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MATHEMATICAL SIMULATION OF THE STRESS-STRAIN STATE OF THE WINDING OF A CLOSED MAGNETIC SYSTEM

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Mathematical modeling of the stress-strain state of the winding of a closed magnetic system was carried out, which consists in the development of a three-dimensional geometric model of the helical winding and the determination of the values of the characteristics of the magnetic system, which are best from the point of view of meeting the requirements of the technical task of the designed object.

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INTRODUCTION

The most important property of the magnetic configuration of the system designed to contain high-temperature plasma is the presence of magnetic surfaces, which are formed by the trajectories of the lines of force of the magnetic field at multiple rotations around the main axis of the torus. Obtaining three-dimensional geometric models for mathematical modeling of the calculation of the stress-strain state of the winding of a closed magnetic system is the most important and time-consuming design task. Its solution is associated with several simplifications and assumptions: the poles of magnetic windings are modeled by infinitely thin conductors with current; the task of determining the lines of force is solved under the condition that several hundreds of revolutions around the main axis of the torus are sufficient; the traces of the lines of force of the magnetic field in a fixed meridional section are located on a closed curve; the increment of the length element along the line of force of the magnetic field in the direction of the magnetic induction vector, which ensures sufficient accuracy of the estimation calculations, is taken from a few millimeters to tens of millimeters. There are also characteristics that, together with the boundary magnetic surface, decisively affect the retention of the plasma, these are: the magnitude of the angle of rotation transformation; the rate of fall of the specific magnetic volume across the magnetic surfaces (magnetic pit); rate of change of rotational transformation along the radius (width); modulation of the magnetic field strength along the force line. Searching for the optimal combination of these parameters is the main task in mathematical modeling and calculations of the stress-strain state (SSS) of the winding of a closed magnetic system.

1. THREE-DIMENSIONAL GEOMETRIC MODEL OF SCREW WINDING

The method of kinematic modeling was used to obtain a three-dimensional geometric model. The essence of the method is that for the assignment of the surface it is necessary to describe its frame (creating and guiding curve), a family of planes that determine the location of sections and boundary conditions. The description of the kinematic surface of the helical winding (HW) consists of the description of the change in the shape of the drawing curve and the description of the law of movement of this curve in the plane and the law of the change of the curve in space.

Coordinates of the winding line located on the surface of the torus. Consider the torus (Fig. 1) with parameters R_0 and $(a_0 + h/2)$, here h is the height of the pole [1]. The medial line of a normal section is called the line of its section with an "overblown" torus. Denote through φ_-; ϑ_- i φ_+; ϑ_+ toroidal coordinates of the ends of the medial line. The dimensions of the pole are determined by the following parameters – the height h and the difference in coordinates $\vartheta_+ - \bar{\vartheta}$ i $\bar{\vartheta} - \vartheta_-$. The following shows how to find the angular width of the normal pole section, i.e. the difference, from these data $\vartheta_+ - \vartheta_-$.

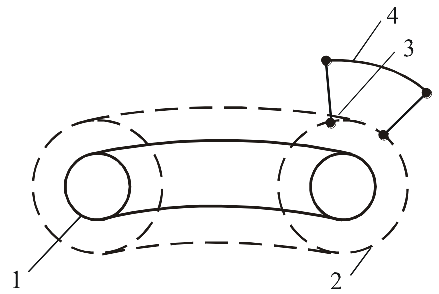

Fig. 1. Determination of the length of the medial line: 1 – torus (R_0, a); 2 – "overblown" torus (R_0, a + h/2); 3 – medial line; 4 – normal section

Suppose that $\vartheta_+ - \bar{\vartheta} = \bar{\vartheta} - \vartheta_-$. The *x*, *y*, *z* coordinates of any point of the "overblown" torus are determined by its angles ϑ and φ by formulas (1)–(3), if in them the radius *a* is replaced by the radius $(a + h/2)$:

$$x = [R_0 + (a + h/2)\cos\vartheta]\cos\varphi;$$
$$y = [R_0 + (a + h/2)\cos\vartheta]\sin\varphi; \qquad (1)$$
$$z = (a + h/2)\sin\vartheta.$$

If the points with coordinates *x*, *y*, *z* are in the given normal plane $\nu(\bar{\vartheta}, \bar{\varphi})$, which passes through the point *M* with coordinates $\bar{\vartheta}, \bar{\varphi}$, then the radius vector of this point *R*, offset from the point, lies in the plane $\nu(\bar{\vartheta}, \bar{\varphi})$ and, therefore, is orthogonal to the previously introduced vector $\vec{\nu}$. Their scalar product is zero

$$(R, \nu) = R_x \nu_x + R_y \nu_y + R_z = 0, \qquad (2)$$

where $R_x = x - x_м$; $R_y = y - y_м$; $R_z = z - z_м$.

The points located on the medial line of the normal section, by definition, lie on the "overblown" torus and in the normal plane. Therefore, their coordinates, firstly, can be written in the form of a system of equations (1), and, secondly, they satisfy condition (2).

Then

$$[R_0 + (a + h/2)\cos\vartheta]\cos\varphi\nu_x + [R_0 + (a + h/2)\cos\vartheta]\sin\varphi\nu_y + (a + h/2)\sin\vartheta\nu_z = x_M\nu_x + y_м\nu_y + z_м\nu_z. \quad (3)$$

The equation connects the angles ϑ and φ points of the medial line lying in the normal section, which is determined by the angles $\overline{\vartheta}$ i $\overline{\varphi}$. To find the derivative $d\varphi/d\vartheta$, we differentiate equation (3) by ϑ, considering φ as a function of ϑ:

$$-\nu_x(a+h/2)\sin\vartheta\cos\varphi-\nu_y(a+h/2)\sin\vartheta\sin\varphi+\nu_z(a+h/2)\cos\vartheta+$$
$$+\frac{d\varphi}{d\vartheta}\left\{-\nu_x R_0+(a+h/2)\cos\vartheta\sin\varphi+\nu_y\left[R_0+(a+h/2)\cos\vartheta\right]\cos\varphi\right\}=0.$$

Then

$$\frac{d\varphi}{d\vartheta}=\frac{\sin\vartheta[\nu_x\cos\varphi+\nu_y\sin\varphi]+\nu_x\cos\vartheta}{[R_0+(a+h/2)\cos\vartheta][\nu_y\cos\varphi-\nu_x\sin\varphi]}(a+h/2). \quad (4)$$

The length of the arc of the medial line is determined by the formula

$$dS^2=dx^2+dy^2+dz^2=[-(a+h/2)\sin\vartheta\cos\vartheta-$$
$$-[R_0+(a+h/2)\cos\vartheta]\sin\varphi d\varphi]^2+[-(a+h/2)\sin\vartheta\sin\varphi d\vartheta+$$
$$+[R_0+(a+h/2)\cos\vartheta]\cos\varphi d\varphi]^2+(a+h/2)^2\cos^2\vartheta d\vartheta^2=$$
$$=(a+h/2)^2 d\vartheta^2+[R_0+(a+h/2)\cos\vartheta]^2 d\varphi^2,$$

from which

$$\frac{dS}{d\vartheta}=\sqrt{(a+h/2)^2+[R_0+(a+h/2)\cos\vartheta]^2\left(\frac{d\varphi}{d\vartheta}\right)^2}. \quad (5)$$

Substituting the value of the derivative into expression (5) $\frac{d\varphi}{d\vartheta}$ we get the differential dependence between the angle ϑ and the arc of the medial line S

$$\frac{d\vartheta}{dS}=\frac{1}{a+h/2}\frac{1}{\sqrt{1+\left[\dfrac{\sin\vartheta[\nu_x\cos\varphi+\nu_y\sin\varphi]+\nu_z\cos\vartheta}{\nu_y\cos\varphi+\nu_x\sin\varphi}\right]^2}}.$$

From here

$$\frac{d\varphi}{dS}=\frac{d\varphi}{d\vartheta}\frac{d\vartheta}{dS}=\frac{\sin\vartheta(\nu_x\cos\varphi+\nu_y\sin\varphi)+\nu_z\cos\vartheta}{[R_0+(a+h/2)\cos\vartheta](\nu_y\cos\varphi-\nu_y\sin\varphi)}\cdot\frac{1}{1+\left[\dfrac{\sin\vartheta(\nu_x\cos\varphi+\nu_y\sin\varphi)+\nu_z\cos\vartheta}{\nu_y\cos\varphi-\nu_z\sin\varphi}\right]^2}.$$

After performing the appropriate mathematical transformations, we obtain:

$$\frac{d\vartheta}{dS}=\frac{1}{a+h/2}\cdot\frac{\nu_y\cos\varphi+\nu_x\sin\varphi}{\sqrt{(\nu_y\cos\varphi-\nu_x\sin\varphi)^2+\left\{\sin\vartheta[\nu_x\cos\varphi-\nu_y\sin\varphi]+\nu_z\cos\vartheta\right\}^2}}; \quad (6)$$

$$\frac{d\varphi}{dS}=\frac{1}{R_0+(a+h/2)\cos\vartheta}\cdot\frac{\sin\vartheta(\nu_x\cos\varphi+\nu_y\sin\varphi)+\nu_z\cos\vartheta}{\sqrt{\nu_y(\cos\varphi-\nu_x\sin\varphi)^2+\left\{\sin\vartheta[\nu_x\cos\varphi+\nu_y\sin\varphi]+\nu_z\cos\vartheta\right\}^2}}. \quad (7)$$

Relations (6), (7) can be considered as a system of two nonlinear differential equations of the first order. Solving this system under initial conditions $\vartheta=\overline{\vartheta}$, $\varphi=\overline{\varphi}$ for $S = 0$, we find dependencies $\vartheta = \vartheta(S)$ and $\varphi = \varphi(S)$ and with their help we will find the desired value of S from the condition $\vartheta_+ = \vartheta(S)$.

You can greatly simplify the task if you use a raster representation of the surface. In this case, the model of the HW surface is presented in the form of a grid consisting of characteristic intersecting lines belonging to the surface. These lines are meridional cross-sections and segments connecting characteristic points when dividing the closed lines limiting the cross-sections. Intersection points of closed curves, which limit the cross-section and segments, form raster nodes, and a set of such points on the modeled surface is a raster. If the distance between the raster nodes is small, then the raster points describe the surface of the HW quite accurately. The three-dimensional geometric model of the HW is shown in Figs. 2a, 2b.

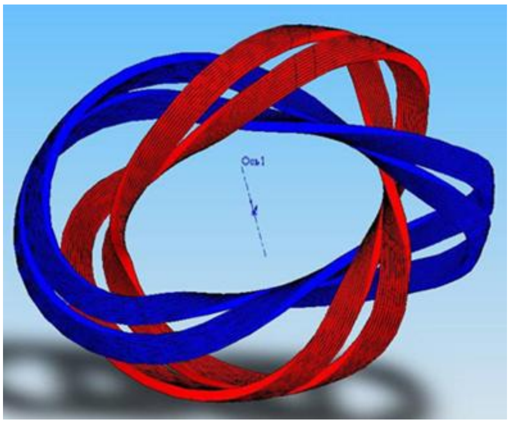

Fig. 2a. Three-dimensional geometric model of the HW (front view)

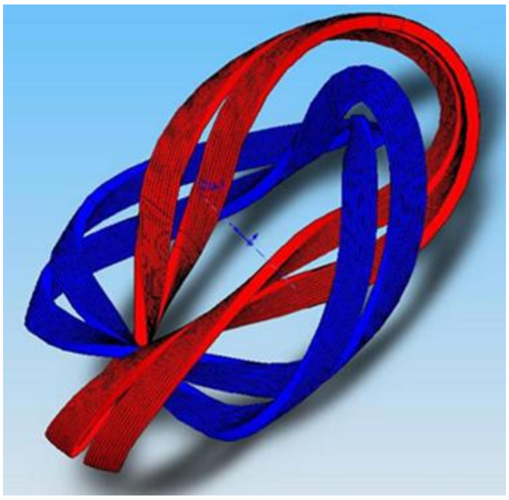

Fig. 2b. Three-dimensional geometric model of the HW (arbitrary direction of gaze)

The formation of the complex surface of the HW is carried out by a set of meridional sections (Fig. 3) [2, 3]. Infinitely thin conductors, considered as calculated, are divided into 720 discrete currents of the element. The length of each element ranges from 10 to 16 mm. The division is carried out by a cutting plane φ = const.

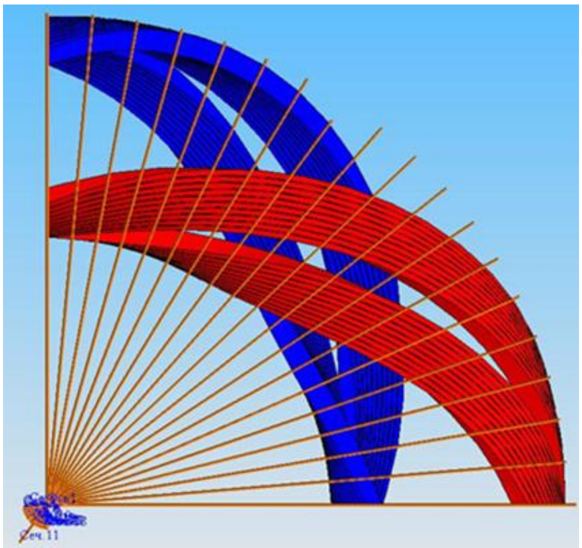

Fig. 3. Formation of the surface of the HW by a set of meridional sections

2. DETERMINATION OF THE CHARACTERISTIC VALUES OF THE MAGNETIC CONFIGURATION

Formulation of the problem. The optimization problem of determining the values of the characteristics of the magnetic configuration is a multi-criteria problem of nonlinear programming due to the presence of five initial parameters and is formulated as the problem of determining the values of the characteristics of the field, which are the best from the point of view of meeting the requirements of the technical task with an unchanged structure of the designed object. Among the initial parameters, we will single out the most critical (separate criterion) – the topology of the magnetic field lines [4] and, thereby, reduce the solution of the multi-criteria problem to a single-criterion one, and the conditions for the performance of the other initial parameters (rotational transformation angle, width, magnetic pit, voltage modulation along of the line of force of the magnetic field [2]) can be attributed to the limitations of the problem, which are determined analytically.

To solve the problem of mathematical modeling of SSS, we will introduce partial criteria for the optimality of characteristic[3]

$$Q_{k_i}(x_1, x_2, \ \dots, x_5),$$

where k = 1, 2, 3, 4; i = 1, 2, 3; $Q_{1_i} = \sigma_{1_i}$ – main stresses acting on the faces of the selected pole element of the magnetic windings; $Q_{2_i} = \tau_{2_i}$ – tangential stresses on a selected element; $Q_{3_i} = \varepsilon_{3_i}$– deformations in the directions of the axes, which are applied to the characteristic points of the end elements; $Q_{4_i} = \varphi_{4_i}$– angles of rotation of the calculated points of the final elements in the direction of the coordinate axes.

The solution to the task of determining the partial optimality criteria of SSS characteristics is reduced to the determination of parameters $(x_1, x_2, \ \dots, x_5)$, which are in the range of available values D and at the same time provide a minimum of all optimality criteria Q_k. For optimality based on a set of criteria, we will introduce a vector optimality criterion $Q(X)$. Then the objective function can be written as

$$\min_{X \in D} Q(X),$$

where $X = (x_1, x_2, \ \dots x_5)$ – vector of controlled parameters; $Q(X)$ – objective function or system performance criterion.

Controlled parameters include:

– geometric characteristics of the torus;

– the law of winding poles on a toroidal surface $\phi = \phi(\vartheta, \alpha, \beta, m_{\square}, \ell)$, where ϑ – the angle characterizing the position of the winding line point in the meridional section; α i β – modulation coefficients; $m_{\square}$ – the number of steps of the helical conductor along the length of the torus; ℓ – the number of occurrences of helical conductors;

– quantities included in the Biot-Savart law

$$\vec{B} = f\left(\vec{J}, d\ell, \vec{S}\right),$$

where $\vec{S}$ – radius vector drawn from the element $\vec{J}\,d\ell$ to the point at which the magnetic field vector is calculated;

– a number of constants characterizing the properties of the materials that make up the winding pole;

– the square of thecross section of winding poles.

The main requirement for the mathematical model of the SSS is that the mathematical description should accurately reflect the magnitudes of stresses and strains occurring in the elements of the magnetic system. The accuracy of the model is determined by the reliability of the results obtained during the tests.

3. CALCULATION MODEL OF SSS

1. It is assumed that the currents flowing through the conductors of the magnetic windings are concentrated in an infinitely thin conductor located in the center of the magnetic windings.

2. Infinitely thin conductors are divided into 720 discrete current cells. The length of each element varies from 10 mm to 16 mm. The division is carried out by a cutting plane (Fig. 4).

3. In the selected coordinate system, the coordinates of the ends of the elements and the coordinates of the midpoints of the segments of each discrete element are calculated.

4. Using the analytical expression of the Biot-Savart-Laplace law

$$\vec{B} = \frac{J}{C}\oint \frac{\left[d\vec{l}\cdot\vec{S}\right]}{S^3},$$

where $\vec{S} = \vec{r} - \vec{r}_0$, radius vector drawn from the current element *Jdl* to the point of observation, the vector of the magnetic field $\vec{B}$ and forces acting on the middle of the discrete element of each *conductor* of the helical winding is calculated.

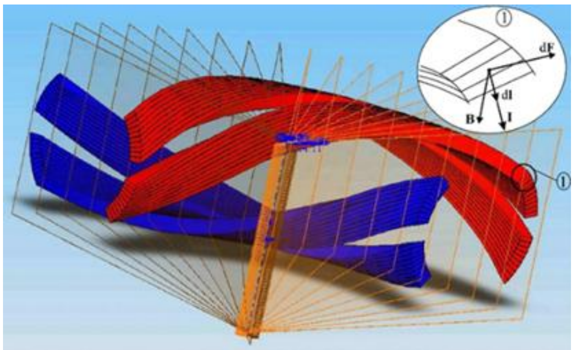

Fig. 4. Vector representation of the calculation of forces acting on discrete current elements

The results of the calculations of the distribution of the magnetic field and the forces acting on the discrete current elements (n is the number of the current element) are shown in Figs. 5–9.

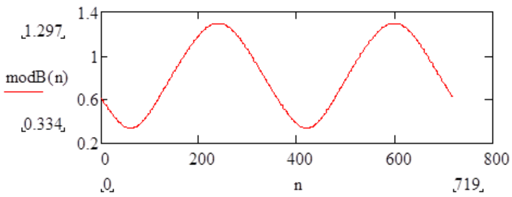

Fig. 5. Distribution of the modulus of the induction vector of the magnetic field Bn(1) along the length of the 1st half-pole

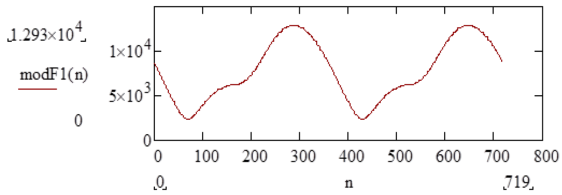

Fig. 6. Distribution of the modulus of forces dFn(1) along the length of the 1st conductor of the 1st half-pole

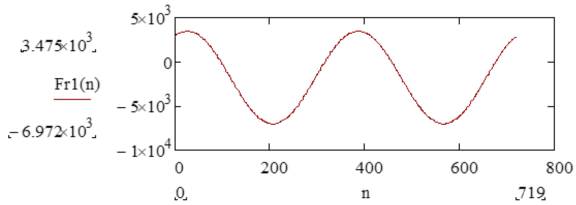

Fig. 7. Distribution of the radial force component dFn(1) along the length of the 1st conductor of the 1st half-pole

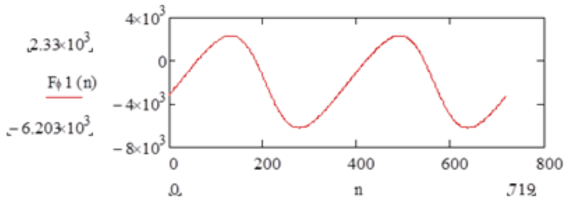

Fig. 8. Distribution of the azimuthal component of the force dFn(1) along the length of the 1st conductor of the 1st half-pole

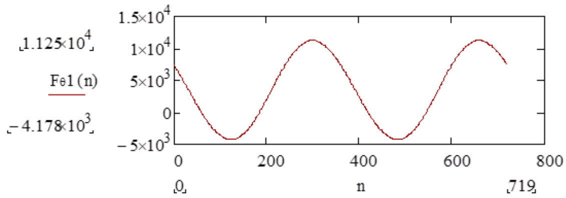

Fig. 9. Distribution of the poloidal force component dFn(1) along the length of the 1st conductor of the 1st half-pole

CONCLUSIONS

Mathematical models and methods of determining the stress-strain state of the winding of a closed magnetic system have been developed. The practical value of the obtained results is that the developed models and methods can be used to calculate electrodynamic forces in the conductors of the helical winding of a closed magnetic system, which allows solving a number of design, technological and operational tasks, namely: – at which currents the forces reach extreme values; in which deformations of the poles lead to distortions of the geometry of the winding poles and how these perturbations in the geometry are reflected in the properties of the magnetic field that hold the plasma.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ОБМОТКИ ЗАМКНУТОЇ МАГНІТНОЇ СИСТЕМИ

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Проведено математичне моделювання напружено-деформованого стану обмотки замкнутої магнітної системи, котре полягає в розробці тривимірної геометричної моделі гвинтової обмотки та визначенні значень характеристик магнітної системи, найкращих з точки зору задоволення вимог технічного завдання проектованого об'єкта.