$.

2. The frequency ω^i_{rot} (3) itself decreases when the parameter q decreases in the area $q<q_{res}$. The frequency of unstable oscillations ω should also decrease with it. For this reason, the frequency of unstable oscillations also can become comparable with ω_{ci}.

The above estimations and reasoning are of a qualitative nature. For a quantitative analysis, it would be necessary to solve the problem about the stability of non-neutral plasma partially filling a waveguide taking into account for adequate kinetics of ions produced in crossed fields by ionization of the working (residual) gas, similarly to how it was done in [11] for a waveguide completely filled with plasma. To do this, it is necessary to generalize, to the case of a waveguide partially filled with plasma, the technique for solving the kinetic equation for ions, developed in [14].

Note that in estimations similar to those given above, it is necessary correctly to take into account the finite value of k_z because of the sharp dependence of the frequency of the diocotron mode ω_1 (5) on k_z.

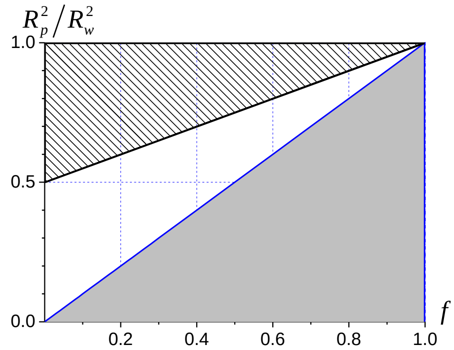

Fig. 4. Area on the plane of parameters (f $(R_p/R_w)^2$), inside which $\omega|_{res}/\omega_{ci} \le 1$ (shaded). Calculation by formula (13). The gray shaded area, located at the bottom of the picture, in which $(R_p/R_w)^2 < f$, is not considered. Inequality (7) is not satisfied in it

4. PECULIARITIES OF SLH MODE

The family of slow lower hybrid (SLH) modes also passes through the area of low (ion) frequencies and through zero frequency (see [1, 2]). However, in the long-wavelength limit ($k_z R_p << 1$), it passes at much smaller values q than the diocotron mode. Accordingly, the growth rates of the electron-ion instability associated with SLH modes are slower than the growth rates associated with diocotron mode. As calculations in [11] showed, in the case of complete filling of the waveguide with non-neutral plasma, the maximum growth rate of SLH modes is less than the cyclotron frequency ($\gamma \sim \omega_{pi} < \omega_{ci}$), and when $f<<1$ it is much less ($\gamma << \omega_{ci}$).

In addition, all radial modes of the SLH family pass through the low-frequency area and can be unstable, while in the experiment only oscillations having a radial dependence of the amplitude corresponding to the lowest radial mode are observed.

For these reasons, interaction of diocotron mode with ions seems to be more preferable for explaining the nature of the low-frequency instability observed in experiments [4-9] and others than interaction of SLH modes with ions.

5. REMARKS

The frequencies of low frequency oscillations observed in experiments are often compared with the ion cyclotron frequency ω_{ci}. However, in non-neutral plasma, the cyclotron frequency is not a characteristic frequency of either ions or electrons. Characteristic frequencies are the "modified" cyclotron frequencies $\Omega_{e,i}$ (4). They are equal to the frequencies of radial oscillations of electrons/ions in crossed magnetic and electric fields. The value Ω_i (4) is expressed through the used variables and parameters q, f and m_e/m_i. In a radial electric field that is weak for ions ($\omega_{ci}^2 >> |4eE_r/m_i r|$ or otherwise $q(1-f) << m_e/m_i$), the frequency Ω_i is close to the cyclotron frequency, $\Omega_i \approx \omega_{ci}$. In a strong electric field ($\omega_{ci}^2 << |4eE_r/m_i r|$ or $m_e/m_i << q(1-f) \le 1$), the frequency Ω_i significantly exceeds ω_{ci}. The maximum value Ω_i (4) is reached in the Brillouin limit for electrons ($q=q_{max}=1/(1-f)$). For example, for argon we have:

$(\Omega_i)_{max} \approx (m_e/m_i)^{½} |\omega_{ce}| \approx (m_e/m_i)^{½} |\omega_{ce}| \approx 271\ \omega_{ci}$. The "modified" cyclotron frequency of electrons Ω_e depends on parameters q, f. It decreases to zero as the parameter q increases. It is interesting that when

$$q \approx (1-m_e/m_i)/(1-f) = q_{max}(1-m_e/m_i), \qquad (16)$$

the frequencies Ω_e and Ω_i become equal. At larger q than (16), the frequency Ω_e becomes even less than the "modified" cyclotron frequency of ions Ω_i.

CONCLUSIONS

The discussed electron-ion instability has the same peculiarities as the low-frequency oscillations have, observed in experiments [4-9] and others:

1. The instability can be low-frequency in the long-wavelength limit $k_z R_p$<<1, like the mode observed in the experiment.

2. The instability should take place in a wide range of changing of the parameter q, as well as the unstable oscillations are observed in the experiment.

3. The amplitude of unstable mode has a dependence on the radius corresponding to the lowest radial mode (i.e., it has no zeros within the interval $0<r<R_p<R_w$). (This was shown in [15] for q<<1 and f<<1). Oscillations, observed in experiments, have a radial dependence of such a form only.

4. Measurements [5, 6] show that low-frequency oscillations grow rapidly in experiments, so that the instability should have a fast growth rate: $\gamma \geq \omega_{ci}$. Such growth rates of the discussed instability are possible at sufficiently large values of q or/and f. If (according to [10, 11] and (15)), we estimate the growth rate of diocotron mode as $\gamma \sim \omega_{pi}$, then from (15) it follows that the growth rate γ will exceed ω_{ci} in the area, where $qf > 2m_e/m_i$. Unstable low-frequency oscillations in experiment arise at field strengths and plasma densities at which the parameter q satisfies this inequality.

The considered instability, that arises due to the relative azimuth motion of electrons and ions of non-neutral plasma, as well as anisotropy and the non-Maxwellian nature of the ion distribution function, can be low-frequency and can claim to explain the nature of unstable low-frequency oscillations observed in experiments with non-neutral plasma [4-9] and others.

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МОДИ ТРАЙВЕЛПІСА-ГУЛДА ХВИЛЕВОДУ, ЧАСТКОВО ЗАПОВНЕНОГО ЗАРЯДЖЕНОЮ ПЛАЗМОЮ. ЧАСТИНА 3

Ю.М. Єлісеєв

Обговорюється можливість пояснити природу низькочастотних коливань, які спостерігаються у пристроях із зарядженою плазмою, нестійкістю відносного руху по азимуту компонентів плазми і анізотропією функції розподілу іонів. Досліджено умову резонансу іонів з діокотронною модою з кінцевим значенням поздовжнього хвильового вектору k_z. Проведено числові оцінки для параметрів плазми та нестійких коливань, характерних для експериментів. Оцінено частоти, інкременти зростання та інші характеристики та особливості очікуваної електрон-іонної нестійкості. Зроблено висновок про можливість пояснити нею природу низькочастотних коливань, що спостерігаються в установках із зарядженою плазмою.