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POTENTIAL PART OF ELECTRIC FIELD OF ICP COIL

N.A. Azarenkov, A.V. Gapon

V.N. Karazin Kharkiv National University, Kharkov, Ukraine

E-mail: gapon@univer.kharkov.ua

ICP devices are widespread in modern technology, so the capacitive coupling in ICP is well known. It does not make a sufficient impact to the power deposition in plasma, and mostly it assumed to be a harmful effect [1]. Fortunately, making use of Faraday shield allows one to avoid a detail investigation of the capacitive coupling. However, in some cases it would be useful to account both coupling types of the inductor with a plasma [2]. There are many ways for analytical and numerical simulation of the inductors and their coupling with a plasma. In the most simulating tools, the problem comes to FEM PDE model, which includes as minima the body of an inductor wire and the surrounding space. In the paper, we use some other approach, based on integral solutions for electrodynamics potentials. Electric fields of some inductors are studied in the framework of the approach.

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1. CURRENT DENSITY EQUATION

In this paper the workflow of an electric field finding consist in 1) numerical solution of some integral equation for electric current density in the conductive bodies 2) calculation of the electric field from the obtained currents. Let us find the integral equation. An electric field strength is expressed through the electrodynamics potentials as

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\varphi . \qquad (1)$$

In the stationary approximation,

$$\varphi = \int \frac{\rho dV}{R}, \qquad (2)$$

$$\boldsymbol{A} = \frac{1}{c}\int \frac{\boldsymbol{j}dV}{R}. \qquad (3)$$

We assume, that in the conductive bodies Ohm's law is correct

$$\boldsymbol{E} + \boldsymbol{E}_0 = \frac{\boldsymbol{j}}{\sigma}. \qquad (4)$$

First term represents an electric field, induced by the current $\boldsymbol{j}$ flowed in the conductors under consideration, whereas $\boldsymbol{E}_0$ is the electric field of an external sources, for example, of the transmission line. From (1-4) and charge conservation law (5)

$$\frac{\partial\rho}{\partial t} + \nabla\boldsymbol{j} = 0 , \qquad (5)$$

the required equation for the current density can be obtained:

$$\int \frac{\boldsymbol{R}}{R^3}\nabla\boldsymbol{j}\, dV + \int \frac{1}{R}\frac{\partial^2 \boldsymbol{j}}{\partial t^2} dV + \frac{\partial}{\partial t}\boldsymbol{E}_0 = \frac{1}{\sigma}\frac{\partial}{\partial t}\boldsymbol{j}. \qquad (6)$$

2. FINITE-DIFFERENCE SCHEME

The scheme is based on the fact that HF electric field strength vanishes inside an ideal conductor or obeys to Ohm's law, if the conductivity is finite. As the currents flow near the surface, in this section we shall mean by $\boldsymbol{j}$ a surface current density, and by σ a surface conductivity. In assumption of sinusoidal time dependence of $\boldsymbol{j}$ (6), the equation to be discretize take the form

$$\int \frac{\boldsymbol{R}}{R^3}\nabla\boldsymbol{j}\, ds + \frac{\omega^2}{c^2}\int \frac{\boldsymbol{j}}{R} ds - i\omega\frac{\boldsymbol{j}}{\sigma} = -i\omega\boldsymbol{E}_0 . \qquad (7)$$

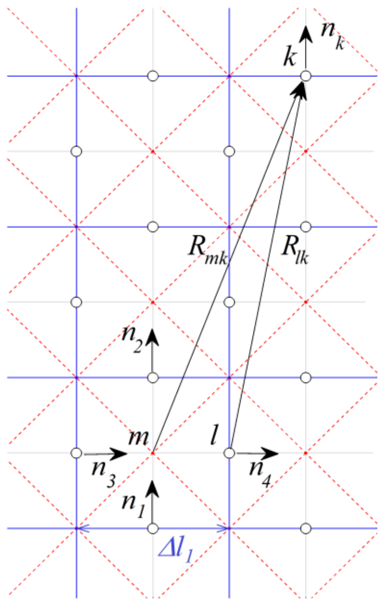

Fig. 1. Meshing of the flat surface

At first, consider a planar surface. A treatment of the first and second terms, which represent impacts of the charge and current distributions, is implemented on two different meshes. We refer to them as *q_mesh* and *j_mesh*. On the Fig.1 these are shown by dashed and solid lines, respectively. The elements of the *q_mesh* are right-angled. Generally, the elements of the *j_mesh* are diamond-shaped.

Let us require the equation (7) to satisfy for a set of certain control points numbered by the index k. Formally, control points can be taken elsewhere inside the conductor, but in the presented scheme the control points set consists of the centers of elements of the *j_mesh*.

The discretization consists in an integration of equation (7) for the set of the control points under following assumptions:

1) the charge density (i.e., $\nabla \boldsymbol{j}$) is distributed uniformly over the element of the *q_mesh*;

2) the current density $\boldsymbol{j}_l$ is constant within the element of the *j_mesh* and is parallel to the normal **n** to the edge of the overlapped element of the *q_mesh*.

Representing the integrals in (7) as summa over all the elements of both the meshes, and multiplying left and right parts by $\boldsymbol{n}_k$, we obtain equation set for j_l:

$$\sum_{m,l} \int_m \frac{\boldsymbol{R n}_k}{R^3} \nabla \boldsymbol{j}\, ds + \frac{\omega^2}{c^2} j_l \int_l \frac{\boldsymbol{n}_l \boldsymbol{n}_k}{R} ds - \frac{i\omega}{\sigma} j_l \boldsymbol{n}_l \boldsymbol{n}_k = -i\omega \boldsymbol{E}_0 \boldsymbol{n}_k . \quad (8)$$

Assumptions 1), 2) allows us to reduce the integro-differential equation (7) to the set of the linear equations

$$a_{kl}\, j_l = b_k , \quad (9)$$

where b_k corresponds to the external electric field strength in the control point k, the coefficient a_{kl} reflects the contribution of the element l from the *j_mesh* in the electric field strength in the point k and depend only on the mesh geometry data, and j_l is the current density in the element l.

Both the integrals in (8) contribute in the coefficient a_{kl}, though the contribution of the certain element of the *q_mesh* is redistributed among the adjacent elements of the *j_mesh*, as it is expressed through the variables j_k. To perform integration in cases when control points are outside of the elements, we neglect the variation of R within the element. For the element m and the control point k (Fig. 1), we have

$$\int_m \frac{\boldsymbol{R n}_k}{R^3} \nabla \boldsymbol{j} ds \to Q \frac{\boldsymbol{R}_{mk} \boldsymbol{n}_k}{R_{mk}{}^3} , \qquad Q = j_2 \Delta l_2 - j_1 \Delta l_1 + j_4 \Delta l_4 - j_3 \Delta l_3 \quad (10)$$

if the control point k is outside of the element l. In (10), position vector $\boldsymbol{R}_{mk}$ is directed from the element center to the control point. The surface currents components normal to the correspond element boundary segments $\Delta l_{1…4}$ are denoted by $j_{1…4}$. The normals to the boundary segments are shown on the Fig. 1 as vectors $\boldsymbol{n}_{1…4}$. Directions of the normals to the certain boundary segment are the same for both of the adjacent elements and is determined by an indexing order of the *j_mesh* nodes. If the point k belongs to the border a of the rectangular element m with the edges a and b, then

$$\int_m \frac{\boldsymbol{R n}_k}{R^3} \nabla \boldsymbol{j} ds \to \frac{Q}{\Delta s} \frac{\boldsymbol{R}_{mk} \boldsymbol{n}_k}{R_{mk}} \left(b \ln \frac{c+a}{c-a} + a \ln \frac{c+2b}{a} \right) \quad (11)$$

$$c = \sqrt{4b^2 + a^2}$$

In (11) Δs is the element surface. The second term in (8) transforms into

$$j_l \int_l \frac{\boldsymbol{n}_l \boldsymbol{n}_k}{R} ds \to j_l \frac{\boldsymbol{n}_l \boldsymbol{n}_k}{R_{lk}} \Delta s_l \quad (12)$$

or into

$$\int_l \frac{\boldsymbol{j}}{R} ds \to \boldsymbol{j}_l \frac{2(a+b)}{d} \left(a \ln \frac{d+b}{a} + b \ln \frac{d+a}{b} \right) , \quad (13)$$

$$d = \sqrt{b^2 + a^2}$$

dependently on the control point k is outside of the element l (12), or in the center of the element (13).

After the current densities on the *j-mesh* had been found from (9), values of the current densities on the *q-mesh* are evaluated as (see Fig. 1)

$$\boldsymbol{j}_m = \frac{1}{4} (j_1 \boldsymbol{n}_1 + j_2 \boldsymbol{n}_2 + j_3 \boldsymbol{n}_3 + j_4 \boldsymbol{n}_4) . \quad (14)$$

Relation (16) gives an approximation to the true values of the current density on the conductive surface.

With the minimal tuning the difference scheme for a flat surface can be adopted to solve the problem on an arbitrary smooth surface, if it is possible to cover the surface by the quadrilateral elements properly.

It is desirable also that the ratio *a/b* does not deviate from unity sufficiently. As a rule, it is not so hard to fulfil.

3. REPRESENTATION OF SKIN LAYER

Elaborated schema can be used in modeling of the stationary ICP discharges under conditions (15)-(18):

$$\lambda_i \le L , \quad (15)$$

$$\lambda_{e\varepsilon} \gg L , \quad (16)$$

$$\delta \ll L , \quad (17)$$

$$\nu_m \ll \omega . \quad (18)$$

In (15)-(18) λ_i is the free path of ions, $\lambda_{e\varepsilon}$ is the energy length of free electron,, is the depth of the skin layer, L is the characteristic size of the discharge volume, ν_m is the frequency of elastic collisions of electrons with neutral particles, ω is the frequency of the generator. Under these conditions, the stochastic heating of electrons can be comparable or greater than the ohmic one. Both types of heating determine the effective conductivity of plasma σ_{eff} in the skin layer by the mean of relations (19)-(22) [1]

$$\sigma_{eff} = \frac{e^2 n_s}{m(\nu_m + \nu_{stoc} + i\omega)} , \quad (19)$$

$$\nu_{stoc} = \frac{v_e}{4\delta(\ell(\alpha)+\alpha/4)}, \qquad (20)$$

$$\ell(\alpha) = \frac{1}{\pi}\left(e^{\alpha}(1+\alpha)E_1(\alpha)-1\right), \qquad (21)$$

$$\alpha = \frac{4\omega^2\delta^2}{\pi v_e^2}. \qquad (22)$$

Since a significant electron current occurs only in the skin layer, inequality (17) allows us to consider it localized on the plasma surface, and write Ohm's law as

$$j_s = E_s \delta\sigma_{eff}, \qquad (23)$$

where j_s is the surface current density. From (23) the energy flux density of plasma associated with heating is equal to

$$\Pi = \frac{1}{4}\Re\frac{|j_s|^2}{\delta\sigma_{eff}}. \qquad (24)$$

4. RESULTS

Consider as an example the inductor coupled with the cylindrical skin-layer (Fig. 2) under the following parameters: working gas argon, pressure $P = 20$ mTor, inductor diameter $D_i = 0.28$ m, inductor conductor diameter $D_w = 10$ mm, skin layer diameter $D_p = 0.26$ m, electron density at the plasma boundary $n_s = 10^{18}$ m^{-3}, electron temperature $T_e = 5$ eV, generator frequency $f = 13.56$ MHz, input inductor current $I = 10$ A.

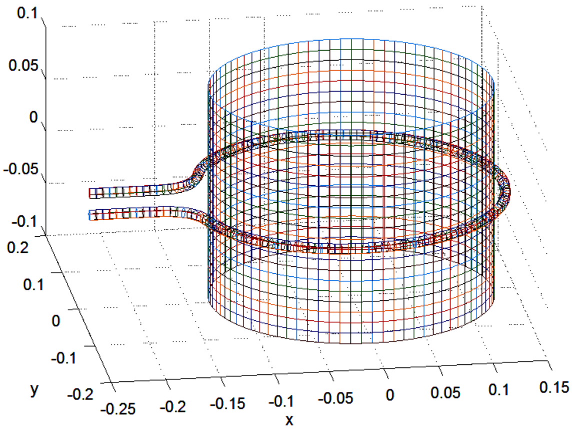

Fig. 2. Discretization of the one-inductor and the cylindrical skin layer

We obtain voltage on the inductor $U = 81 + 187i$ V, and power input to the plasma $W = 179...90i$ W. As the number of turns of N increases, the power generally increases, but at some values of N local minima can be observed. Under the example given, but with $N = 2$, the impedance is doubled, while the active power introduced into the plasma is almost unchanged (decreases by 1.5 percent). These features are shown in Figs. 3, 4

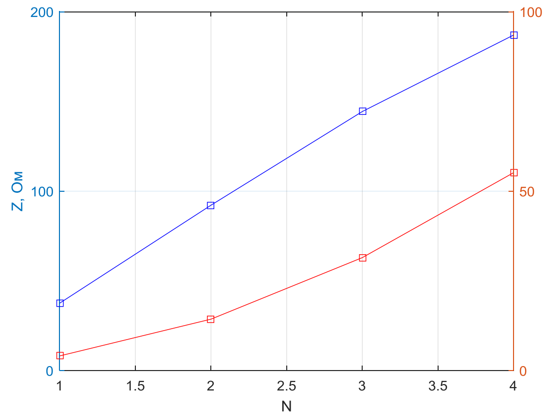

Fig. 3. Dependency of impedance of the inductor from the fig.2 on the number of turns

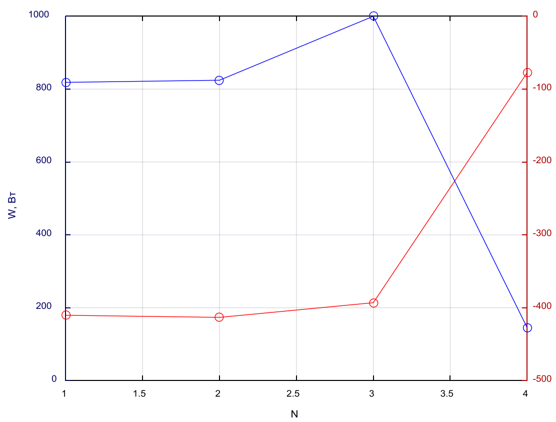

Fig. 4. Dependences of impedance of the inductor from the Fig. 2 on the number of turns

In Fig. 5 the distribution of the energy flux density (24) in plasma are shown. Distributions of the active and reactive parts of the surface current density are presented on the Figs. 6, 7.

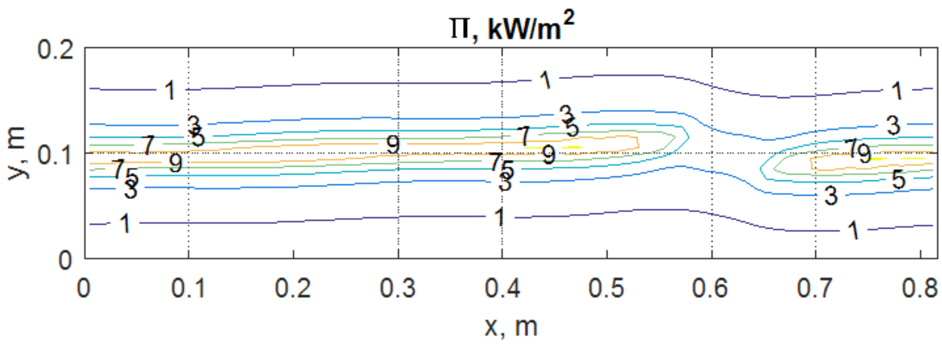

Fig. 5. Contour lines of energy flux density in plasma

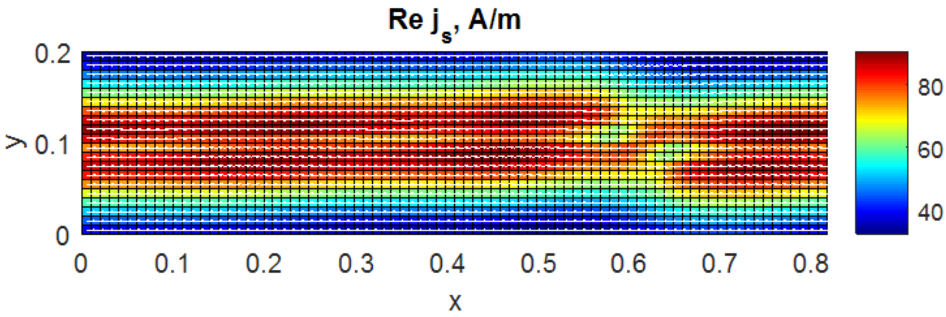

Fig. 6. Distribution of active part of the surface current density

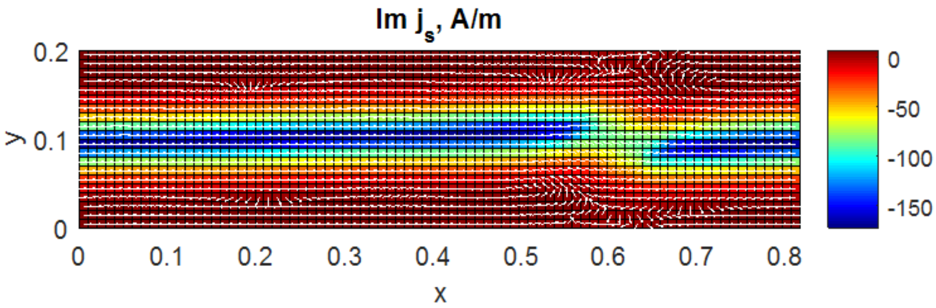

Fig. 7. Distribution of reactive part of the surface current density

CONCLUSIONS

So, the problem of finding of electric fields of an inductor coupled with a plasma skin layer is examined and the proper finite difference scheme is elaborated. The latter has the advantage that it is based on integral equations for potentials and does not require a large volume of surrounding space to model the boundary at infinity.

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ПОТЕНЦИАЛЬНАЯ ЧАСТЬ ЭЛЕКТРИЧЕСКОГО ПОЛЯ ИНДУКТОРА ИНДУКЦИОННОГО РАЗРЯДА

Н.А. Азаренков, А.В. Гапон

Устройства ICP широко распространены в современной технике, поэтому емкостная связь в ICP хорошо известна. Она не оказывает значительного влияния на ввод энергии в плазму, и в большинстве случаев считается вредным эффектом [1]. Использование экрана Фарадея позволяет избежать подробного исследования емкостной связи. Однако в некоторых случаях полезно учитывать оба типа связи индуктора с плазмой [2]. Существует множество способов аналитического и численного моделирований индукторов и их связи с плазмой. В большинстве САПР проблема сводится к решению дифференциальных уравнений методом конечных элементов. В этой статье мы используем другой подход, основанный на интегральных уравнениях для электродинамических потенциалов. В рамках этого подхода исследуются электрические поля различных индукторов.

ПОТЕНЦІЙНА ЧАСТИНА ЕЛЕКТРИЧНОГО ПОЛЯ ІНДУКТОРА ІНДУКЦІЙНОГО РОЗРЯДУ

М.О. Азаренков, О.В. Гапон

Пристрої ICP дуже поширені в сучасній техніці, тому ємнісний зв'язок в ICP добре відомий. Він суттєво не впливає на введення енергії в плазму, і в більшості випадків вважається шкідливим ефектом [1]. Використання екрану Фарадея дозволяє уникнути докладного дослідження ємнісного зв'язку. Однак у деяких випадках корисно враховувати обидва типи зв'язку індуктора з плазмою [2]. Існує багато способів аналітичного і числового моделювання індукторів та їх зв'язку з плазмою. У більшості САПР проблема зводиться до розв’язку диференційних рівнянь методом скінченних елементів. У цій статті ми використовуємо інший підхід, заснований на інтегральних рівняннях для електродинамічних потенціалів. У рамках цього підходу досліджуються електричні поля різних індукторів.