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SIMULATION OF PLASMA WAKEFIELD FOCUSING AND SELF-FOCUSING OF A SHORT SEQUENCE OF ELECTRON BUNCHES DEPENDING ON THE BUNCH LENGTH, SHAPE AND DISTANCE BETWEEN BUNCHES

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By using two-dimensional numerical simulation, the ratio between the effects of wakefield focusing and self-focusing during the propagation of a short sequence of electron bunches in plasma has been simulated. Cases of dominant wakefield focusing have been demonstrated. In addition, the collection data is presented on the parameters of the bunch length, shape and distance between bunches correspond to certain ratios of wakefield focusing and self-focusing that can be used in further studies.

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INTRODUCTION

The issues of focusing electron bunches and, in particular, relativistic electron bunches, do not lose their relevance [1]. The focusing of electron bunches has been the subject of research in a number of publications [2-11]. The intensity of focusing of electron bunches in plasma often significantly exceeds the intensity and degree of magnetic focusing [12]. Two focusing effects take place in the case of plasma. Firstly, it is well known self-focusing mechanism based on the compensation of the space charge of a bunch injected into plasma. Secondly the use of a transverse plasma wakefield can additionally strengthen focusing effect. Some preliminary studies of focusing by an excited wakefield has been presented in [13]. Homogeneous focusing of electron bunches by an excited wake-field has been studied in [6].

The question of the dependence of the ratio of wakefield focusing and self-focusing (focusing of shielded bunch by the magnetic field of its own current) on the length of sequence bunches and the distance between them has not been sufficiently studied. The paper considers this issue using numerical simulation by code LCODE [14].

The cylindrical coordinate system (r, z) was taken. The time tis normalized to ω_{pe}^{-1}, all the distances – to $c\omega_{pe}^{-1}$, the density – to the unperturbed plasma electron density n_{0e}, the beam current I_b to $mc^3/e = 17$ kA, the fields – to $mc\omega_{pe}/e$, where m is the electron mass, e – is the electron charge, c – is the speed of light, ω_{pe} – is the plasma electron frequency. These normalisations are used also in the figures.

We present the results of numerical simulation of plasma wakefield excitation by a sequence of relativistic electron bunches, obtained with the 2.5D quasistatic code LCODE that treats the plasma as a cold electron fluid, and the bunches as ensembles of macroparticles. The parameters were similar to those of the plasma wakefield experiments, in which the electron beam represented by a regular sequence of electron bunches excites the wakefield in the uniform plasma. The maximum simulation time is $80.1T_0$, here $T_0 = \omega_{pe}^{-1}$. The distribution of bunches is Gaussian in the transversal directions. The plasma ions represent the immobile background.

Spatial step equals $0.1cT_0$. Time step for plasma electrons equals $0.1T_0$. Time step for beam electrons equals $0.1\gamma_b^{1/2}T_0$, here $\gamma_b = 5$ is relativistic factor of bunch electrons.

We presenttemporal dependences in selected points of observation.

RESULTS OF NUMERICAL SIMULATION

To study the ratio between wakefield focusing and self-focusing, we consider sequences of two bunches with different current distributions along the bunch length (we consider a two-dimensional simulation picture). Considering the indicated simulation pattern and the fact that we assume a fixed bunch length, we can assert that the charge of the bunch's changes in accordance with the indicated principles. We consider sequences of two bunches, for each of which the current is along the bunch: remains constant with a given amplitude along the bunch; changes according to the law of cosine; increases linearly along the bunch; decreases linearly along the bunch. The length of the bunches and the distance between them is measured in plasma wavelength λ. Let us first consider the case of a sequence of two bunches, the current of which is distributed uniformly along the entire length of the bunch. Thanks to this example, we will find out the ratio of wake focusing and self-focusing, the method for determining this ratio. The base length of the sequence bunches in this case was the 0.5 λ. The base distance between bunches is also 0.5 λ. Fig. 1 shows results of numerical simulation of wakefield excitation in plasma by a sequence of two bunches. The current distribution along the length of the bunches is uniform (homogeneous). The following describes the logic for determining the ratio of wake focusing and self-focusing, which were applied for other cases. In Fig. 1 shows the radial electric force E_r. When the bunch just flies into the plasma, its space charge is not compensated, since the plasma electrons have not yet had time to scatter out from the bunch region.

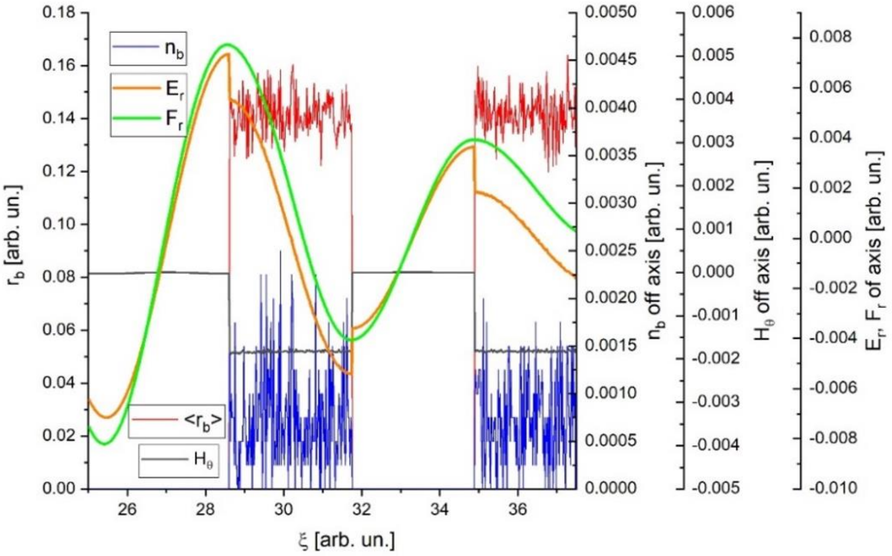

Fig. 1. $<r_b>$ – average bunch radius; n_b – off axis average electron density of bunches; H_θ – off axis magnetic field; E_r – off-axis transverse electric field; F_r – average focusing force; t = 3. Cosine current distribution of bunches

This force, due to the action of E_r, pushes the bunch electrons apart. In addition, there is another 2nd force, which focuses the bunch by its own magnetic field H_θ of the bunch, created by bunches current. The total force F_r is equal to $F_r = E_r - H_\theta$.

If the bunch does not change, then its current does not change and H_θ does not change, and H_θ is different from zero only in the region of a bunch. Outside the bunch (along ξ) $H_\theta=0$. E_r created by the bulk charge of the bunch is $E_r=H_\theta$. Therefore, as seen in Fig. 1, in the head of the bunch, where the plasma electrons have not yet had time to scatter from the bunch $F_r \approx 0$.

By the way, H_θ acts only on the bunch, H_θ does not act on plasma electrons. And E_r acts on plasma electrons. Plasma electrons under the action of E_r scatter with time to the sides, and thus charge screening of the bunch arises with time. In Fig. 1 to the middle of the bunch, charge screening occurs and $E_r = 0$ is reached. Focusing is carried out only because of the H_θ and $E_r = H_\theta$.

Here, one can say, only self-focusing occurs, that is, only focusing by its own magnetic field. Further along the bunch, the plasma electrons diverge by inertia due to the momentum received from the space charge. And this discrepancy by inertia means the appearance of a wake field E_r of the opposite sign from repulsive to focusing.

Here only self-focusing occurs (only focusing by bunches) own magnetic field. Further along the bunch, the plasma electrons diverge by inertia due to the momentum received from the space charge. And this discrepancy by inertia means the appearance of a wake field E_r of the opposite sign from repulsive to focusing.

As seen in Fig. 1, $E_r=H_\theta$ is reached by the end of the 1st bunch. So, $F_r=2H_\theta$. From $E_r=H_\theta$, one can conclude that by the end of the 1st bunch, the ratio of self-focusing and wake focusing is 50/50%.

From $E_r=H_\theta$, one can conclude that by the end of the 1st bunch, the ratio of self-focusing and wake focusing is 50/50%. As can be seen from Fig. 1, the heads of all bunches, except for the 1st one, are defocused, and the tails are focused.

Fig. 2 shows the excitation of the wake field by a sequence of bunches, the current distribution in which changes according to the cosine law.

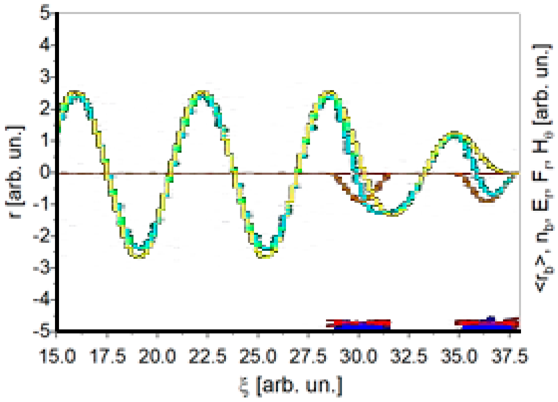

Fig. 2. $<r_b>$ – average bunch radius (red line); n_b – off axis average electron density of bunches (blue line); H_θ – off axis magnetic field (brown line); E_r – off axis transverse electric field (light blue line); F_r – average focusing force (yellow line); t=3. Cosine current distribution of bunches

For Fig. 1, it is easy to calculate the ratio of self-focusing and wakefield focusing in accordance with the graph. However, it is clear that absolute values are not important for determining the ratio. Only their ratios make sense. For all other figures, the ratios were put on using graphical data.

For the graphs in Fig. 1. $F_r^{self\text{-}focus} \approx H_\theta = 0.00175$. At the end of the first bunch the ratio of focusing power to self-focusing $F_r/F_r^{self\text{-}focus} = 2.3$. $F_r^{wake}/F_r^{self\text{-}focus} \approx 1.3$, where $F_r^{wake} = F_r - F_r^{self\text{-}focus}$. Thus, the approximate ratio of wakefield focusing and self-focusing is 50/50%, as predicted, taking into account all the features and errors of numerical simulation. For the second bunch $F_r/F_r^{self\text{-}focus} \approx 4.46$, $F_r^{wake}/F_r^{self\text{-}focus} = 3.46$. Thus, it is obvious that a change in the bunch current profile from uniform to one that changes according to the cosine law leads to an increase in the ratio of wakefield focusing to self-focusing, and an increase in the influence of wake focusing in the process.

In Fig. 3 the process of excitation of the wakefield was shown by a sequence of bunches with a linearly increasing current distribution. It can be seen that according to the data in Fig. 3, there is no wakefield focusing in the region of the first bunch.

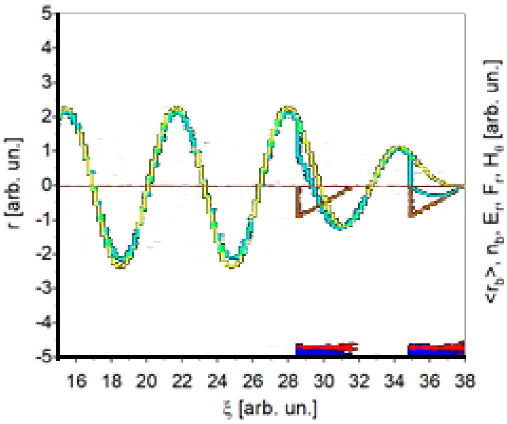

Fig. 3. <r_b> *– average bunch radius (red line);* n_b *– off axis average electron density of bunches (blue line);* H_θ *– off axis magnetic field (brow nline);* E_r *– off axis transverse electric field (light blue line);* F_r *– average focusing force (yellow line); t=3.*
Linear growth of current distribution of bunches

In the case of the second bunch of the sequence, the following relation are observed: $F_r^{wake}/F_r^{self\text{-}focus} = 1.5$.

Fig. 4 shows the process of excitation of the wakefield by a sequence of two bunches, the current (and charge) of which decreases linearly. In this case, it is obvious that for both bunches, the predominant wakefield focusing of the bunches is observed. In this case, the heads of the bunches are defocused, while the tails of the bunches are subject to wake focusing. In the case of rectangular bunches (see Fig. 1), the bunch tails are also mostly focused. A similar result is also observed in the case of bunches, the current of which is distributed according to the cosine law. The absence of focusing as such (only inhomogeneous with an amplitude less than the maximum) is observed only for the first bunch of the sequence for the distribution of the growing bunch current. For Fig. 5, the distance between bunches is used in the case of a sequence of two bunches as the distance between the end of the first bunch and the beginning of the second bunch. In the case of a uniform distribution of each bunch with changing both the bunch lengths and the distance between the bunches the ratio of wakefield focusing and self-focusing for the second bunch does not exceed 3.2 and reaches a minimum, equal to 1.15, both at bunch lengths and at a distance between bunches equal to the wavelength.

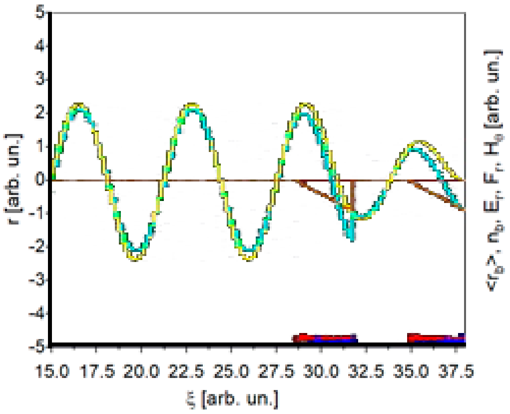

Fig. 4. <r_b> *– average bunch radius (red line);* n_b *– off axis average electron density of bunches (blue line);* H_θ *– off axis magnetic field (brown line);* E_r *– off axis transverse electric field (light blue line);* F_r *– average focusing force (yellow line); t=3.*
Linear decrease of current distribution of bunches

In the case of linearly decreasing and cosine current profiles, only wakefield focusing in the region of bunch tails is predominantly observed due to the features of these distributions. Closer to the tails of the bunches, the amplitude of the intrinsic current of the bunch approaches 0. For a linearly increasing distribution, the situation is similar, except that the ratio is more inhomogeneous on the graph, and a maximum is reached at a bunch length of 0.8. Fig. 5 shows the relationship between wakefield focusing and self-focusing as a function of the distance between the bunches and the length of the bunches. From the analysis of the data, it is obvious that, regardless of the shape of the bunches, especially with an increase in their length and especially in the region of the second bunch of the sequence.

It is possible to identify areas in which only self-focusing predominates.

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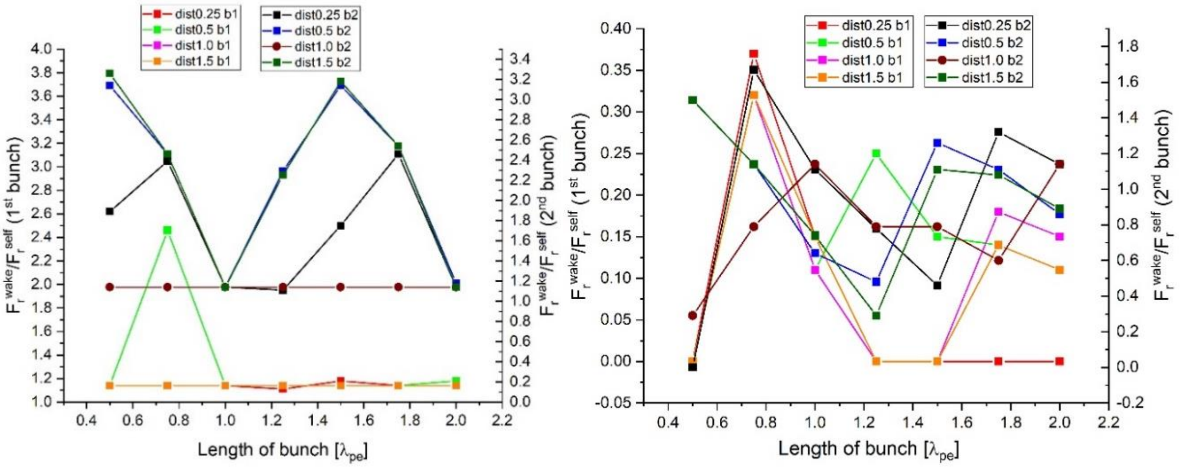

Fig. 5. Dependence of the ratio of wakefield focusing and self-focusing as a function of the lengths and distance between the bunches for a uniform (left fig.) and linearly increasing (right fig.) bunch current distribution

In addition, there are optimal ratios of the length of the bunches and the distance between them, at which the maximum ratio of wakefield focusing and self-focusing is observed. In addition, there are obvious cases in which, almost regardless of the length and distance between the bunches, the same ratio between wakefield focusing and self-focusing is observed. In particular, such cases are considered in Fig. 5 for uniform bunches. For example, in Fig. 5 (right) shows the results for a linearly increasing current density of relativistic electron bunches. An extremum can be observed at a bunch length of 0.75 of the plasma wavelength. In this case, at different distances between the bunches, the ratio of wake focusing and self-focusing for the first bunch is about 0.375 and for the second bunch about 1.8. The data obtained from figures can be applied in further studies.

CONCLUSIONS

The ratio between the effects of wakefield focusing and self-focusing during the propagation of a short sequence of electron bunches in plasma has been simulated. It is shown that the intensity of focusing strongly depends on the shape of the bunches, their lengths, the distance between the bunches, and their number. For most parameters, the intensity of the wakefield focusing exceeds the intensity of self-focusing. In particular, for the end of second relativistic electron bunch, the current of which is distributed uniformly along the bunch and which follows through the period of the plasma wave after the first bunch, the intensity of wakefield focusing exceeds the intensity of self-focusing more than in three times.

In the case of a uniform distribution of each bunch with changing both the bunch lengths and the distance between the bunches the ratio of wakefield focusing and self-focusing for the second bunch does not exceed 3.2 and reaches a minimum, equal to 1.15, both at bunch lengths and at a distance between bunches equal to the wavelength. For the middle of the second bunch, the current of which decreases linearly along the bunch, the intensity of wakefield focusing exceeds the intensity of self-focusing in two times.

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МОДЕЛЮВАННЯ КІЛЬВАТЕРНОГО ФОКУСУВАННЯ ТА САМОФОКУСУВАННЯ КОРОТКОЇ ПОСЛІДОВНОСТІ ЕЛЕКТРОННИХ ЗГУСТКІВ У ЗАЛЕЖНОСТІ ВІД ДОВЖИНИ, ФОРМИ ЗГУСТКІВ ТА ВІДСТАНІ МІЖ НИМИ

Д.С. Бондар, В.І. Маслов, І.М. Оніщенко

За допомогою двовимірного чисельного моделювання змодельовано співвідношення між ефектами кільва-терного фокусування та самофокусування при поширенні короткої послідовності електронних згустків у плазмі. Продемонстровані випадки домінуючого кільватерного фокусування. Крім того, представлені збірні дані про параметри довжини, форми згустку та відстані між згустками, що відповідають певним співвідношенням фокусування кільватерним полем та самофокусування, які можуть бути використані в подальших дослідженнях.