https://doi.org/10.46813/2022-140-033

THE ELASTICITY THEORY PROBLEM FOR A HOLLOW SPHERE WITH A DISPLACED CAVITY CENTER

M.I. Kopp[1], P.N. Ostapchuk[2], V.V. Yanovsky[1,3]
[1]Institute for Single Crystals, National Academy of Science Ukraine, Kharkiv, Ukraine;
[2]Institute of Electrophysics and Radiation Technologies, National Academy of Science Ukraine, Kharkiv, Ukraine;
[3]V.N. Karazin Kharkiv National University, Kharkiv, Ukraine
E-mail: ostapchuk@kipt.kharkov.ua

In this paper, the elastic state of a sphere is studied, inside which there is a spherical cavity (spherical pore) at an arbitrary distance from the center of the sphere. Expressions for the displacement components, strain, and stress tensors depending on the geometrical parameters of the problem and the pressure values on the surfaces of the outer and inner spheres were obtained.

PACS: 62.20.-x, 61.46.+w

INTRODUCTION

The classical problem of the theory of elasticity associated with the deformation of an elastic hollow sphere (the Lame problem) has been considered in many papers (see, for example, [1–3]). Interest in this problem has not weakened for several decades due to the wide aspect of applying the results of the theory of hollow elastic spheres to various problems in materials science, biomechanics, nanophysics, etc. In articles [4–6], mathematical models of the sclera of the eyeball and glaucoma based on the classical (linear) theory of elasticity were developed. Good agreement with the experimental data [7] was shown by the nonlinear theory of hollow sphere deformation as a mechanical model of the sclera of the eyeball [8]. Another critical task is to model the processes that occur in the vicinity of a micropore in a metal under high pressure. This problem is important since the operational qualities of the metal grow with the «healing» of continuous micro defects [9]. In [10] dealt with the problem of spherically symmetric compression of a ball having a micropore in the center. In [10], a technique for estimating the loading force that creates a particular deformed condition was provided based on the stated laws of change in the displacement field.

In [11] symmetric deformation of a hollow ball made of a two-component composite under the action of uniformly distributed internal and external pressures was considered. In the same place, a numerical calculation of microstrains in the viscoelastic components of the structure of the ball under consideration was carried out. The results of the classical theory of the deformation of an elastic hollow ball were applied in [12] to study the deformation of a gas-filled pore of radius R located at the center of a spherical body of radius R_B.

Micropores or spherical cavities can be formed at any distance from the center of a spherical body. The situation gets considerably more problematic from a mathematical point of view. A bispherical coordinate system is useful for solving it. The boundary conditions are specified on spheres with centers on the same straight line. The use of a bispherical coordinate system in [13–14] allowed the description of the diffusion evolution of vacancy and gas-filled pores in spherical nanoparticles.

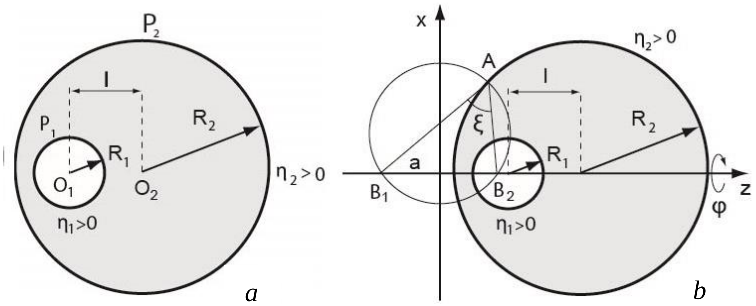
The elastic sphere is looked at in this research for the case of an arbitrary position of a spherical cavity (spherical pore) under the impact of a uniform distribution of external and internal pressures.

FORMULATION OF THE PROBLEM

Let's consider an elastic sphere of radius R_2 inside which there is a spherical cavity (pore) of radius R_1, and $R_1 < R_2$. The center of the cavity O_1 is displaced by a distance l from the center of the sphere O_2. The condition that the cavity is located inside the sphere leads to a purely geometric inequality:

$$R_1 / R_2 + l / R_2 < 1 \ . \quad (1)$$

Pressure P_1 acts inside the cavity, and pressure P_2 acts outside the sphere (Figure, a). The purpose of the work is to determine the elastic state of a hollow sphere with a displaced center of the cavity. For this, it is convenient to use a bispherical coordinate system. In it, each point A in space is assigned a triple of numbers (η, ξ, φ), where $\eta = \ln(\frac{|AB_1|}{|AB_2|})$, $\xi = \angle B_1 A B_2$, φ is the polar angle (see Figure, b).



A spherical cavity (pore) of radius R_1 in an elastic sphere of radius R_2 is displaced from the center of sphere by a distance l (a); the surfaces of the cavity and sphere in the bispherical coordinate system are coordinate planes: $\eta = const$ (b)

The relationship between bispherical and Cartesian coordinates has the form [15]:

$$x = \frac{a \cdot \sin\xi \cdot \cos\varphi}{\cosh\eta - \cos\xi}; \qquad y = \frac{a \cdot \sin\xi \cdot \sin\varphi}{\cosh\eta - \cos\xi};$$

$$z = \frac{a \cdot \sinh\eta}{\cosh\eta - \cos\xi}, \tag{2}$$

and surfaces of the cavity and sphere are given by the following relations:

$$\eta_1 = arsinh\left(\frac{a}{R_1}\right), \qquad \eta_2 = arsinh\left(\frac{a}{R_2}\right), \tag{3}$$

where the parameter a is determined through the values of their radii R_1, R_2, and the distance between their centers l:

$$a = \frac{\sqrt{[(l-R_1)^2 - R_2^2][(l+R_1)^2 - R_2^2]}}{2 \cdot l}.$$

The deformations caused by forces applied to the body surface are described by the equilibrium equation in vector form [1, 2]:

$$2(1-\sigma) grad\ div\, \vec{u} - (1-2\sigma) rot\ rot\, \vec{u} = 0, \tag{4}$$

where $\vec{u}$ is the displacement vector of the deformed body, σ is Poisson's ratio. The general solution of equation (4) was found by Papkovich (see, for example, [16])

$$\vec{u} = \vec{\Phi} - \frac{1}{4(1-\sigma)} grad(\vec{r} \cdot \vec{\Phi} + \Phi_0), \tag{5}$$

where $\vec{\Phi}$ is any harmonic vector satisfying the equation

$$\Delta\vec{\Phi} = grad\ div\, \vec{\Phi} - rot\ rot\, \vec{\Phi} = 0,$$

Φ_0 is an arbitrary harmonic scalar whose equation

$$\Delta\Phi_0 = div\ grad\ \Phi_0 = 0$$

is satisfied.

We assume the sphere is isotropic, i.e., the symmetry is retained along the coordinate φ, and the displacement vector of the sphere and spherical hollow has only the η component:

$$u_\eta = u_\eta(\xi, \eta); \qquad u_\xi = u_\varphi = 0. \tag{6}$$

In the case of a coaxial (central) location of the spherical hollow, the displacements $\vec{u} = u_\eta(\xi,\eta) \cdot \vec{e}_\eta$ satisfy the following equation:

$$rot\, \vec{u} = 0. \tag{7}$$

Therefore, taking into account the vector identity $\Delta\vec{u} = grad\ div\, \vec{u} - rot\ rot\, \vec{u}$ and relation (7), from (4) we obtain the equation for the displacement vector

$$\Delta\vec{u} = grad\ div\vec{u} = 0$$

or its components $u_\eta(\xi,\eta)$

$$\frac{\partial}{\partial\eta}\left(\frac{1}{\operatorname{ch}\eta - \cos\xi}\frac{\partial u_\eta}{\partial\eta}\right) + \frac{1}{\sin\xi}\frac{\partial}{\partial\xi}\left(\frac{\sin\xi}{\operatorname{ch}\eta - \cos\xi}\frac{\partial u_\eta}{\partial\xi}\right) = 0 \tag{8}$$

with boundary conditions

$$u_\eta(\xi,\eta)|_{\eta=\eta_1} = u_{R_1}, \qquad u_\eta(\xi,\eta)|_{\eta=\eta_2} = u_{R_2}. \tag{9}$$

The general solution (8) in the bispherical coordinate system has the form:

$$\begin{aligned} u_\eta(\xi,\eta) = \sqrt{2(\operatorname{ch}\eta - \cos\xi)}\Bigg\{ u_{R_1} \sum_{k=0}^{\infty} \frac{\operatorname{sh}(k+1/2)(\eta-\eta_2)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)} \exp(-(k+1/2)\eta_1) P_k(\cos\xi) - \\ - u_{R_2} \sum_{k=0}^{\infty} \frac{\operatorname{sh}(k+1/2)(\eta-\eta_1)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)} \exp(-(k+1/2)\eta_2) P_k(\cos\xi) \Bigg\}, \end{aligned} \tag{10}$$

where $P_k(x)$ are the Legendre polynomials, u_{R_1} and u_{R_2} the constant coefficients and are to be determined from the boundary conditions on the surfaces of the inner and outer spheres:

$$\sigma_{\eta\eta}|_{\eta=\eta_1} = -P_1, \qquad \sigma_{\eta\eta}|_{\eta=\eta_2} = -P_2. \tag{11}$$

Here $\sigma_{\eta\eta}$ is the component of the stress tensor σ_{ik} (see Appendix (A.2)):

$$\sigma_{\eta\eta} = \frac{(2\mu+\lambda)}{a}(\operatorname{ch}\eta - \cos\xi) \cdot \frac{\partial u_\eta}{\partial\eta} - \frac{2\lambda}{a}\operatorname{sh}\eta \cdot u_\eta. \tag{12}$$

Substituting (12) into the boundary conditions (11), we obtain equations for required coefficients u_{R_1} and u_{R_2} :

$$\frac{(2\mu+\lambda)}{a}(\operatorname{ch}\eta-\cos\xi)\cdot\frac{\partial u_\eta}{\partial\eta}|_{\eta=\eta_1}-\frac{2\lambda}{a}\operatorname{sh}\eta\cdot u_\eta|_{\eta=\eta_1}=-P_1, \tag{13}$$
$$\frac{(2\mu+\lambda)}{a}(\operatorname{ch}\eta-\cos\xi)\cdot\frac{\partial u_\eta}{\partial\eta}|_{\eta=\eta_2}-\frac{2\lambda}{a}\operatorname{sh}\eta\cdot u_\eta|_{\eta=\eta_2}=-P_2.$$

The order of further calculations is as follows. Using the general solution (10), we calculate the derivative $\partial u_\eta/\partial\eta$ and the displacement component u_η at $\eta=\eta_1$ and $\eta=\eta_2$. Then we substitute the notation $t=\cos\xi$ in (13) and integrate the resulting equations with respect to the variable t. The calculations, although not complicated, are cumbersome. The result is given by the following expressions:

$$u_{R_1}=\frac{2\left[(\beta_2-\alpha_2)\,P_1-\gamma_1P_2\right]}{\gamma_1\gamma_2-(\alpha_1+\beta_1)(\alpha_2-\beta_2)},\qquad u_{R_2}=\frac{2\left[\gamma_2P_1-(\alpha_1+\beta_1)\,P_2\right]}{\gamma_1\gamma_2-(\alpha_1+\beta_1)(\alpha_2-\beta_2)}, \tag{14}$$

where notations are introduced

$$\alpha_1=\frac{2\mu-3\lambda}{R_1};\quad \beta_1=\frac{3(2\mu+\lambda)}{2a}\cdot\Phi(\eta_1,\eta_2);\quad \gamma_1=\frac{3(2\mu+\lambda)}{a}\cdot\tilde{\Phi}(\eta_1,\eta_2);$$
$$\alpha_2=\frac{2\mu-3\lambda}{R_2};\quad \beta_2=\frac{3(2\mu+\lambda)}{2a}\cdot G(\eta_1,\eta_2);\quad \gamma_2=\frac{3(2\mu+\lambda)}{a}\cdot\tilde{G}(\eta_1,\eta_2).$$

The functions Φ, $\tilde{\Phi}$ and G, $\tilde{G}$ are infinite series containing exponents (see Appendix A. (3)).

Thus, the coefficients u_{R_1} and u_{R_2} depend on the elastic moduli of the material (λ,μ), geometric dimensions (radii $R_{1,2}$ and parameter a), as well as on internal P_1 and external pressure P_2.

STRESS TENSOR OF ELASTIC SPHERE WITH CAVITY

Using a solution to the equilibrium equation (10) and Hooke's law (see Appendix), we give explicit expressions for non-zero components of the stress tensor of an elastic sphere with a cavity:

$$\sigma_{\eta\eta}=\frac{2\mu-\lambda}{a}\operatorname{sh}\eta\sqrt{2(\operatorname{ch}\eta-\cos\xi)}\sum_{k=0}^{\infty}\frac{P_k(\cos\xi)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)}\cdot U+\frac{(2\mu+\lambda)\sqrt{2}}{a}(\operatorname{ch}\eta-\cos\xi)^{3/2}\sum_{k=0}^{\infty}\frac{(k+1/2)P_k(\cos\xi)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)}\cdot V;$$
$$\sigma_{\xi\xi}=-\frac{2(\mu+\lambda)}{a}\operatorname{sh}\eta\sqrt{2(\operatorname{ch}\eta-\cos\xi)}\sum_{k=0}^{\infty}\frac{P_k(\cos\xi)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)}\cdot U+\frac{\lambda\sqrt{2}}{a}(\operatorname{ch}\eta-\cos\xi)^{3/2}\sum_{k=0}^{\infty}\frac{(k+1/2)P_k(\cos\xi)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)}\cdot V;$$
$$\sigma_{\xi\eta}=\frac{2\mu}{a}\sin\xi\sqrt{2(\operatorname{ch}\eta-\cos\xi)}\sum_{k=0}^{\infty}\frac{P_k(\cos\xi)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)}\cdot U+\frac{\mu\sqrt{2}}{a}(\operatorname{ch}\eta-\cos\xi)^{3/2}\sum_{k=0}^{\infty}\frac{P_k'(\cos\xi)}{\operatorname{sh}(k+1/2)(\eta_1-\eta_2)}\cdot V;$$
$$\sigma_{\xi\xi}=\sigma_{\varphi\varphi},\qquad \sigma_{\xi\eta}=\sigma_{\eta\xi},\qquad P_k'\equiv dP_k/d\xi, \tag{15}$$

where the following notation is introduced

$$U=u_{R_1}\cdot\operatorname{sh}(k+1/2)(\eta-\eta_2)e^{-(k+1/2)\eta_1}-u_{R_2}\cdot\operatorname{sh}(k+1/2)(\eta-\eta_1)e^{-(k+1/2)\eta_2};$$
$$V=u_{R_1}\cdot\operatorname{ch}(k+1/2)(\eta-\eta_2)e^{-(k+1/2)\eta_1}-u_{R_2}\cdot\operatorname{ch}(k+1/2)(\eta-\eta_1)e^{-(k+1/2)\eta_2}$$

and the coefficients u_{R_1} and u_{R_2} are given by expressions (14). In contrast to the coaxial position of spherical pore [3], we found the stress state of sphere (15) depends on the elastic moduli of the material.

Similar calculations for the case $l=0$, when the hollow sphere is located in the center of the elastic sphere but already in spherical symmetry, are given by:

$$u_r(r)=\frac{u_{R_2}R_2^2-u_{R_1}R_1^2}{R_2^3-R_1^3}r+\frac{(u_{R_1}R_2-u_{R_2}R_1)}{R_2^3-R_1^3}\frac{R_1^2R_2^2}{r^2};\qquad \sigma_{rr}=\lambda\left(\frac{du_r}{dr}+\frac{2u_r}{r}\right)+2\mu\frac{du_r}{dr}; \tag{16}$$
$$u_{R_1}=\frac{P_1\left(2(1-2\nu)\,R_1^3+(1+\nu)\,R_2^3\right)-3(1-\nu)P_2\,R_2^3}{2E}\cdot\frac{R_1}{R_2^3-R_1^3};\; u_{R_2}=\frac{3(1-\nu)P_1\,R_1^3-P_2\left(2(1-2\nu)R_2^3+(1+\nu)R_1^3\right)}{2E}\cdot\frac{R_2}{R_2^3-R_1^3},$$

where E – Young's modulus, ν – Poisson's coefficient.

CONCLUSIONS

1. The problem of the elastic state of a sphere with a spherical cavity (spherical pore) at an arbitrary distance from its center is exactly solved. It is assumed that the sphere is elastically isotropic.

2. Expressions for the displacement components, strain and stress tensors depending on the geometric parameters of the problem and the pressure values on the surfaces of the outer and inner spheres are obtained.

3. Similar calculations were performed when the spherical hole was located in the center of the elastic sphere.

APPENDIX

Strain tensor components in a bispherical coordinate system (BCS)

$$\varepsilon_{\eta\eta} = \frac{\operatorname{ch}\eta - \cos\xi}{a}\cdot\frac{\partial u_\eta}{\partial\eta} - \frac{\sin\xi}{a}\cdot u_\xi\,;\quad \varepsilon_{\xi\xi} = \frac{\operatorname{ch}\eta - \cos\xi}{a}\cdot\frac{\partial u_\xi}{\partial\xi} - \frac{\operatorname{sh}\eta}{a}\cdot u_\eta\,;\quad \varepsilon_{\varphi\varphi} = \frac{\cos\xi\cdot\operatorname{ch}\eta - 1}{a\cdot\sin\xi}\cdot u_\xi - \frac{\operatorname{sh}\eta}{a}\cdot u_\eta\,; \tag{A.1}$$

$$\varepsilon_{\eta\xi} = \frac{\operatorname{ch}\eta - \cos\xi}{2a}\cdot\left(\frac{\partial u_\xi}{\partial\eta} + \frac{\partial u_\eta}{\partial\xi}\right) + \frac{\operatorname{sh}\eta}{2a}\cdot u_\xi + \frac{\sin\xi}{2a}\cdot u_\eta\,;\qquad \varepsilon_{\eta\xi} = \varepsilon_{\xi\eta}\,;$$

$$\varepsilon_{\eta\varphi} = \frac{\operatorname{ch}\eta - \cos\xi}{2a}\cdot\frac{\partial u_\varphi}{\partial\eta} + \frac{\operatorname{sh}\eta}{2a}\cdot u_\varphi\,;\qquad \varepsilon_{\eta\varphi} = \varepsilon_{\varphi\eta}\,;$$

$$\varepsilon_{\xi\varphi} = \frac{\operatorname{ch}\eta - \cos\xi}{2a}\cdot\frac{\partial u_\varphi}{\partial\xi} + \frac{1 - \cos\xi\cdot\operatorname{ch}\eta}{2a\cdot\sin\xi}\cdot u_\varphi\,;\qquad \varepsilon_{\xi\varphi} = \varepsilon_{\varphi\xi}\,.$$

Hooke's law has the following form in the isotropic approximation $\sigma_{ik} = 2\mu\varepsilon_{ik} + \lambda\varepsilon_{ll}\delta_{ik}$:

$$\sigma_{\eta\eta} = \frac{2\mu}{a}\left((\operatorname{ch}\eta - \cos\xi)\cdot\frac{\partial u_\eta}{\partial\eta} - \sin\xi\cdot u_\xi\right) + \lambda\cdot\Theta\,;\qquad \sigma_{\xi\xi} = \frac{2\mu}{a}\left((\operatorname{ch}\eta - \cos\xi)\cdot\frac{\partial u_\xi}{\partial\xi} - \operatorname{sh}\eta\cdot u_\eta\right) + \lambda\cdot\Theta\,; \tag{A.2}$$

$$\sigma_{\varphi\varphi} = \frac{2\mu}{a}\left(\frac{\cos\xi\cdot\operatorname{ch}\eta - 1}{\sin\xi}\cdot u_\xi - \operatorname{sh}\eta\cdot u_\eta\right) + \lambda\cdot\Theta\,;$$

$$\sigma_{\eta\xi} = \frac{\mu}{a}\left((\operatorname{ch}\eta - \cos\xi)\cdot\left(\frac{\partial u_\xi}{\partial\eta} + \frac{\partial u_\eta}{\partial\xi}\right) + \operatorname{sh}\eta\cdot u_\xi + \sin\xi\cdot u_\eta\right);\qquad \sigma_{\eta\xi} = \sigma_{\xi\eta}\,;$$

$$\sigma_{\eta\varphi} = \frac{\mu}{a}\left((\operatorname{ch}\eta - \cos\xi)\cdot\frac{\partial u_\varphi}{\partial\eta} + \operatorname{sh}\eta\cdot u_\varphi\right);\qquad \sigma_{\eta\varphi} = \sigma_{\varphi\eta}\,;$$

$$\sigma_{\xi\varphi} = \frac{\mu}{a}(\operatorname{ch}\eta - \cos\xi)\cdot\frac{\partial u_\varphi}{\partial\xi} + \frac{\mu}{a}\left(\frac{1 - \cos\xi\cdot\operatorname{ch}\eta}{\sin\xi}\right)\cdot u_\varphi\,;\qquad \sigma_{\xi\varphi} = \sigma_{\varphi\xi}\,.\qquad ;$$

$$\Phi(\eta_1,\eta_2) = \sum_{k=0}^{\infty}\frac{1}{e^{(2k+1)(\eta_1-\eta_2)} - 1}\left[\frac{e^{-2\eta_1}\cdot e^{-(2k+1)\eta_2} + e^{-(2k+3)\eta_1}}{(2k+3)(2k+5)} - \frac{2\left(e^{-(2k+1)\eta_2} + e^{-(2k+1)\eta_1}\right)}{(2k-1)(2k+3)} + \frac{e^{2\eta_1}\cdot e^{-(2k+1)\eta_2} + e^{-(2k-1)\eta_1}}{(2k-1)(2k-3)}\right];$$

$$\tilde{\Phi}(\eta_1,\eta_2) = \sum_{k=0}^{\infty}\frac{e^{-(2k+1)\eta_2}}{e^{(2k+1)(\eta_1-\eta_2)} - 1}\left[\frac{e^{-2\eta_1}}{(2k+3)(2k+5)} + \frac{e^{2\eta_1}}{(2k-1)(2k-3)} - \frac{2}{(2k-1)(2k+3)}\right]; \tag{A.3}$$

$$G(\eta_1,\eta_2) = \sum_{k=0}^{\infty}\frac{1}{e^{(2k+1)(\eta_1-\eta_2)} - 1}\left[\frac{e^{(2k+1)\eta_1 - 4(k+1)\eta} + e^{-(2k+3)\eta_2}}{(2k+3)(2k+5)} - \frac{2\left(e^{(2k+1)\eta_1 - 2(2k+1)\eta_2} + e^{-(2k+1)\eta_2}\right)}{(2k-1)(2k+3)} + \frac{e^{(2k+1)\eta_1 - 4k\eta_2} + e^{-(2k-1)\eta_2}}{(2k-1)(2k-3)}\right];$$

$$\tilde{G}(\eta_1,\eta_2) = \sum_{k=0}^{\infty}\frac{e^{-(2k+1)\eta_2}}{e^{(2k+1)(\eta_1-\eta_2)} - 1}\left[\frac{e^{-2\eta_2}}{(2k+3)(2k+5)} + \frac{e^{2\eta_2}}{(2k-1)(2k-3)} - \frac{2}{(2k-1)(2k+3)}\right].$$

REFERENCES

1. L.D. Landau, E.M. Lifshitz. *Theory of Elasticity*. Pergamon Press, 1989.
2. A.I. Lurie. *Theory of Elasticity*. Springer-Verlag Berlin Heidelberg, 2005.
3. Yu. Dahl // *Vestn. St. Peterbg. Univ. Prikl. Mat. Inform. Protsessy Upr*. 2009, v. 10, p. 55.
4. S.M. Bauer, P.E. Tovstik, A.B. Kachanov *// Russ. J. Biomech*. 1999, v. 2, p. 27.
5. S.M. Bauer, B.A. Zimin, P.E. Tovstik. *The simplest models of the theory of shells and plates in ophthalmology*. St. Petersburg: Publishing House of St. Petersburg. un-ta, 2000, 92 p.
6. Yu.M. Dahl, A.A. Morshchinina // *Vestn. St. Petersburg. university*. 2008, v. 10, p 47.
7. E.N. Iomdina. *Biomechanics of the sclera of the eye in myopia: diagnosis of disorders and their experimental correction*: Abstract dis. Dr. biol. Sciences. M., 2000, 48 p.
8. A.A. Morshchinina // *Vestn. St. Petersburg. university*. 2009, v. 1, p. 84.
9. V.I. Gorelov // *J. Appl. Mech. Tech. Phys.* 1984, v. 5, p. 157.
10. E.V. Murashkin // *Izv. Sarat. university. New ser.*

Ser. Maths. Mechanics. Informatics. 2014, v. 14, N 1, p. 99.

11. S.E. Omarov, Yu.N. Antipov // *Scientific Journal "KSTU News".* 2014, N 35, p. 239.

12. F.V. Nolfi and H. Wiedersich // *Acta Metallurgica*. 1974, v. 22, p. 1087.

13. V.V. Yanovsky, M.I. Kopp, M.A. Ratner // *Functional Materials*. 2019, v. 26, N 1, p. 131.

14. V.V. Yanovsky, M.I. Kopp, M.A. Ratner // *Functional Materials*. 2020, v. 27, N 3, p. 533.

15. G. Arfken. *Mathematical Methods in Physics*. M.: "Atomizdat", 1970, 712 p.

16. V.V. Novozhilov. *Theory of Elasticity*. Pergamon Press, London, 1961, 448 p.

Article received 20.07.2022

ЗАДАЧА ТЕОРІЇ ПРУЖНОСТІ ДЛЯ ПОРОЖНЬОЇ СФЕРИ ЗІ ЗМІЩЕНИМ ЦЕНТРОМ ПОРОЖНИНИ

М.І. Копп, П.М. Остапчук, В.В. Яновський

Досліджується пружній стан кулі, усередині якої знаходиться сферична порожнина (куляста пора) на довільній відстані від центра кулі. Отримано вирази для компонентів зсувів, тензорів деформацій та напруг залежно від геометричних параметрів задачі та значень тиску на поверхнях зовнішньої та внутрішньої сфер.