ANOMALOUS MODULATION OF THE WAVE FIELD IN A PLASMA WAVEGUIDE

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The consequences of the development of the modulation instability of an intense wave field in a waveguide filled with plasma are considered. It is shown that in the mode of developed instability, the amplitude of the main wave under conditions of its weak absorption decreases by three to four times. The average energy density of the instability spectrum is almost twice the energy density of the main wave. The amplitude of the modulation envelope is three times the average amplitude of the main wave. The frequency of occurrence of anomalous modulation bursts is estimated.

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INTRODUCTION

Regular wave motion in a medium with cubic nonlinearity is subject to modulation. In [1], it was shown that in a one-dimensional case, in the absence of noise (fluctuations) in a conservative environment, for certain initial conditions, a single autowave can appear − the Peregrin soliton, which is an envelope of wave motion, whose amplitude is about three times larger than the average wave amplitude. In the case of a non-conservative medium with a source that supports wave motion and under arbitrary initial conditions, that is, in the presence of noise, the appearance of envelopes of anomalous amplitude is also possible [2] as a result of the development of modulation instability. Moreover, if the set of realizations in space of the initial conditions of a single envelope, the Peregrin soliton, has a measure of zero, then in the case of the development of the modulation instability of wave motion under arbitrary conditions, the appearance of envelopes of anomalous amplitude is more than likely. However, there are in the set and envelopes of smaller amplitude, which exchange energy with each other. Below, for the case of a weakly absorbing medium with a source generating the main wave, we will discuss the conditions for the appearance of modulation and estimate the frequency of appearance of the envelopes of the anomalous amplitude.

1. DESCRIPTION OF THE PROCESS OF MODULATION OF A WAVE OF LARGE AMPLITUDE

Consider the instability of a monochromatic wave

$$E(x,t)\cdot\exp\{i\omega t - ikx\}, \qquad (1)$$

in a wave medium with a weak dispersion and local cubic nonlinearity, where $E(x,t)$ is its complex amplitude which is slowly varying. Consider a wave with the following characteristics

$$\omega = \omega_0 + \vec{k}^2 - |E|^2. \qquad (2)$$

Such a dispersion is characteristic of Langmuir waves in a plasma and oscillations in plasma waveguides in the corresponding normalization. The Lighthill − NSE equation, which describes the slow variation of the oscillation envelope under these conditions, takes the form

$$\frac{\partial E}{\partial t} = -\delta E - i\frac{\partial^2 E}{\partial x^2} - iE|E|^2 + g. \qquad (3)$$

Let us discuss the case of the presence of a consistent source and drain (absorption or dissipation) of the wave energy (here, δ – the absorption decrement and
g – the external source of wave energy). Lighthill Criterion [3]

$$(\partial^2\omega/\partial k^2)^{-1}\cdot\partial\omega(|E|^2)/\partial|E|^2 < 0, \qquad (4)$$

at which the wave is modulationally unstable in the direction of its propagation for (1), turns out to be satisfied. The oscillation amplitude $E(x,t)$ slowly varying with time can be represented as

$$E = u_0(t)\exp\{i\phi_{k_0}(t) - ik_0x\} + \sum_{n\neq 0} u_n(t)\exp\{i\phi_{k_n}(t) - ik_nx\}. \qquad (5)$$

That is, the instability is understood as the excitation of a perturbation spectrum

$$\sum_{n\neq 0} u_n(t)\cdot\exp\{i\phi_{k_n}(t)\}\cdot\exp\{i\omega_0 t - ik_nx\},$$

where $u_n(t)\cdot\exp\{i\phi_{k_n}(t)\}$ is the slowly varying complex amplitude of the n-th spectrum mode. The real field is a modulated wave at a frequency ω_0. Therefore, to restore the form of the wave field, expression (5) should be multiplied by $\exp\{i\omega_0 t\}$. Separating the "fast" phase factor $\exp\{i\omega_0 t - ik_0x\}$ corresponding to the main wave, in this case we obtain the oscillation field

$$E = \exp\{i\omega_0 t - ik_0x\}\cdot\{u_0\exp[i\phi_{k_0}] + \sum_{n\neq 0} u_n\exp[i\phi_{k_n} - i(k_n - k_0)x]\}. \qquad (6)$$

In the review [4] and the book [5], the authors, based on taking into account only two interaction diagrams, namely

$$2\omega_0 = \omega(k) + \omega(-k)$$

and

$$\omega(k) + \omega(-k) = \omega(k') + \omega(-k'),$$

formulated approaches to the description of the nonlinear stage of modulation instability, which later became known as S-theory. The evolution of instability can be traced using a more convenient modified S-theory [6]. Note that the description in the framework of the modified S-theory is consistent with the results of the direct calculation of equation (3) at least at the initial stage of the non-linear mode of the process (see, for example, [7]).

The system of equations for the main wave can be written as

$$\frac{d\phi_0}{dt} = -u_0{}^2 - 2\sum_{m>0}^{N}(u_m{}^2 + u_{-m}{}^2) - \qquad (7)$$
$$-2\sum_{m>0}^{N} u_m u_{-m} cos\Phi_m ,$$

$$u_0 = -g\{-\delta - 2\sum_{m>0}^{N} u_m u_{-m} sin\Phi_m\}^{-1}, \qquad (8)$$

For spectrum modes, the following equations are valid

$$\frac{dv_S}{dt} = v_s\{-\delta + u_0^2\frac{v_{-s}}{v_s} sin\Phi_S + \qquad (9)$$
$$+2\frac{v_{-s}}{v_s}\sum_{n>0}^{N} u_n u_{-n} sin\Psi_{sn}\},$$

$$\frac{d\phi_S}{dt} = K_S^2 - 2[u_0^2 + \frac{1}{2}v_S^2 + \sum_{n>0}^{N}(u_n^2 + u_{-n}^2)] - \qquad (10)$$
$$-u_0^2\frac{v_{-s}}{v_s}cos\Phi_s - 2\frac{v_{-s}}{v_s}[-u_s u_{-s} + \sum_{n>0}^{N} u_n u_{-n} cos\Psi_{sn}].$$

Slowly varying amplitude of the electric field of a packet in the rest wave system of the main wave can be written as

$$E(\xi,t) = u_0 + \sum_{m>0}^{N}[u_m \exp\{-iK_m\xi + i(\phi_m - \phi_0)\} + \qquad (11)$$
$$+u_{-m}\exp\{iK_m\xi + i(\phi_{-m} - \phi_0)\}],$$

here

$$K_n^2 = 1 + (\frac{2|n| - N}{N})\sqrt{1-\delta}\ ,\ k_{\pm n} = k_0 \pm \mathrm{K}_n,$$

$k_0 >> |\mathrm{K}_n|$, $\gamma = -\delta + \sqrt{\mathrm{K}^4 - 2\mathrm{K}^2|u_0|^2}$ – the linear increment of instability, the phases of the growing modes with time are in the interval

$$0 < \Phi_n < \pi\ , \Phi_n = 2\phi_0 - \phi_n - \phi_{-n} = \Phi_{-n},$$

$\Psi_{sn} = \Phi_s - \Phi_n$, $v_s\ u_n$ correspond to the actual amplitudes of the modes of the instability spectrum.

2. NUMERICAL ANALYSIS OF THE SYSTEM OF EQUATIONS

For the number of modes N = 25, we choose the absorption parameters and the source characteristic g = δ = 0.05, which correspond to a weakly absorbing medium, the calculation step is Δ*t* = 0.05, the total number of steps – 2000 corresponds to the time interval 0…100.

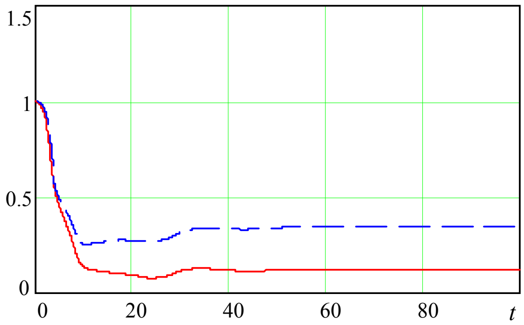

Fig. 1. Behavior of the value $u_0^2 + \sum_{m\neq 0}^{N} u_i^2$ *– the upper curve and* u_0^2 *– the lower curve*

It can be verified that the relative intensity (square of the amplitude) of the field of the main wave decreases significantly in the nonlinear mode and then varies rather weakly. You can define averages $\overline{u}_0$=0.352 and $\overline{u}_0^2$=0.124. The intensity of the spectrum in the developed instability mode is more than twice the intensity of the main wave (Fig. 1).

The energy of the spectrum $\sum_{m\neq 0}^{N} u_i^2$ increases and then changes slightly in the developed instability mode (Figs. 2, 3).

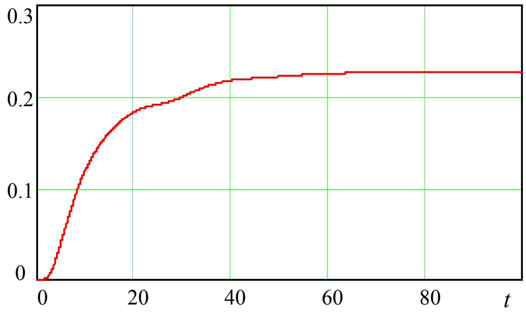

Fig. 2. The relative energy of the instability spectrum without the main wave $\sum_{m\neq 0}^{N} u_i^2$ *as a function of time*

The field amplitudes can be distinguished, which are three times (upper dotted line) and 3.5 (dash-dotted line) times the average amplitude of the main wave $\overline{u}_0$ = 0.352 (lower dotted line). Such envelopes with an amplitude equal to $3\cdot\overline{u}_0$ or greater than occur in one or two in a spatial interval equal to 10 wavelengths ($20\pi / k_0$) over time $\Delta t \approx 8$ in used units of calculation.

N	t	Δt
1	19.5	-
2	26.5	7
3	34.5	8
4	42.4	7.9
5	50.6	8.2

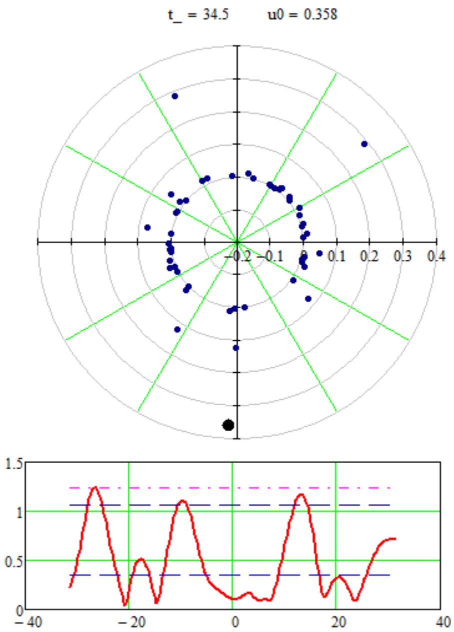

Fig. 3. Phase portrait of the mode spectrum in the amplitude-phase space (above) and the field amplitude in interval –40<ξ< 40 over the time t= 34.5 (bottom). Upper dotted line $-3\overline{u}_0$, *dash-dotted line* $-3.5\overline{u}_0$, *lower dotted line* $-\overline{u}_0$

Similar results are obtained with the absorption parameter and the source characteristic $g = \delta = 0.01$ (Figs. 4-6). The mode of developed instability comes later.

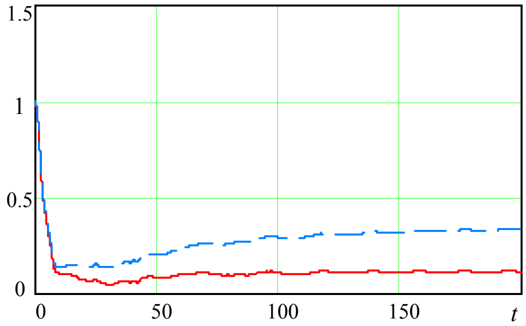

Fig. 4. Behavior of the value $u_0^2 + \sum_{m\neq 0}^{N} u_i^2$ *– the upper curve and* u_0^2 *– the lower curve*

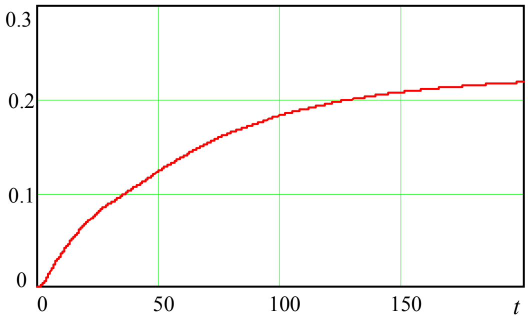

Fig. 5. The relative energy of the instability spectrum without the main wave $\sum_{m\neq 0}^{N} u_i^2$ *as a function of time*

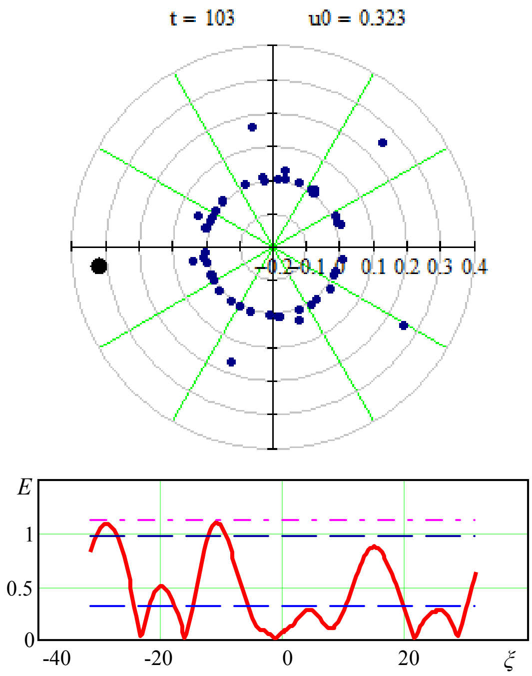

Fig. 6. Phase portrait of the mode spectrum in the amplitude-phase space (above) and the field amplitude in interval –40<ξ< 40 over the time t= 34.5 (bottom). Upper dotted line $-3\bar{u}_0$*, dash-dotted line* $-3.5\bar{u}_0$*, lower dotted line* $-\bar{u}_0$

CONCLUSIONS

Thus, with weak absorption in a medium with cubic nonlinearity, a large amplitude wave is modulationally unstable with the formation of strong modulation. Even with a relatively small absorption (here $\mathrm{Im}\,\omega/\omega_0 \approx 0.05$), the integral losses of the excited spectrum turn out to be significant and the average energy of the modulated wave in the developed instability mode does not exceed 40% of its initial energy.

In this case, the amplitude of the main wave in the spectrum is stabilized at a level of approximately three times less ($\bar{u}_0 = 0.352$) than the initial one. Under these conditions, at intervals of time $\Delta t \approx 6$ in the relative units under consideration, modulation bursts arise with an amplitude of three times and more than the average amplitude of the main wave. Moreover, on a spatial interval exceeding a dozen of the main wavelengths, there are one or two such envelopes of anomalous amplitude.

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ОБ АНОМАЛЬНОЙ МОДУЛЯЦИИ ВОЛНОВОГО ПОЛЯ В ПЛАЗМЕННОМ ВОЛНОВОДЕ

В.М. Куклин, Е.В. Поклонский

Рассмотрены последствия развития модуляционной неустойчивости интенсивного волнового поля в волноводе, заполненном плазмой. Показано, что в режиме развитой неустойчивости амплитуда основной волны в условиях ее слабого поглощения уменьшается в три-четыре раза. Средняя плотность энергии спектра неустойчивости почти в два раза превосходит плотность энергии основной волны. Амплитуда огибающей модуляции в три раза больше средней амплитуды основной волны. Проведены оценки частоты появления аномальных всплесков модуляции.

ПРО АНОМАЛЬНУ МОДУЛЯЦІЮ ХВИЛЬОВОГО ПОЛЯ В ПЛАЗМОВОМУ ХВИЛЕВОДІ

В.М. Куклін, Є.В. Поклонський

Розглянуто наслідки розвитку модуляційної нестійкості інтенсивного хвильового поля в хвилеводі, заповненому плазмою. Показано, що в режимі розвинутої нестійкості амплітуда основної хвилі в умовах її слабкого поглинання зменшується в три-чотири рази. Середня щільність енергії спектра нестійкості майже в два рази перевершує щільність енергії основної хвилі. Амплітуда обвідної модуляції в три рази більше середньої амплітуди основної хвилі. Проведено оцінки частоти появи аномальних сплесків модуляції.