https://doi.org/10.46813/2022-139-091

RADIATION FRICTION OF RELATIVISTIC CHARGED PARTICLES MOVING IN A PERIODIC FIELD

V.V. Ognivenko
National Science Center "Kharkov Institute of Physics and Technology", Kharkiv, Ukraine
E-mail: ognivenko@kipt.kharkov.ua

The motion of relativistic charged particles beam in an external periodic field is considered, taking into account the influence of incoherent fields produced by particles on this motion. On the basis of the dynamics of individual particles motion under the action of the pair interaction forces each of them we derived the coefficient of friction. The expression for the friction force, which describes the average change in the momentum of charged particles per unit time, in the case of motion of an initially monoenergetic particle beam, is obtained.

PACS: 41.60.−m, 52.25.Gj

INTRODUCTION

As is known, charged particles moving in external periodic fields emit electromagnetic radiation. For particles with very high energy, the main part of the intensity of the radiation is concentrated in the region of high frequencies [1]. The effect of this radiation on the motion of charged particles leads to a change in the rms value of the momentum of the particles. These effects are significant in the case when the system of charged particles is homogeneous in space, and the self-consistent fields are negligible. The flow of relativistic electrons moving in an external periodic field (undulator), which is a source of intense ultrashort-wavelength electromagnetic radiation, satisfies just such conditions. The change in the rms value of the longitudinal momentum of electrons moving in an external static and periodic in space magnetic field is considered in [2, 3]. These studies are of particular importance in connection with the work aimed at creating X-ray free electron lasers (FEL) operating in the mode of self-amplified of spontaneous emission (see, for example, [4]). The question naturally arises of how the incoherent field of spontaneous electromagnetic radiation affects the change in the mean value of the electron momentum. This work is devoted to the study of this issue.

1. BASIC EQUATIONS

We consider an external field that is periodic along the z-axis of the Cartesian coordinate system and uniform in the transverse direction (perpendicular to the z-axis). Let the flow of relativistic charged particles move along the z axis in this periodic field. We will assume that the forces of pair interaction between particles are known. The equations of longitudinal motion of an individual charged particle can be written in the form:

$$\frac{d}{dt}p_z = F_z[x(t),t] = \sum_s \mathrm{F}_z^{(s)}[X_s(t,x_{0s})], \qquad (1)$$

$$\frac{d}{dt}z_s = \frac{p_z}{m\gamma}, \qquad (2)$$

where p_z and z are the momentum and the longitudinal coordinate of a particle, $\mathrm{F}_z^{(s)}$ is the force of pair interaction between particles, $X_s(t,x_{0s}) = \{x(t),t;x_s(t,x_{0s})\}$, $x=(\mathbf{r},\mathbf{p})$ is the set of coordinates and momentum of a particle, $x_0=(\mathbf{r}_0,\mathbf{p}_0)$ are the initial values of coordinates and momentum of a particle, γ is the relativistic factor.

We will consider the motion of particles in the time Δt, when the trajectory of the particles does not change significantly Then, on the right-hand side of equation (1), expanding the expression for the force in a series in small parameters $\Delta z = z_s - z_s^{(0)}(t)$ and $\Delta p_z = p_z - p_z(0)$, we obtain the following equation

$$\frac{d}{dt}p_z = F_z[x_0(t),t] + \left(\Delta\xi\frac{\partial}{\partial\xi}F_z[x(t),t]\right)_{x(t)=x^{(0)}(t)}, \qquad (3)$$

where $\xi=(z,p_z)$, $\Delta\xi=(\Delta z,\Delta p_z)$.

The change in the momentum and coordinate, according to (1) and (2) is determined by the equations

$$\Delta p_z = \int_{t_0}^{t} dt' F_z\left(x^{(0)},t\right), \qquad (4)$$

$$\Delta z_s = \frac{1}{m}\int_{t_0}^{t}\frac{dt'}{\gamma^3}(t-t')F_z\left(x^{(0)},t\right). \qquad (5)$$

Substituting (4) and (5) into (3), we obtain the following equation describing the change with time in the average value of the momentum of the particle

$$\frac{d}{dt}\langle p_z\rangle = \langle F_z[x_0(t),t]\rangle + A_z, \qquad (6)$$

where

$$A_z = \int_{t_0}^{t} dt' \left\langle F_z\left(x^{(0)}(t'),t'\right)\hat{L}(x,t-t')F_z\left(x^{(0)}(t),t\right)\right\rangle, \qquad (7)$$

$\hat{L}(x,t) = \frac{t}{m\gamma^3}\frac{\partial}{\partial z} + \frac{\partial}{\partial p_z}$, the angle brackets denote ensemble averaging.

The first term in equation (6) describes the average force acting on the test particle, or the deceleration force of an individual charge by the field of its own radiation. The second term in equation (6) is the radiation friction force.

Averaging in the right-hand side of Eq. (7) is performed using the distribution function of dynamic states of the particles in the phase space of the coordinates and momentum of the particles at initial time t_0 (in the coordinate $z=0$) [5, 6]. We consider the influence of average forces on the motion of particles insignificant if these forces are not equal to zero.

Averaging in the integrand of Eq. (7) in the same way as it was done in [2, 6], neglecting the particle correlation at the initial moment of time, and taking into account Eq. (1) for the microscopic force, we obtain

$$A_z = \int_{t_0}^{t} dt' \int F_z^{(s)}\left[X_s\left(t', x_{0s}\right)\right] \times \\ \times \hat{L}\left(x, t-t'\right) F_z^{(s)}\left[X_s\left(t, x_{0s}\right)\right] f\left(x_{0s}\right) dx_{0s} \quad , \qquad (8)$$

where $f\left(x_{0s}\right)$ is the one-particle distribution function.

To calculate A_z we note that the force of pair interaction in the integrand (8) is determined by the motion of the charged particles along equilibrium trajectories in an external periodic field.

2. FLOW OF PARTICLES IN A PERIODIC FIELD

Let charged particles move in a static and periodic in space magnetic field

$$\mathbf{H} = H_0 \left[\mathbf{e}_x \cos(k_u z) + \mathbf{e}_y \sin(k_u z)\right], \qquad (9)$$

where H_0, λ_u are the amplitude and period of the magnetic field, $k_u = 2\pi/\lambda_u$.

We consider the interaction of charged particles via the electromagnetic field produced by them. The transverse equilibrium velocity of charged particles in the magnetic field (9) has the form

$$\mathbf{v}_{\perp s}^{(0)}(t) = -\mathbf{e}_x \, \mathrm{v}_\perp \cos\left[k_u z_s(t)\right] - \mathbf{e}_y \, \mathrm{v}_\perp \sin\left[k_u z_s(t)\right].$$

The expression for the force of pair interaction between particles under the assumption of a small value of the amplitude of the external periodic field ($eH_0\lambda_0/2\pi mc^2 \ll 1$) can be written in the form [3]

$$\mathrm{F}_z^{(s)}(x,t) = -\frac{e^2 K^2 \gamma_{zs} k_u}{\gamma R_*(r,t)} G(r,t;q_{0s}), \qquad (10)$$

where

$$G(r,t;q_{0s}) = \left(\beta_{zs} + \frac{R_{0z}}{R_*} - \frac{\beta_{zs}}{k_{0s}^2 R_*} - \frac{\beta_{zs} R_{0\perp}^2}{2R_*^2 \gamma_{zs}^2}\right)\sin\psi + \\ + \left(\beta_{zs} + \frac{R_{0z}}{R_*}\right)\frac{\cos\psi}{k_{os} R_*} \quad ,$$

$$\psi = \gamma_{zs}^2 k_u \left(R_{0z} + \beta_{zs} R_*\right), \; R_* = \sqrt{R_{0z}^2 + R_{0\perp}^2/\gamma_{zs}^2} \, ,$$

$$R_{0\perp} = \sqrt{\left(x - x_{0s}\right)^2 + \left(y - y_{0s}\right)^2} \; , \; R_{0z} = z - z_s(t),$$

$$k_{0s} = \beta_{zs}\gamma_{zs}^2 k_u \, , \mathrm{v}_\perp = \frac{cK}{\gamma_0} \, \mathbf{,} \; K = \frac{|e| H_0}{mc^2 k_u}.$$

Let's consider the initial (pre-Brownian) stage of charged particles diffusion in momentum space, assuming that particles are monoenergetic at the entrance to the external periodic field. Furthermore, we are interested in small changes in the energy of charged particles due to their radiation friction $\Delta\gamma \ll \gamma_0$.

Substituting the expression for the force (10) into right-hand side of Eq. (8), after integration over the momentum, we obtain the following equation for the radiation friction force

$$A_z = -e^4 K^4 k_u^2 \int_{t_0}^{t} dt' \times \\ \times \int_{\Omega(t')} dq_{0s} n_b \frac{G\left[X_s\left(t', x_{0s}\right)\right] G\left[X_s\left(t, x_{0s}\right)\right]}{\gamma R_*\left(t', x_{0s}\right) R_*\left(t, x_{0s}\right)} \, , \qquad (11)$$

where $dq_{0s} = \mathrm{v}_{z0s}\, dx_{0s} dy_{0s} dt_{0s}$, Ω is the range of integration over the initial coordinates of the emitting charges.

We assume that the beam is a solid cylindrical of radius r_b with a uniform average density n_b.

On the right-hand side of equation (11), it is convenient to pass from the integration variables x_{0s}, y_{0s}, t_{0s} to new variables r', φ, θ, using the formulas: $\mathrm{x}_{0s} = \mathrm{x}_0 - \gamma_{zs} r' \cos\varphi \sin\theta$, $\mathrm{y}_{0s} = \mathrm{y}_0 - \gamma_{zs} r' \sin\varphi \sin\theta$, $\Delta z_s(z_1) = r' \cos\theta$, $0 \le \varphi \le 2\pi$, $0 \le \theta \le \pi$.

The limits of integration over the initial coordinates in equation (11) can be written in the form (cf. [2]):

at $z < z_r$

$$r'_{\max}(z_1,\theta) = r_1(z_1,\theta) \equiv \frac{z_1}{\gamma_{zm}^2 \left(\beta_{zm} + \cos\theta\right)}, \qquad (12)$$

at $z > z_r$

$$r'_{\max}(z_1,\theta) = \begin{cases} r_1(z_1,\theta), & 0 \le \theta \le \theta_*(z_1) \\ \dfrac{r_b}{\gamma_{zm}\sin\theta}, & \theta_*(z_1) \le \theta \le \pi/2 \end{cases}, \qquad (13)$$

where $\theta_*(z) = 2arctg(z_r/z)$, $z_r = \beta_{zm}\gamma_{zm} r_b$.

Substituting the expression (10) into equation (11) and integrating over the initial coordinates using boundary conditions (12), we obtain the following expression for the radiation friction force at $z<z_r$

$$A_z = -\frac{15\pi}{32} e^2 r_0 \,\mathrm{K}^4 k_u^2 n_b \frac{z^2}{\gamma_z}, \qquad (14)$$

where $r_0 = \dfrac{e^2}{mc^2}$.

For $z>z_r$, calculating the integrals on the right-hand side of Eq. (11), taking into account the limits of integration (13), we obtain

$$A_z = -\pi e^2 r_0 \,\mathrm{K}^4 k_u^2 n_b r_b z B\left(\frac{z}{z_r}\right), \qquad (15)$$

where

$$B(x) = \frac{5}{3}\left(1 - \frac{1}{x}\right) + \frac{35}{16}\left(arctgx - \frac{\pi}{4}\right) - \\ - \frac{2}{x}\ln\left(\frac{1+x^2}{2}\right) + \frac{15}{32x}.$$

Let's note that in the limiting case $z \gg z_r$ $B(x) = \dfrac{5}{3} + \dfrac{35}{16}\dfrac{\pi}{2}$.

CONCLUSIONS

Thus, an expression for the force of radiation friction of relativistic charged particles passing in an external periodic field is obtained in this work. The pre-Brownian stage of charged particles motion is investigated, when the difference in the initial velocities of particles at the entrance to the undulator can be neglected. It should also be noted that expression for the friction force in the case of the pre-Brownian motion of Coulomb interacting charged particles was obtained in [7].

As follows from (14, 15), the radiation friction force is proportional to the beam density. This force increases as the beam of particles moves in a periodic field.

At distances $z>z_r$ this force increases in proportion to the distance in the first degree. At distances $z<z_r$, the radiation friction force increases in proportion to the square of the distance passed by the beam. This dependence of the force on the distance is due to an increase in the number of particles in the field of which the test particle is located, as the beam passes through an external periodic field.

From formulae (14, 15) and the expression for the diffusion coefficient D_z, obtained in [2], the relation follows

$$A_z = -\frac{D_z}{p_z},$$

which relates the force of radiation friction and the diffusion coefficient.

As follows from formulae (14, 15) for ultrarelativistic electron beams, which are used to obtain ultrashort-wavelength radiation in the FELs, the radiation friction force must be taken into account along with the radiation damping force of an individual charge. In addition, explicit expressions for the radiation friction force make it possible to kinetically describe the relaxation of relativistic flows of charged particles in external periodic fields.

REFERENCES

1. L.D. Landau, E.M. Lifshitz. *The classical theory of fields.* Pergamon Press, Oxford, 1968.
2. V.V. Ognivenko. Momentum spread in a relativistic electron beam in an undulator // *J. Exp. Theor. Phys.* 2012, v. 115, № 5, p. 938-946.
3. V.V. Ognivenko // *J. Exp. Theor. Phys.* 2021, v. 132, № 5, p. 766-775.
4. Ye.N. Ragozin, I.I. Sobel'man. Lazernyye istochniki v myagkoy rentgenovskoy oblasti spectra // *UFN*. 2005, № 12, p. 1339-1341.
5. N.N. Bogolyubov. *Problems of Dynamical Theory in Statictical Physics.* M.: “Gostekhteorizdat”, 1946; Interscience, New York, 1962.
6. V.V. Ognivenko. Dynamical derivation of momentum diffusion coefficients at collisions of relativistic charged particles // *J. Exp. Theor. Phys*. 2016, v. 122, № 1, p. 203-208.
7. V.V. Ognivenko. The pair interaction forces and the friction and diffusion coefficients of particles in momentum space // *Problems of Atomic Science and Technology. Series “Plasma Physics”.* 2017, № 1, p. 195-198.

Article received 11.01.2022

РАДІАЦІЙНЕ ТЕРТЯ РЕЛЯТИВІСТСЬКИХ ЗАРЯДЖЕНИХ ЧАСТИНОК, ЩО РУХАЮТЬСЯ В ПЕРІОДИЧНОМУ ПОЛІ

В.В. Огнівенко

Розглянуто рух пучка релятивістських заряджених частинок у зовнішньому періодичному полі, з урахуванням впливу на цей рух некогерентних полів, створюваних частинками. На основі динаміки руху окремих частинок під дією сил парної взаємодії кожної з них отриманий коефіцієнт тертя. Отримано вираз для сили тертя, що описує середню зміну імпульсу заряджених частинок за одиницю часу у випадку руху початково моноенергетичного пучку частинок. Установлений взаємозв'язок між средньоквадратичним розкидом по імпульсах і силою гальмування частинок.

РАДИАЦИОННОЕ ТРЕНИЕ РЕЛЯТИВИСТСКИХ ЗАРЯЖЕННЫХ ЧАСТИЦ, ДВИЖУЩИХСЯ В ПЕРИОДИЧЕСКОМ ПОЛЕ

В.В. Огнивенко

Рассмотрено движение пучка релятивистских заряженных частиц во внешнем периодическом поле, с учетом влияния на это движение некогерентных полей, создаваемых частицами. На основе динамики движения отдельных частиц под действием сил парного взаимодействия каждой из них получен коэффициент трения. Получено выражение для силы трения, описывающее изменение среднего значения импульса заряженных частиц за единицу времени в случае движения первоначально моноэнергетического потока частиц. Установлена взаимосвязь между среднеквадратическим разбросом по импульсам и силой торможения частиц.