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EXCITATION OF WAKE FIELDS BY A RELATIVISTIC ELECTRON BUNCH IN A POLAR SEMICONDUCTOR WAVEGUIDE

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The process of wake fields excitation by a relativistic electron bunch in polar semiconductors is studied. Cylindrical semiconductor waveguide, in which relativistic electron bunch moves along axis, is considered. It is shown that the excited wake field in the terahertz and infrared frequency ranges consists of the field of longitudinal HF and LF hybrid plasmon-phonon oscillations and the field of the HF and LF transverse polaritons, which are a set of eigen electromagnetic waves of the polar semiconductor waveguide. The spatio-temporal structure of the total excited wake field is obtained, the intensity of the excited wake waves is determined.

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INTRODUCTION

Plasma phenomena in semiconductors and semimetals are an inherent property of these condensed media [1 - 7]. The density of electron-hole plasma in semiconductors with intrinsic conductivity (for example, silicon, germanium and other) is relatively low, on the order of $10^{10}...10^{13}\,\mathrm{cm}^{-3}$, and is determined primarily by the width of the energy gap between the valence band and conduction band, and by the temperature of the material too [1, 2]. In gapless semiconductors and semimetals concentration of intrinsic carriers can be higher [8]. In doped semiconductors, the concentration of carriers (electrons in *n*-type semiconductors and holes in *p*-type semiconductors) can be significantly increased (within limits $10^{13}...10^{18}\,\mathrm{cm}^{-3}$) and it depends mainly on the dopant concentration [9].

All crystalline compounds have a mixed covalent-ionic bond. For example, in gallium arsenide $GaAs$, the contribution of the ionic bond to the total bond energy is 32%. The presence of ionic bond in semiconductor compounds will influence on the polarization properties of semiconductors and, accordingly, the frequency dispersion of the dielectric constant of semiconductors [10, 11]. The appearance of a phonon component in the dielectric constant, in turn, will lead to a significant change spectra of longitudinal (potential) and transverse (vortex) electromagnetic oscillations in semiconductors.

Since solid-state plasma is characterized by a high concentration of carriers and by a degree of homogeneity and stability, it seems very promising to use plasma of semiconductors and semimetals to realization wake methods of relativistic charged particles (primarily electrons and positrons) acceleration [12]. In such scheme an intense relativistic electron bunch passes through vacuum channel of a semiconductor (semimetal) and excites wake eigen oscillations (plasmons, optical electromagnetic waves in the infrared range), which are in Cherenkov synchronism with a relativistic electron bunch $\omega = k(\omega)\beta_0 c$, ω is wave frequency, $k(\omega)$ is longitudinal wavenumber, $\beta_0 = v_0 / c \approx 1$, c is speed of light in vacuum. Excited wake waves can be used to accelerate charged particles [13, 14].

It is possible to talk about long-lived excitations in a solid-state plasma, as in any other medium, only if their eigen frequencies significantly exceed the frequencies of collisions with phonons and impurity atoms. Further we will assume that this condition is satisfied.

In the present work, the process of excitation of wake electromagnetic fields in the polar semiconductor waveguide by a relativistic electron bunch is investigated. The wake field includes the longitudinal hybrid plasmon-phonon oscillations of a polar semiconductors, and a set of eigen bulk electromagnetic waves (bulk polaritons) of the polar semiconductor waveguide. Our aim is to obtain wake wave intensity, the frequency spectrum and a spatio-temporal structure of excited wake fields.

1. STATEMENT OF THE PROBLEM

Let's consider the homogeneous polar semiconductor cylinder of radius *b*, the side surface of which is covered with a perfectly conductive metal film. Along the axis of the polar semiconductor waveguide, an axisymmetric relativistic electron bunch moves uniformly and rectilinearly. Below we will only talk about isotropic media. These are primarily crystals with a cubic lattice.

For the polar semiconductors the permittivity have the form [10, 11]

$$\varepsilon(\omega) = \varepsilon_{opt}\left(\frac{\omega_L^2 - \omega^2}{\omega_T^2 - \omega^2} - \frac{\omega_p^2}{\omega^2}\right), \qquad (1)$$

ε_{opt} is optical permittivity, ω_L and ω_T are frequencies of the longitudinal and transverse phonons, $\omega_p = \omega_{Len} / \sqrt{\varepsilon_{opt}}$ is frequency of plasma oscillation, $\omega_{Len} = \sqrt{\dfrac{4\pi n_e e^2}{m_e}}$ is Langmuir frequency, n_e is free carrier concentration, m_e is effective mass of electrons, e is electron charge. The first term in the expression for the permittivity (1) is due to the contribution to the total polarization of the ionic subsystem of the crystal, and the second term takes into account the polarization of free carriers (plasma).

2. DISPERSION PROPERTIES OF POLAR DIELECTRIC WAVEGUIDE

Let us now briefly discuss the question of the propagation of electromagnetic waves in an ion dielectric waveguide. Dispersion equations for potential longitudinal oscillations and electromagnetic waves have the forms

$$\varepsilon(\omega)=0\,, \tag{2}$$

$$\frac{\omega^2}{c^2}\varepsilon(\omega)-k_z^2-\frac{\lambda_n^2}{b^2}=0\,, \tag{3}$$

k_z is longitudinal wave number. The dielectric constant is described by the formula (1). The solution of the equation (2) is two frequencies

$$\omega_{lp}^{(\pm)2}=\frac{\omega_L^2+\omega_p^2}{2}\pm\sqrt{\left(\frac{\omega_L^2+\omega_p^2}{2}\right)^2-\omega_T^2\omega_p^2}\,. \tag{4}$$

As a result of the interaction of longitudinal optical phonons and plasma oscillations, high-frequency (HF) (sign +) and low-frequency (LF) (sign -) hybrid plasmon-phonon oscillations are formed. At that, in the case $\omega_L>\omega_p$ the frequency of HF hybrid oscillations always exceeds the frequency of longitudinal optical phonons $\omega_{lp}^{(+)}>\omega_L$, and the frequency of LF hybrid oscillations is always lower than the plasma frequency $\omega_{lp}^{(-)}<\omega_p$. At a low concentration of free carriers $\omega_L^2>>\omega_p^2$, one of them ($\omega_{lp}^{(+)}$) is close to the frequency of longitudinal optical phonons

$$\omega_{lp}^{(+)2}=\omega_L^2+\omega_p^2\frac{\varepsilon_{st}-\varepsilon_{opt}}{\varepsilon_{st}}\,,$$

and the other ($\omega_{lp}^{(-)}$) to the frequency

$$\omega_{lp}^{(-)}=\omega_p\sqrt{\frac{\varepsilon_{opt}}{\varepsilon_{st}}}\,,$$

where ε_{st} is static permittivity ($\varepsilon_{st}>\varepsilon_{opt}$).

If $\omega_p>\omega_L$, then the frequencies ω_L and ω_p change places. The frequency $\omega_{lp}^{(+)}$ is higher than the plasma frequency, and the frequency $\omega_{lp}^{(-)}$ is lower than the frequency of longitudinal optical phonons. At high plasma density $\omega_p^2>>\omega_L^2$, the frequency $\omega_{lp}^{(+)}$ is close to the plasma frequency

$$\omega_{lp}^{(+)2}=\omega_p^2+\omega_L^2\frac{\Delta\varepsilon}{\varepsilon_{st}},\ \Delta\varepsilon=\varepsilon_{st}-\varepsilon_{opt}\,,$$

and the frequency $\omega_{lp}^{(-)}$ is approximately equal to the frequency

$$\omega_{lp}^{(-)}=\omega_L\sqrt{\frac{\varepsilon_{opt}}{\varepsilon_{st}}}\,.$$

Let us now consider the dispersion properties of transverse (vortex) electromagnetic waves in polar semiconductors (transverse polaritons), which are described by equation (3). Taking into account the expression for the dielectric constant (1), this equation can be reduced to the following

$$\omega^2\frac{\omega_L^2-\omega^2}{\omega_T^2-\omega^2}=\omega_{np}^2+k^2v_s^2\,, \tag{5}$$

where $v_s=c/\sqrt{\varepsilon_{opt}}$,

$$\omega_{np}^2=\omega_p^2+\frac{\lambda_n^2c^2}{b^2\varepsilon_{opt}}\,.$$

Roots of the biquadratic equation (5)

$$\omega_{ntp}^{(\pm)2}=\frac{\omega_L^2+\omega_{np}^2+k^2v_s^2}{2}\pm$$

$$\pm\sqrt{\left(\frac{\omega_L^2+\omega_{np}^2+k^2v_s^2}{2}\right)^2-\omega_T^2(\omega_{np}^2+k^2v_s^2)} \tag{6}$$

describe the dispersion of HF (sign +) and LF (sign -) of transverse polaritons. The dispersion curve of HF polaritons starts at the cutoff frequency

$$\omega_{nc}^{(+)2}=\frac{\omega_L^2+\omega_{np}^2}{2}+\sqrt{\left(\frac{\omega_L^2+\omega_{np}^2}{2}\right)^2-\omega_T^2\omega_{np}^2}\,.$$

Further, with an increase in the longitudinal wave number k, the frequency increases and at $k\to\infty$, the dispersion curve asymptotically approaches from above to the straight line $\omega=kv_s$. The dispersion curve of low-frequency polaritons $\omega_{ntp}^{(-)}(k)$ also begins with cutoff frequency

$$\omega_{nc}^{(-)2}=\frac{\omega_L^2+\omega_{np}^2}{2}-\sqrt{\left(\frac{\omega_L^2+\omega_{np}^2}{2}\right)^2-\omega_T^2\omega_{np}^2}\,,$$

then enters on the section of a straight line $\omega=kc/\sqrt{\varepsilon_{st}}$ and, at $k\to\infty$ asymptotically approaches from below to the frequency of transverse optical phonons.

The presence of free carriers in a polar semiconductor raises the cutoff frequencies, and in general, the qualitative picture of the dispersion of transverse polaritons in polar semiconductor waveguides is the same as in ionic dielectric waveguides [15].

2.1. DETERMINATION OF THE GREEN FUNCTION

We will solve the problem of wake field excitation by an axisymmetric relativistic electron bunch in the polar semiconductor waveguide with permittivity (1) as follows [12, 16]. We will find wake field $\vec{E}_G$ (Green function) of elementary charge, having the form of a thin ring with charge dQ. Elementary charge density of an infinitely thin ring has the form

$$d\rho_b=-\frac{dQ(r_0,t_0)}{v_0}\frac{\delta(r-r_0)}{2\pi r_o}\delta(t-t_0-\frac{z}{v_0})\,, \tag{7}$$

$$dQ=j_0(t_0,r_0)2\pi r_0dr_0dt_0\,,$$

$$j_0(r_0,t_0)=\frac{Q}{s_{eff}t_{eff}}R\left(r_0/r_b\right)T\left(t_0/t_b\right), \tag{8}$$

where Q is full charge of bunch, t_0 is time of entry of elementary charge, r_0 is radius ring, v_0 is bunch velocity, t_b,r_b are characteristic duration and transverse bunch size, $R\left(r_0/r_b\right)$ is function described transversal profile of bunch density, s_{eff} is characteristic square of bunch transverse section,

$$s_{eff}=\pi r_b^2\hat{\sigma},\ \hat{\sigma}=2\int\limits_0^{b/r_b}R(\rho_0)\rho_0 d\rho_0 .$$

The function $T(t_0/t_b)$ describes longitudinal profile, t_{eff} is effective bunch duration,

$$t_{eff}=\hat{\tau}t_b,\ \hat{\tau}=2\int\limits_0^{\infty}T(\tau_0)d\tau_0 .$$

Let us represent the electromagnetic field excited by an elementary ring charge (7) as

$$\vec{E}_G(r,r_0,z,t-t_0)=dQ\vec{\mathrm{E}}(r,r_0,z,t-t_0)\,.$$

Then the full electromagnetic field, excited by an electron bunch of finite dimensions, is found by summing (integrating) the fields of elementary ring bunches

$$\vec{E}(r,z,t)=\int\limits_0^b 2\pi r_0 dr_0\int\limits_{-\infty}^{t}dt_0\, j(r_0,t_0)\vec{\mathrm{E}}(r,r_0,z,t-t_0)\,.$$

Taking into account relation (8), this expression can be written as follows

$$\vec{E}(r,z,t)=\frac{2\pi Q}{s_{eff}t_{eff}}\int\limits_0^b R\left(\frac{r_0}{r_b}\right)r_0 dr_0\times$$

$$\times\int\limits_{-\infty}^{t}T\left(\frac{t_0}{t_b}\right)\vec{\mathrm{E}}(r,r_0,z,t-t_0)dt_0\,.$$

The Green's function for the considered dielectric waveguide with a permittivity $\varepsilon(\omega)$ was obtained in [12, 16] in the form of a series in Bessel functions

$$E_{Gz}(r,\bar{t})=\frac{2i}{\pi b^2}dQ\sum_{n=1}^{\infty}\frac{J_0\left(\lambda_n\frac{r_0}{b}\right)J_0\left(\lambda_n\frac{r}{b}\right)}{J_1^2(\lambda_n)}S_n(\bar{t})\,,\quad(9)$$

where

$$S_n(\bar{t})=\int\limits_{-\infty}^{\infty}e^{-i\omega\bar{t}}\,\frac{d\omega}{\omega}\frac{k_\perp^2(\omega)}{\varepsilon(\omega)D_n(\omega)}\,,\quad(10)$$

$$D_n(\omega)=k_0^2\varepsilon(\omega)-k_l^2-\frac{\lambda_n^2}{b^2}\,,\quad(11)$$

$k_l=\omega/v_0$, $k_0=\omega/c$, λ_n are the roots of the Bessel function $J_0(x)$, $\bar{t}=t-t_0-z/v_0$.

The zeros of the dielectric constant $\varepsilon(\omega)=0$ are the poles of the integrand (10). Calculating the residues at the poles $\omega=\pm\omega_{lp}^{(+)}-i0,\ \ \omega=\pm\omega_{lp}^{(-)}-i0$, we find the potential part of the Green's function

$$E_{Gz}^{(l)}(r,\bar{t})=2\frac{dQ}{\varepsilon_{opt}}\vartheta(\bar{t})\left[k_{lp}^{(+)2}L_{lp}^{(+)}G(k_{lp}^{(+)}r,k_{lp}^{(+)}r_0)\cos\omega_{lp}^{(+)}\bar{t}+\right.$$

$$\left.+k_{lp}^{(-)2}L_{lp}^{(-)}G(k_{lp}^{(-)}r,k_{lp}^{(-)}r_0)\cos\omega_{lp}^{(-)}\bar{t}\right],\quad(12)$$

$$L_{lp}^{(+)}=\frac{\omega_{lp}^{(+)2}-\omega_T^2}{\omega_{lp}^{(+)2}-\omega_{lp}^{(-)2}}\,,\quad L_{lp}^{(-)}=\frac{\omega_T^2-\omega_{lp}^{(-)2}}{\omega_{lp}^{(+)2}-\omega_{lp}^{(-)2}}\,,$$

$$\vartheta(\bar{t})=\begin{cases}1,\ \bar{t}>0,\\0,\ \bar{t}<0,\end{cases}\quad k_{lp}^{(\pm)}=\omega_{lp}^{(\pm)}/v_0\,,$$

$$G(kr,kr_0)=\begin{cases}\dfrac{I_0(kr_0)}{I_0(kb)}\Delta_0(kr,kb),\ r>r_0,\\[2mm]\dfrac{I_0(kr)}{I_0(kb)}\Delta_0(kr_0,kb),\ r<r_0,\end{cases}\quad(13)$$

$$\Delta_0(kr,kb)=I_0(kb)K_0(kr)-I_0(kr)K_0(kb)\,.$$

In the limiting case $kb>>1$, the expression for the function $G(kr,kr_0)$ is simplified

$$G(kr,kr_0)=\begin{cases}I_0(kr_0)K_0(kr),\ r>r_0,\\I_0(kr)K_0(kr_0),\ r<r_0.\end{cases}$$

The potential wake field, excited by an ring bunch, contains two waves: HF and LF plasmon-phonon waves, the frequencies of which are determined by expression (4).

The integrand in (10) also has poles that are the roots of the equation

$$D_n(\omega)=k_0^2\varepsilon(\omega)-k_l^2-\frac{\lambda_n^2}{b^2}=0\,.\quad(14)$$

Equation (14) determines the frequency spectrum of the radial harmonic with the number n of electromagnetic waves excited by the relativistic electron bunch in ion dielectric waveguide. With respect to the square of the frequency ω^2, the spectrum equation (14) reduces to determining the roots of the quadratic equation. The frequencies $\omega_{ntp}^{(\pm)}$, corresponding to the roots of this equation, lie in the microwave and infrared ranges. The spectrum equation (19) can be written as follows

$$\omega^2\left(\varepsilon_{opt}\frac{\omega_L^2-\omega^2}{\omega_T^2-\omega^2}-\frac{1}{\beta_0^2}\right)=\omega_n^2\equiv\omega_{Len}^2+\frac{\lambda_n^2c^2}{b^2}\,,$$

where $\beta_0=v_0/c$. The roots of this equation are of the form

$$\omega_{ntp}^{(\pm)}=\frac{1}{2d_{opt}}\left[\omega_T^2 d_{st}+\omega_n^2\pm\right.$$

$$\left.\pm\sqrt{\left(\omega_T^2 d_{st}+\omega_n^2\right)^2-4\omega_n^2\omega_T^2 d_{st}}\right].\quad(15)$$

Here

$$d_{opt}=\varepsilon_{opt}-\beta_0^{-2},\ d_{st}=\varepsilon_{st}-\beta_0^{-2}\,.$$

For the frequency $\omega_{ntp}^{(+)}$ it is always $\omega_{ntp}^{(+)}>\omega_L$, and for the frequency $\omega_{ntp}^{(-)}$ we have $\omega_{ntp}^{(-)}<\omega_T$. In the limiting case

$$\omega_T^2 d_{st}>>\omega_n^2$$

expression for frequencies (15) are simplified

$$\omega_{ntp}^{(-)^2}=\frac{\omega_n^2}{d_{st}}\equiv\frac{\beta_0^2}{\beta_0^2\varepsilon_{st}-1}\left(\omega_{Len}^2+\frac{\lambda_n^2c^2}{b^2}\right),\quad(16)$$

$$\omega_{ntp}^{(+)^2}=\omega_T^2\frac{d_{st}}{d_{opt}}+\omega_n^2\frac{\Delta\varepsilon}{d_{opt}d_{st}}\,.\quad(17)$$

In the limiting case

$$\omega_n^2>>\omega_L^2=\omega_T^2\frac{\varepsilon_{st}}{\varepsilon_{opt}}$$

expressions for the frequencies of the eigen waves of the semiconductor waveguide follow from (15) and have the form

$$\omega_{ntp}^{(-)^2} = \omega_T^2,$$

$$\omega_{ntp}^{(+)^2} = \frac{\omega_n^2}{d_{opt}} \equiv \frac{\omega_n^2 \beta_0^2}{\beta_0^2 \varepsilon_{opt} - 1}.$$

Frequency $\omega_{ntp}^{(-)}$ (16) is known in the theory of wake fields excitation by relativistic electron bunch in semiconductor waveguides [12] and is in the microwave (terahertz) range. The frequency $\omega_{ntp}^{(+)}$ (17) lies in the infrared range and the process of wake fields excitation at this frequency, as it seems to us, has not been previously studied.

For further analysis, the Fourier integral (10) is conveniently represented as

$$S_n(\bar{t}) = \frac{1}{d_{opt}} \int_{-\infty}^{\infty} \frac{k_\perp^2(\omega)\omega}{k_0^2 \varepsilon(\omega)} \frac{\left(\omega^2 - \omega_T^2\right) e^{-i\omega\bar{t}}}{\left(\omega^2 - \omega_{ntp}^{(-)^2}\right)\left(\omega^2 - \omega_{ntp}^{(+)^2}\right)} d\omega .$$

By calculating the residues in the poles $\omega = \pm\omega_{ntp}^{(-)} - i0$, $\omega = \pm\omega_{ntp}^{(+)} - i0$, we find the electromagnetic part of the Green's function

$$E_{Gz}^{(t)}(r, r_0, \bar{t}) = dE_{tz}^{(+)}(r, r_0, \bar{t}) + dE_{tz}^{(-)}(r, r_0, \bar{t}), \quad (18)$$

$$dE_{tz}^{(\pm)}(r, r_0, \bar{t}) = \frac{dE_w}{d_{opt}} \sum_{n=1}^{\infty} \alpha_n^{(\pm)} \Pi_n(r, r_0) \vartheta(\bar{t}) \cos \omega_{ntp}^{(\pm)} \bar{t}, \quad (19)$$

$$\alpha_n^{(+)} = \frac{\lambda_n^2}{\lambda_n^2 + k_{ntp}^{(+)^2} b^2} \frac{\omega_{ntp}^{(+)^2} - \omega_T^2}{\omega_{ntp}^{(+)^2} - \omega_{ntp}^{(-)^2}},$$

$$\alpha_n^{(-)} = \frac{\lambda_n^2}{\lambda_n^2 + k_{ntp}^{(-)^2} b^2} \frac{\omega_T^2 - \omega_{ntp}^{(-)^2}}{\omega_{ntp}^{(+)^2} - \omega_{ntp}^{(-)^2}}, \quad k_{ntp}^{(\mp)} = \omega_{ntp}^{(\mp)} / v_0,$$

$$\Pi_n(r, r_0) = \frac{J_0(\lambda_n r / b) J_0(\lambda_n r_0 / b)}{J_1^2(\lambda_n)}, \quad dE_w = \frac{4dQ}{b^2}.$$

Expressions (18), (19) describe bulk wake electromagnetic field excited by the infinitely thin electron ring bunch in polar semiconductor waveguide.

Thus, we obtained the Green function, which contains the longitudinal (potential) and electromagnetic (vortex) parts. The potential part is a field of longitudinal bulk plasmon-phonon oscillations. As for the electromagnetic part of the Green function, it contains a set of radial electromagnetic waves of polar semiconductor waveguide.

2.2. EXCITATION OF WAKEFIELDS BY AN ELECTRON BUNCH OF FINITE SIZE

The resulting electromagnetic field $\vec{E}(r, \tau)$ of the electron bunch can be determined by summing the fields $\vec{E}_G$ of elementary electron ring charges. We first consider the excitation of wake plasmon-phonon oscillations by the relativistic electron bunch of finite dimensions.

For the wake field of plasmon-phonon oscillations we obtain the following expression

$$E_z^{(l)}(r, \tau) = \frac{2Q}{\varepsilon_{opt}} \Big[k_{lp}^{(+)2} L_{lp}^{(+)} \Gamma(k_{lp}^{(+)} r) Z(\omega_{lp}^{(+)} \tau) + $$
$$+ k_{lp}^{(-)2} L_{lp}^{(-)} \Gamma(k_{lp}^{(-)} r) Z(\omega_{lp}^{(-)} \tau) \Big], \quad (20)$$

where $\tau = t - z / v_0$,

$$Z(\omega\tau) = \frac{1}{t_{eff}} \int_{-\infty}^{\tau} T\left(\tau_0 / t_b\right) \cos\omega(\tau - \tau_0) d\tau_0, \quad (21)$$

$$\Gamma(kr) = \frac{2\pi}{s_{eff}} \int_0^b R\left(r_0 / r_b\right) G(kr, kr_0) r_0 dr_0. \quad (22)$$

The function $Z(\omega\tau)$ describes the distribution of the wake field at a frequency ω in the longitudinal direction at each moment of time and function $\Gamma(kr)$ describes the distribution of the wake field by cross section of the waveguide. We will consider an electron bunch with a symmetric longitudinal profile $T(\tau_0) = T(-\tau_0)$.

Behind a bunch $\tau \to \infty$, the wake field (20) of plasma oscillations has the form of a monochromatic wave

$$E_z^{(l)}(r, \tau) = \frac{2Q}{\varepsilon_{opt}} \vartheta(\tau) \Big[k_{lp}^{(+)2} L_{lp}^{(+)} \hat{T}(\omega_{lp}^{(+)}) \Gamma(k_{lp}^{(+)} r) \cos\omega_{lp}^{(+)} \tau + $$
$$+ k_{lp}^{(-)2} L_{lp}^{(-)} \hat{T}(\omega_{lp}^{(-)}) \Gamma(k_{lp}^{(-)} r) \cos\omega_{lp}^{(-)} \tau \Big], \quad (23)$$

where

$$\hat{T}(\omega) = \frac{2}{t_{eff}} \int_0^{\infty} T\left(\frac{t}{t_b}\right) \cos(\omega t) dt \quad (24)$$

is the Fourier component of a function $T\left(t / t_b\right)$ on frequency ω. We present the expressions for the Fourier component $\hat{T}(\omega)$ for Gaussian longitudinal profile of the electron bunch

$$T(\tau_0 / t_b) = e^{-\tau_0^2 / t_b^2}, \quad \hat{T}(\omega) = e^{-(\omega t_b)^2 / 4}.$$

Wake HF and LF plasmon-phonon waves is most efficiently radiated when the coherence condition is fulfilled $(\omega_{lp}^{(\pm)} t_b)^2 < 1$. If the inequality $(\omega_{lp}^{(\pm)} t_b)^2 >> 1$ holds, then the electron bunch radiates incoherently and the amplitude of the wake plasma wave is exponentially small.

Let's consider an electron bunch with a Gaussian transverse profile

$$R(r) = e^{-r^2 / r_b^2}. \quad (25)$$

When the condition $k_L b >> 1$ is satisfied on the axis $r = 0$ the function $\Gamma(kr)$ takes on the value

$$\Gamma(0) = -\frac{1}{2} e^{\rho_b} Ei\left(-\rho_b\right), \quad \rho_b = \frac{k^2 r_b^2}{4}, \quad (26)$$

$$Ei\left(z\right) = \int_{-\infty}^{z} \frac{e^t}{t} dt$$

is integral exponential function. For thin $\rho_b << 1$ and wide $\rho_b >> 1$ bunches the asymptotic representations for function (26) are

$$\Gamma(0) = \begin{cases} \dfrac{1}{2} \ln\left(\dfrac{1}{\rho_b}\right), & \rho_b << 1, \\ \dfrac{1}{\rho_b}, & \rho_b >> 1. \end{cases}$$

Thus, with the full coherence of the Cherenkov excitation of wake plasma wave $\omega_{lp}^{(\pm)} t_b \le 1$, $k_{lp}^{(\pm)} r_b \le 1$ the wake field on the axis of the waveguide takes the maximum value

$$E_z^{(l)}(r,\tau)=\frac{2Q}{\varepsilon_{opt}}\vartheta(\tau)\Big[k_{lp}^{(+)2}L_{lp}^{(+)}\ln\frac{2}{k_{lp}^{(+)}r_b}\cos\omega_{lp}^{(+)}\tau+$$

$$+k_{lp}^{(-)2}L_{lp}^{(-)}\ln\frac{2}{k_{lp}^{(-)}r_b}\cos\omega_{lp}^{(-)}\tau\Big]. \quad (27)$$

Let us now consider the excitation of wake electromagnetic waves by an electron bunch in the dielectric waveguide. Using the electromagnetic Green function (23), we obtain the wake electromagnetic field as a superposition of radial modes

$$E_z^{(t)}(r,\tau)=\frac{4Q}{b^2}\sum_{n=1}^{\infty}\Gamma_n\frac{J_0\left(\lambda_n\frac{r}{b}\right)}{J_1^2(\lambda_n)}\Big[\alpha_n^{(+)}Z(\omega_{ntp}^{(+)}\tau)+$$

$$+\alpha_n^{(-)}Z(\omega_{ntp}^{(-)}\tau)\Big], \quad (28)$$

$$\Gamma_n=\frac{2\pi}{s_{eff}}\int_0^b R\left(\frac{r_0}{r_b}\right)J_0\left(\lambda_n\frac{r_0}{b}\right)r_0dr_0.$$

For a symmetric electron bunch in the "wave zone" $\omega_{ntp}^{(\pm)}\tau>>1$, the wake field (28) is a superposition of radial monochromatic modes of the semiconductor waveguide

$$E_z^{(t)}(r,\tau)=\frac{4Q}{b^2}\sum_{n=1}^{\infty}\Gamma_n\frac{J_0\left(\lambda_n\frac{r}{b}\right)}{J_1^2(\lambda_n)}\Big[\alpha_n^{(+)}\hat{T}_n^{(+)}\cos\omega_{ntp}^{(+)}\tau+$$

$$+\alpha_n^{(-)}\hat{T}_n^{(-)}\cos\omega_{ntp}^{(-)}\tau\Big], \quad (29)$$

where $\hat{T}_n^{(\pm)}\equiv\hat{T}(\omega_{ntp}^{(\pm)})$ is Fourier component (22).

We also present an expression for the power of the wake electromagnetic radiation, which we define as the component of the total Poiting vector along the dielectric waveguide axis

$$P=\frac{c}{4\pi}\int_0^b\left\langle E_r^{(t)}H_\varphi^{(t)}\right\rangle 2\pi rdr.$$

Angle brackets mean averaging over high-frequency wake field oscillations. As a result, for the radiated power we obtain the following expression

$$P=2Q^2c\sum_{n=1}^{\infty}\frac{\Gamma_n^2}{\lambda_n^2J_1^2(\lambda_n)}\Big[\varepsilon(\omega_{ntp}^{(+)})\,\alpha_n^{(+)2}\hat{T}_n^{(+)2}\frac{\omega_{ntp}^{(+)}k_{ntb}^{(+)}}{c}+$$

$$+\varepsilon(\omega_{ntp}^{(-)})\,\alpha_n^{(-)2}\hat{T}_n^{(-)2}\frac{\omega_{ntp}^{(-)}k_{ntb}^{(-)}}{c}\Big]. \quad (30)$$

For an electron bunch with a Gaussian longitudinal and transverse profiles the coefficients Γ_n and $\hat{T}_n$, which are determined by the specific form of the transverse and longitudinal density profiles of the bunch, have the form

$$\hat{T}_n^{(\pm)}=\exp\left(-\frac{\omega_{ntp}^{(\pm)2}t_b^2}{4}\right), \quad (31)$$

$$\Gamma_n=2\int_0^{1/\eta_b}J_0(\lambda_n\eta_b\rho_0)e^{-\rho_0^2}\rho_0d\rho_0,\quad \eta_b=\frac{r_b}{b}.$$

When the condition $\eta_b<<1$ is satisfied, the expression for the coefficient Γ_n is simplified

$$\Gamma_n=\exp\left(-\frac{1}{4}\lambda_n^2\eta_b^2\right). \quad (32)$$

Accordingly, expression (29) for a Gaussian bunch, taking into account relations (31), (32), takes the form

$$E_z^{(t)}(r,\tau)=\frac{4Q}{b^2}\sum_{n=1}^{\infty}e^{-\frac{\lambda_n^2\eta_b^2}{4}}\frac{J_0\left(\lambda_n\frac{r}{b}\right)}{J_1^2(\lambda_n)}\Bigg[\alpha_n^{(+)}e^{-\frac{\omega_{ntp}^{(+)}t_b^2}{4}}\cos\omega_{ntp}^{(+)}\tau+$$

$$+\alpha_n^{(-)}e^{-\frac{\omega_{ntp}^{(-)}t_b^2}{4}}\cos\omega_{ntp}^{(-)}\tau\Bigg]. \quad (33)$$

For the radiated power (30), we have

$$P=2Q^2c\sum_{n=1}^{\infty}\frac{e^{-\frac{\lambda_n^2\eta_b^2}{2}}}{\lambda_n^2J_1^2(\lambda_n)}\Big[\varepsilon(\omega_{ntp}^{(+)})\,\alpha_n^{(+)2}e^{-\frac{\omega_{ntp}^{(+)2}t_b^2}{2}}\frac{\omega_{ntp}^{(+)}k_{ntb}^{(+)}}{c}+$$

$$+\varepsilon(\omega_{ntp}^{(-)})\,\alpha_n^{(-)2}e^{-\frac{\omega_{ntp}^{(-)2}t_b^2}{2}}\frac{\omega_{ntp}^{(-)}k_{ntb}^{(-)}}{c}\Bigg].$$

From expression (33) it follows that an electron bunch excites finite number of radial modes of electromagnetic radiation, for which the coherence condition $\omega_{ntp}^{(\pm)2}t_b^2\le 1,\ \lambda_n^2r_b^2/b^2\le 1$ for excitation by an electron bunch is satisfied.

CONCLUSIONS

Polar semiconductors are the plasma-like media, in which wake electromagnetic fields can be excited by the relativistic electron bunches. In the semiconductors crystals of this type, there are two groups of oscillation branches of in the infrared and terahertz frequency ranges. These are, first of all, longitudinal HF and LF plasma-phonon oscillations of a polar semiconductor. And also in the infrared range there are the two branches corresponding to HF and LF transverse bulk polaritons of the media, which is a set of electromagnetic eigen waves of a polar semiconductor waveguide. For all of these branches, analytical expressions for the wake fields excited by a relativistic electron bunch are obtained and investigated.

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ЗБУДЖЕННЯ КІЛЬВАТЕРНИХ ПОЛІВ РЕЛЯТИВІСТСЬКИМ ЕЛЕКТРОННИМ ЗГУСТКОМ У ПОЛЯРНОМУ НАПІВПРОВІДНИКОВОМУ ХВИЛЕВОДІ

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Досліджено процес збудження кільватерних полів релятивістським електронним згустком у полярних напівпровідниках. Розглянуто циліндричний напівпровідниковий хвилевод, вздовж вісі якого рухається релятивістський електронний згусток. Показано, що збуджене кільватерне поле в терагерцовому та інфрачервоному діапазонах частот містить у собі поле повздовжніх ВЧ- і НЧ-гібридних плазмон-фононних коливань, а також поле ВЧ- та НЧ-поперечних полярітонів, яке є набором власних електромагнітних хвиль напівпровідникового хвилеводу. Отримана просторово-часова структура результуючого кільватерного поля, визначена інтенсивність збуджених кільватерних хвиль.

ВОЗБУЖДЕНИЕ КИЛЬВАТЕРНЫХ ПОЛЕЙ РЕЛЯТИВИСТСКИМ ЭЛЕКТРОННЫМ СГУСТКОМ В ПОЛЯРНОМ ПОЛУПРОВОДНИКОВОМ ВОЛНОВОДЕ

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Исследован процесс возбуждения кильватерных полей релятивистским электронным сгустком в полярных полупроводниках. Рассмотрен цилиндрический полупроводниковый волновод, по оси которого движется релятивистский электронный сгусток. Показано, что возбуждаемое кильватерное поле в терагерцовом и инфракрасном диапазонах частот состоит из поля продольных ВЧ- и НЧ-гибридных плазмон-фононных колебаний, а также поля ВЧ- и НЧ-поперечных поляритонов, которые представляют собой набор собственных электромагнитных волн полупроводникового волновода. Получена пространственно-временная структура результирующего кильватерного поля, определена интенсивность возбуждаемых кильватерных волн.