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INFLUENCING OF THIN PROTECTIVE COATINGS ON NATURAL FREQUENCIES OF RADIAL OSCILLATIONS OF CLADDINGS OF FUEL RODS OF NUCLEAR REACTORS

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The natural frequencies and the modes of the radial oscillations are computed by using the method of grids for the cylindrical claddings made with the thin protective coatings for the fuel rods of the nuclear reactors. It is received the values more than 150 kHz for first natural frequencies of the radial oscillations of the claddings of fuel rods of the WWER-1000 nuclear reactors. It is shown that the thin protective coatings lead to noticeable increasing of first natural oscillation frequency, but have negligible influencing on the second and higher natural frequencies as well as on the modes of the radial oscillations of the claddings of fuel rods.

INTRODUCTION

The claddings of fuel rods of nuclear reactors have the most worse operation conditions included the extreme irradiations of different physical natures, the chemical corrosive impacts from the surrounding mediums, the noticeable mechanical loadings as well as the significant heat flows, such that operability of the cladding significantly limits the operational time of the nuclear fuel assemblies in the core of a nuclear reactor, as well-known [1]. At present, operational conditions of the claddings of the fuel rods of the nuclear reactors are corresponded to the limit possibilities of the known modern wide-used structural materials and the general problem about developing the structural materials for claddings of fuel rods is of current interests in the modern nuclear science and machinery [2] and using the thin protective coatings is one of effective way to improve the operability of the claddings [3, 4].

All the influencing factors of the cladding of the fuel rods are naturally non-stationary and are significantly depended on the time during operation of the fuel assemblies in the core of the nuclear reactor. It is well-known, that the non-stationary factors, influencing on the cladding of fuel rods, can be represented by the harmonic time dependencies included a lot of the summands with the different frequencies. It is well-known also, that the harmonic time-dependent loadings can lead in some conditions to the more impacting on the mechanical systems than the stationary loadings of same intensities [5], as well as they can induce the specific damaging impacts like the fretting [6]. Due to these circumstances, the problems about oscillations of the fuel rods in assemblies are of current interests, as well as the theme of this research, which deals with the natural frequencies and the modes of the radial oscillations of the claddings of the fuel rods of the WWER-1000 nuclear reactor. The purpose of this research is to develop the approach for evaluating the natural frequencies and the modes of the radial oscillations for the fuel rod's cladding represented as the thick-walled cylinder considering with presence the thin protective coatings, as well as to obtain the quantitative assessments of these natural frequencies and modes of the radial oscillations for the fuel rod's cladding of the WWER-1000 nuclear reactor. These radial oscillations can have impacting on the width of the gap between the cladding and the nuclear fuel pellets and can lead to changes in the temperature state of the nuclear fuel pellets and of the cladding of the fuel rod [1].

MODELLING THE RADIAL FREE VIBRATIONS OF CYLINDRICAL CLADDINGS OF FUEL RODS CONSIDERING THIN PROTECTIVE COATINGS

The cladding of fuel rods made as the long thick-walled cylinder (Fig. 1) represents the typical design [1] which is widely used in the most of nuclear reactors for the power industry. The length L, the internal radius a, the external radius b of the typical cladding for fuel rods are satisfied the conditions:

$$b - a > \frac{1}{8}\frac{b+a}{2}, \tag{1}$$

$$L \gg b. \tag{2}$$

For example, the cladding of the fuel rods used in the WWER-1000 nuclear reactors has the well-known sizes: $L \approx 3800$ mm, $a = 3.9$ mm, $b = 4.55$ mm [7], and it is easy to verify that the conditions (1), (2) are satisfied really.

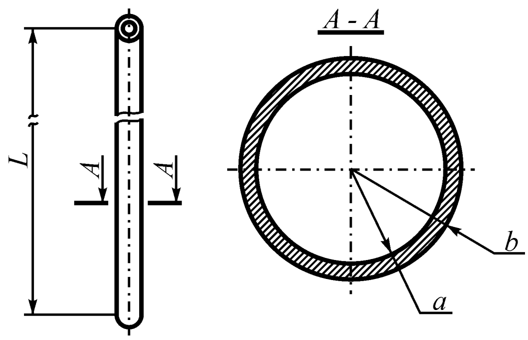

Fig. 1. Typical design of the cylindrical cladding of fuel rods for nuclear reactors

The inequality (1) represents the limiting condition on the sizes for the thick-walled cylindrical structures and it defines the application area for the equations of the theory of elasticity [8]. The condition (2) defines the area of application for the hypotheses of the plane strain problem well-known in the theory of elasticity [9]. Thus, modeling of the radial free oscillations of the typical cylindrical claddings of fuel rods for nuclear reactors can be reduced to the plane strain problem of the theory of elasticity for the cylinder with the side surfaces unloaded and free from any fixings.

Due to the cylindrical shape of the cladding it is suitable to use the cylindrical coordinates, including the radial coordinate r, the circumferential coordinate θ and the axial coordinate z with the corresponding unit vectors $\vec{e}_r$, $\vec{e}_\theta$, and $\vec{e}_z$ as shown on the Fig. 2. The idea of the plane strain is to consider the stress-strain state far from the edges in the central cross-sections of the cladding because such consideration allows neglecting dependence of the stress-strain state on the axial coordinate z due to the condition (2) and it simplifies consideration the problem. Besides, the axial symmetry of the cladding of fuel rods with unloaded side surfaces leads to independence of the strain-stress state on the circumferential coordinate θ and as the result it leads to zero shear stresses and strains. Due to the hypotheses of the plane strain, the stress-strain state of the cladding under the radial axial symmetrical oscillations can be represented using only the radial displacement u, which is depending on the radial coordinate r and the time t only:

$$u = u(r,t). \tag{3}$$

It is well-known in the theory of elasticity [9, 10] that the radial displacement (3) of the thick-walled cylinder considering the plane strain hypotheses must satisfy the differential equation:

$$\rho \frac{1-\nu'^2}{E'} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}, \quad a < r < b, \tag{4}$$

where ρ is the density of the material; $E' = \frac{E}{1-\nu^2}$ and $\nu' = \frac{\nu}{1-\nu}$ are the effective Young's modulus and the Poisson's ratio defined corresponding the plane strain hypothesis thru the values E and ν of the Young's modulus and the Poisson's ratio of the material of the cylinder representing the cladding of fuel rods.

The differential equation (4) must be considered with the initial conditions, defining the state of the cylinder at some given moment $t = t_0$ of the time t:

$$u(r,t_0) = u_0(r), \ \frac{\partial}{\partial t} u(r,t_0) = v_0(r), \ a \le r \le b, \tag{5}$$

where $u_0(r)$ is the given radial displacement field and $v_0(r)$ is given the radial velocity field in the cylinder representing the cladding of fuel rods at the initial moment $t = t_0$ of the time.

The boundary conditions required for considering the differential equation (4) must defining the states of the cylinder representing the cladding on the internal and external side boundary surfaces with coordinates $r = a$ and $r = b$. As was discussed above, in the case of the free oscillations of the cylinder representing the cladding of fuel rods the side surfaces are unloaded and free from any fixings. These types of the boundary conditions for the cylinders representing the cladding of fuel rods without the thin protective coatings is well-known in the theory of elasticity and they are reduced to the condition that the radial stress at the side surfaces are zeroes [9, 10]. The boundary conditions for the cylinder representing the cladding of fuel rods which made with the protective thin coatings had been discussed in the [11] and can be represented in the next form:

$$\frac{E'}{1-\nu'^2}\left(\frac{\partial u}{\partial r} + \nu' \frac{u}{r}\right) - E_a \frac{h_a}{R_a}\frac{u}{r} = 0, \ r = a, \tag{6}$$

$$\frac{E'}{1-\nu'^2}\left(\frac{\partial u}{\partial r} + \nu' \frac{u}{r}\right) + E_b \frac{h_b}{R_b}\frac{u}{r} = 0, \ r = b, \tag{7}$$

where E_a, h_a, and $R_a = a - h_a/2$ are the Young's module, the thickness and the middle surface radius of the internal coating; E_b, h_b, and $R_b = b + h_b/2$ are the Young's module, the thickness and the middle surface radius of the external coating.

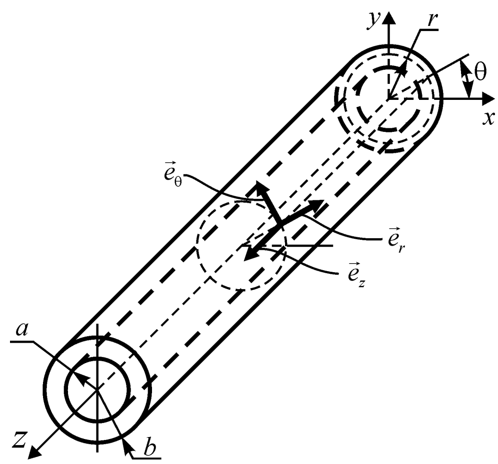

Fig. 2. The cylindrical cladding of fuel rods and corresponded cylindrical coordinates

Summarizing, the mathematical model of the free axial symmetrical radial oscillations of the cladding with the thin protective coatings for fuel rods of nuclear reactor is proposed in the form of the partial differential equation (4) with the initial conditions (5) as well as the boundary conditions (6) and (7).

FINDING THE NATURAL FREQUENCIES

The solution of the problem (4)–(7) about the free radial oscillations of the cylinder representing the cladding of fuel rods can be represented using the imagine value $i^2 = -1$ in the form [10]:

$$u(r,t) = U(r) e^{i(\omega t + \gamma)}, \tag{8}$$

where $U(r)$ is the mode of the oscillation; ω is the cyclic frequency and γ is the initial phase of the oscillation.

Substituting the solution of the form (8) into the equation (4) and into the boundary conditions (6), (7) allows to obtain the differential equation and the boundary conditions for the mode of the oscillation corresponded to the given frequency:

$$\frac{d^2U}{dr^2}+\frac{1}{r}\frac{dU}{dr}-\frac{U}{r^2}-\lambda U=0,\ a<r<b; \quad (9)$$

$$\frac{dU}{dr}+\left(\nu'-\frac{E_a\left(1-\nu'^2\right)}{E'}\frac{h_a}{R_a}\right)\frac{U}{r}=0,\ r=a; \quad (10)$$

$$\frac{dU}{dr}\left(\nu'+\frac{E_b\left(1-\nu'^2\right)}{E'}\frac{h_b}{R_b}\right)\frac{U}{r}=0,\ r=b, \quad (11)$$

where $\lambda=-\rho\frac{1-\nu'^2}{E'}\omega^2$.

Further, the method of grids [12] will be used for approximate solving the differential equation (9) with the boundary conditions (10), (11). Corresponding the idea of the method of grids, the solution represented by continuous function $U(r)$ will be represented by the discrete nodal values of this function in the given nodes (points) of the researched domain $a\le r\le b$, which are defined as (Fig. 3):

$$r_k=a+k\Delta r,\ \Delta r=\frac{b-a}{n+1},\ k=0,1,2,\ldots,n,n+1, \quad (12)$$

where n is the count of the nodes satisfied the condition $a<r<b$ ("internal" nodes); Δr is the step, defined by the distances between any two the nearest points.

Using the grid (12), it is possible to define formally the unknown nodal values of the mode (see Fig. 3):

$$U_k=U\left(r_k\right),\ k=0,1,2,\ldots,n,n+1. \quad (13)$$

To finding the nodal values (13) it is used the finite differences technique [12]; the follows finite differences are used for the internal surface boundary node ($k=0$), for the "internal" nodes ($k=1,2,\ldots,n$) and for the external surface boundary node ($k=n+1$) [12]:

$$\frac{dU_0}{dr}=\frac{-3U_0+4U_1-U_2}{2\Delta r}+o\left(\Delta r^2\right); \quad 14)$$

$$\frac{d^2U_k}{dr^2}=\frac{U_{k-1}-2U_k+U_{k+1}}{\Delta r^2}+o\left(\Delta r^2\right);$$

$$\frac{dU_k}{dr}=\frac{U_{k+1}-U_{k-1}}{2\Delta r}+o\left(\Delta r^2\right),\ k=1,2,\ldots,n; \quad (15)$$

$$\frac{dU_{n+1}}{dr}=\frac{3U_{n+1}-4U_n+U_{n-1}}{2\Delta r}+o\left(\Delta r^2\right). \quad (16)$$

Leading to the method of grids [12], the derivative (14) is substituted into the boundary condition (10), and the derivatives (15) are substituted into the differential equation (9), as well as the derivative (16) is substituted into the boundary condition (11). As the results of these substitutions, the next relations between the nodal values (13) are obtained:

$$\alpha_0U_0+\beta_0U_1+\gamma_0U_2=0; \quad (17)$$

$$\alpha_kU_{k-1}+\beta_kU_k+\gamma_kU_{k+1}-\lambda U_k=0,\ k=1,2,...n; \quad (18)$$

$$\alpha_{n+1}U_{n-1}+\beta_{n+1}U_n+\gamma_{n+1}U_{n+1}=0, \quad (19)$$

where α_k, β_k, and γ_k are the values defines for all numbers of $k=0,1,2,\ldots,n,n+1$ as follows:

$$\alpha_0=\frac{\nu'}{a}-\frac{3}{2\Delta r}-\frac{E_a\left(1-\nu'^2\right)h_a}{E'aR_a},\ \beta_0=\frac{2}{\Delta r},\ \gamma_0=-\frac{1}{2\Delta r};$$

$$\alpha_k=\frac{1}{\Delta r^2}-\frac{1}{2r_k\Delta r};\ \beta_k=-\frac{2}{\Delta r^2}-\frac{1}{r^2};$$

$$\gamma_k=\frac{1}{\Delta r^2}-\frac{1}{2r_k\Delta r};\ k=1,2,\ldots,n;$$

$$\alpha_{n+1}=\frac{1}{2\Delta r};\ \beta_{n+1}=-\frac{2}{\Delta r};$$

$$\gamma_{n+1}=\frac{3}{2\Delta r}+\frac{\nu'}{b}+\frac{E_b\left(1-\nu'^2\right)h_b}{E'bR_b}. \quad (20)$$

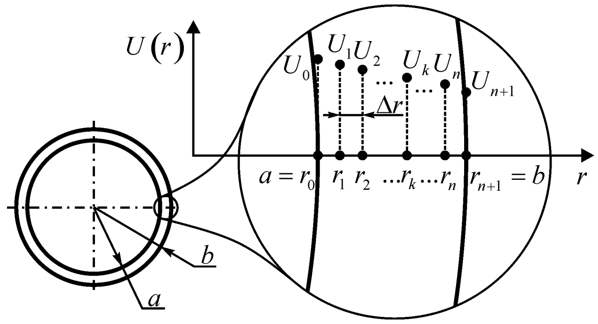

Fig. 3. The cross section of the cladding, as well as the grid nodes and the nodal values of the vibration mode

Using the relations (14) and (16) it is possible to represent the nodal values U_0 and U_{n+1} thru some of the "internal" nodal values:

$$U_0=-\frac{\beta_0}{\alpha_0}U_1-\frac{\gamma_0}{\alpha_0}U_2,$$

$$U_{n+1}=-\frac{\alpha_{n+1}}{\gamma_{n+1}}U_{n-1}-\frac{\beta_{n+1}}{\gamma_{n+1}}U_n. \quad (21)$$

Relations (21) allow excluding the nodal values U_0 and U_{n+1} from the relations (15) and allow representing these relations (15) in the matrix-vector form as follows:

$$\left(\mathbf{A}_n-\lambda\mathbf{I}_n\right)\cdot\mathbf{u}_n=\mathbf{0}_n, \quad (22)$$

where $\mathbf{A}_n$ is the some given matrix and $\mathbf{I}_n$ is the unit diagonal square matrix are with the size $n\times n$; $\mathbf{u}_n$ is the nodal values vector and $\mathbf{0}_n$ is the zero vector are with the size n.

The matrix $\mathbf{A}_n$ and the vector $\mathbf{u}_n$ from the relation (22) are defined as:

$$\mathbf{A}_n=\begin{pmatrix}\beta'_1 & \gamma'_1 & 0 & 0 & \ldots & 0 & 0 & 0\\ \alpha_2 & \beta_2 & \gamma_2 & 0 & \ldots & 0 & 0 & 0\\ 0 & \alpha_3 & \beta_3 & \gamma_3 & \ldots & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \ldots & \alpha_{n-1} & \beta_{n-1} & \gamma_{n-1}\\ 0 & 0 & 0 & 0 & \ldots & 0 & \alpha'_n & \beta'_n\end{pmatrix},$$

$$\mathbf{u}_n=\left(U_1\quad U_n\quad \ldots\quad U_n\right)^T, \quad (23)$$

where β'_1, γ'_1, α'_n, and β'_n are the values defined taking into account the relations (20) as follows:

$$\beta'_1=\beta_1-\beta_0\frac{\alpha_1}{\alpha_0};\ \gamma'_1=\gamma_1-\gamma_0\frac{\alpha_1}{\alpha_0};$$

$$\alpha'_n=\alpha_n-\alpha_{n+1}\frac{\gamma_n}{\gamma_{n+1}};\ \beta'_1=\beta_1-\beta_0\frac{\gamma_n}{\gamma_{n+1}}. \quad (24)$$

Relation (21) represents the homogeneous linear equations for defining the "internal" nodal values, which allow defining the "boundary" nodal values by the relations (21). The condition of existing of the non-zero solution of the homogeneous linear equations (22) defining the "internal" nodal values of the radial oscillation mode has the follows form:

$$\det\left(\mathbf{A}_n - \lambda \mathbf{I}_n\right) = 0 . \qquad (25)$$

The condition (25) represents the non-linear algebraic equation for defining the parameters λ introduced in the differential equation (9); the count of these parameters λ is equal to the number n of the "internal" nodes of the grid (12). By using these parameters $\lambda_1, \lambda_2, \ldots, \lambda_k, \ldots, \lambda_n$, it is possible to define the natural oscillation frequencies of the radial oscillations:

$$\omega_k = \sqrt{-\frac{E'}{\rho}\left(1-\nu'^2\right)\lambda_k}, \; k = 1, 2, \ldots, n . \qquad (26)$$

Solving the algebraic equation represented in the form (25) is well-known as the eigenvalues problem [13]. It is interesting that the matrix $\mathbf{A}_n$ defined in the relations (23) has the Hessenberg's form and the eigenvalue problem for such matrices can be approximately solved using the numerical QR-method directly without the required transformations for the common form matrices [13]. The well-known procedure HQR2 from the handbook [13] is used to solve numerically the problem (25) and to find approximately the eigenvalues and the eigenvectors required for computing the values (25) of the natural frequencies and the natural vibrations modes of the radial vibrations of the cladding of fuel rods. All necessary programs are developed using the FORTRAN programming language which is very suitable for scientific and engineering computing [14].

RESULTS FOR NATURAL OSCILATIONS FREQUENCIES AND MODES OF THE CLADDING OF FUEL RODS

The mathematical formulation (9)–(11) allow us to consider the natural oscillations and modes of the radial vibrations of the cylindrical claddings of fuel rods made with and without the thin protective coatings. Really, influencing the thin protective coatings on the radial oscillations of the cladding is defined by the items with the multipliers $\frac{E_a h_a}{R_a}$ and $\frac{E_b h_b}{R_b}$ presented in the boundary conditions (10) and (11). The particular cases for the zeroes values $E_a h_a = 0$ and $E_b h_b = 0$ are corresponded to the cladding without the internal and external coatings. These circumstances allow us to use the same computing software for evaluating the natural oscillations frequencies and modes both for the claddings with and without the thin protective coatings by the necessary choices of the computing input data. Thus, all possibilities are available for us to research influencing the thin protective coatings on the natural oscillations frequencies of the claddings of fuel rods of nuclear reactors. Next, the quantitative estimations about influencing the thin protective coatings on the natural frequencies and the natural modes of the radial oscillations of the claddings of fuel rods are presented for the WWER-1000 nuclear reactors made without the protective thin coatings and made with these coatings as possible. It is considered the typical cladding of the fuel rods of the WWER-1000 nuclear reactor with the next parameters:

$$a = 3.855\,\text{mm};\, b = 4.55\,\text{mm};$$

$$E = 96\,\text{GPa};\, \nu = 0.33;\, \rho = 6500\,\text{kg}/\text{m}^3 . \qquad (27)$$

Influencing on natural oscillations frequencies and modes of the possible thin protective coatings made from the stainless steel like discussed in [4] with the next value of the Young's modulus:

$$E_a = E_b = 210\,\text{GPa} . \qquad (28)$$

Comparison between the natural oscillation frequencies and the modes for the cladding of fuel rods without the thin protective coatings and with these coatings of different thicknesses h_a and h_b is the methodology basis for estimating the influence of the protective thin protective coatings on the oscillation characteristics of the claddings.

Using the approximate numerical solutions of the eigenvalues and eigenvectors problem (25) to evaluate the natural oscillations frequencies (26) and the modes of radial oscillations of the cladding of fuel rods requires substantiating the accuracy of obtained results. The accuracy of the obtained results for the natural oscillations frequencies and the modes depends on the count n of the "internal" nodes of the grid (see Fig. 3). Increasing the count n of the grid nodes leads to increasing the accuracy of the numerical solutions due to decreasing the approximations errors in the used finite differences (14)–(16) taking into account decreasing the grid step (12) with increasing the nodes number. Due to this depending, substantiating the accuracy of the numerical solutions of the eigenvalues and eigenvectors problem (25) is reduced to substantiating the number n of the grid nodes providing the required accuracy of the results for the natural oscillations frequencies and modes of the cladding of fuel rods. Thus, the accuracy of the approximate numerical solutions of the problem (25) can be estimated by comparing the results obtained by using the different number n of the grid nodes. This comparing (Table) shows that the results with $n \geq 500$ have the error about $2.0 \cdot 10^{-5}\%$, and it is possible to use these results in the further analyses.

To represent the results for the natural frequencies (26) it is used the next values of frequencies:

$$\mu_k = \frac{\omega_k}{2\pi}, \; k = 1, 2, \ldots, n. \qquad (29)$$

The results were obtained for fist natural frequencies of the radial oscillations of the claddings made with the different thin protective coatings are presented on the Fig. 4. It is obtained the large value about 150 kHz of first natural frequency of radial oscillations for the cladding without protective coatings. The protective thin coatings lead to noticeable increasing the value of first natural frequency of the radial oscillations of the cladding of fuel rods. It is shown that the internal coating has the more effect on first natural frequency than the external coating, but effect of presence the both internal and external coatings is approximately equals to superposition of the separate effects from the internal and external coatings.

Convergence of the results for the natural oscillation frequencies with increasing the grid nodes count

Count of the nodes, n	Natural oscillation frequencies, Hz	
	$h_a = h_b = 0$	$h_a = h_b$ = 100 μm
3	154187.1211	193572.9636
10	154608.6745	193873.0273
500	154703.0281	193939.6472
1000	154703.0681	193939.6752

Continuation of the Table

Count of the nodes, n	Natural oscillation frequencies, Hz	
	h_a = 100 μm*	h_b = 100 μm**
3	178571.7676	171302.5818
10	178904.0075	171671.3978
500	178978.2469	171753.5551
1000	178978.2763	171753.5910

*$h_b = 0$; **$h_a = 0$.

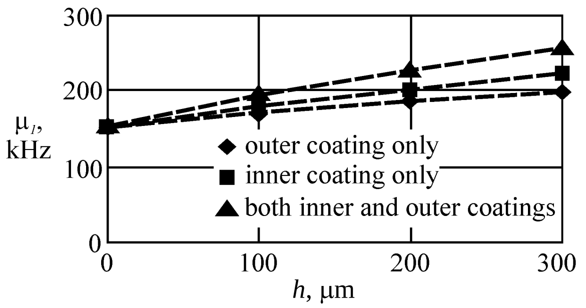

Fig. 4. Influencing the thicknesses h of the coatings on the first natural frequency μ_1 of the radial oscillations of the cylindrical cladding of fuel rods

Let denote $\mu_k^0, k = 1, 2, \ldots, n$ the natural frequencies (29) of the radial oscillations of the cladding of fuel rods with design parameters (27) made without the thin protective coatings. To estimate and to represent the results for influencing the thin protective coatings on the higher natural frequencies of the radial oscillations of the cladding of fuel rods there are used the values of percentile increasing of the frequencies of the claddings with thin coatings comparing with the cladding without the coatings, which are defined as:

$$\mathrm{M}_k = \frac{\mu_k - \mu_k^0}{\mu_k^0} \cdot 100\%, \ k = 1, 2, \ldots, n. \qquad (30)$$

The results of comparing for influencing the thin coatings on first and some higher natural frequencies of the radial oscillations of the cladding of fuel rods are presented on the Fig. 5 using the logarithm coordinates. Due to these results, it is seen (see Fig. 5) that the thin protective coatings are having significant influencing on first natural oscillation frequency only, but influencing on second frequency is ten times smaller than for first frequency and influencing on third frequency is about hundred times smaller than for first frequency.

Let denote as $\mathbf{u}_n^{(k)}, k = 1, 2, \ldots, n$ the vectors representing the solutions of the next homogeneous linear equations:

$$\left(\mathbf{A}_n - \lambda_k \mathbf{I}_n\right) \cdot \mathbf{u}_n^{(k)} = \mathbf{0}_n, \ k = 1, 2, \ldots, n. \qquad (31)$$

Equations (31) are the equations (22) with substituted values $\lambda_1, \lambda_2, \ldots, \lambda_k, \ldots, \lambda_n$ which are the solutions of the equation (25). Due to this circumstance, the linear systems (31) are having the nonzero solutions such that the components of any vector $\mathbf{u}_n^{(k)}, k = 1, 2, \ldots, n$ are defined thru any one of their component. It is suitable to normalize the vectors $\mathbf{u}_n^{(k)}, k = 1, 2, \ldots, n$ such as the absolute maximum component will be equaled to unit. These vectors $\mathbf{u}_n^{(k)}, k = 1, 2, \ldots, n$ are representing the nodal values of the modes $U^{(k)}(r), k = 1, 2, \ldots, n$ of the natural radial oscillations at the grid (12) (see Fig. 3). Each of these natural oscillation modes corresponds to one of the natural oscillation frequencies (26) or (29).

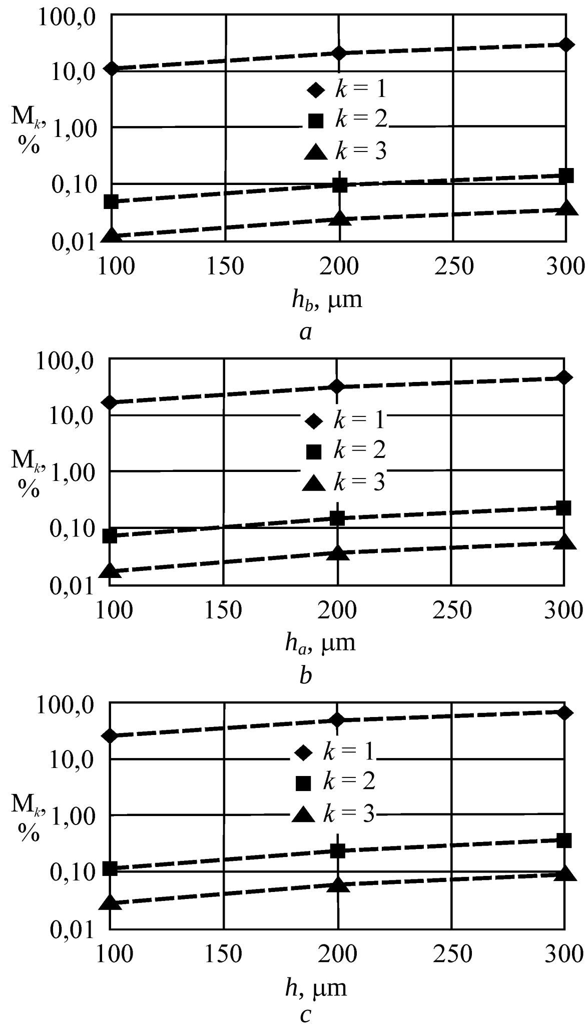

Fig. 5. Influencing the thickness of the coatings on the natural oscillation frequencies of the cladding of fuel rods with the outer (a) and inner coatings (b) only as well as both the inner and outer coatings with the equal thicknesses (c)

Obtained results allow us to conclude that the thin protective coatings are having no noticeable influencing on the natural radial oscillation modes of the cylindrical cladding of fuel rods with design parameters (27). The results for some of the modes of the natural radial oscillations of the cladding of fuel rods are shown on the Fig. 6. The presented results are approved with the well-known fundamental properties [15] of the natural oscillations modes. Really, the each of modes has some number of the crossings with horizontal zero axis: the mode corresponded to first natural frequency has no crossing with zero axis, the mode corresponded to second natural frequency has one crossing with zero axis, the mode corresponded to third natural frequency has two crossing with zero axis and so on (see Fig. 6).

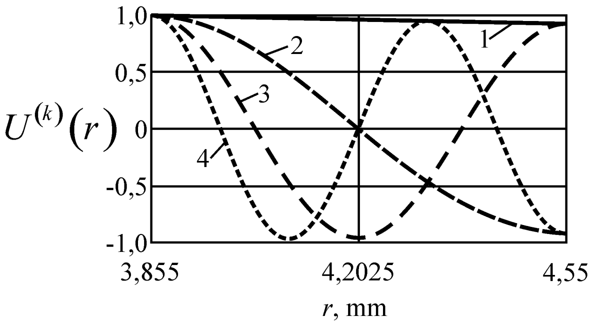

Fig. 6. The modes of the natural radial oscillations of the cladding of fuel rods corresponded to the frequencies with the numbers k :
1 – k = 1; 2 – k = 2; 3 – k = 3; 4 – k = 4

DISCUSSION THE RESULTS

It is obtained the large values about 150 kHz for first natural frequency of the radial oscillations of the typical design of cylindrical cladding of fuel rods. It is well-known in the theory of vibrations [13, 14] that the natural frequencies of oscillations of the structure are defined by relation between the rigidness and the mass of this structure. It is well-known that the elastic cylinder representing the cladding of fuel rods has the high rigidness on the radial direction and it is this high rigidness is the reason for using the claddings with the cylindrical shape, because due to this shape the cladding with small wall thickness has no noticeable strains under the operational pressures from the gaseous fission products and the moving heat carrier. At the same time, the small thickness of the wall leads to the smaller mass of the cladding. Thus, the large values of the natural frequencies are due to the well-known high rigidness on the radial direction of the elastic cylinder representing the cladding of fuel rods with the small thickness of the wall. It is necessary to notice that the model of the thin protective coatings used to formulate the boundary conditions (22), (23) has no considering the inertia of the coating, but considering only the rigidness on the coatings. It seems that the inertia of the claddings is negligible due to the significantly smaller masses, but such neglecting the inertia of the claddings must be substantiated by quantitative results for the natural frequencies considering the inertia of the coatings in further researches.

Obtained numerical results allow us to approve that the thin protective coatings lead to increasing the natural frequencies of the radial oscillations of the typical cylindrical claddings of fuel rods. Increasing first natural frequency of radial oscillations for the typical cylindrical cladding of fuel rods due to using the thin coatings is really noticeable, but increasing the higher natural frequencies is practically negligible comparing with increasing of first frequency. Increasing the natural frequencies due to using the thin protective coating can be explained by increasing the radial rigidness of the cladding through presence the circumferential forces in the coatings as it seen from the boundary conditions (6), (7) or (10), (11) considering the boundary surfaces of the cladding of fuel rods with the thin protective coatings. At the same time, the inertia of the coating is neglected in the boundary conditions (6), (7) modelling of the thin coatings and it is required the additional researches.

CONCLUSIONS

The natural oscillation frequencies of the radial vibrations of the cladding made with the thin protective coatings for the fuel rods of the WWER-1000 nuclear reactors are computed by using the method of grids. It is received the values about 150 kHz for first natural oscillation frequencies of the radial vibrations of the cladding for the fuel rods of the WWER-1000 nuclear reactors. It is shown that the thin protective coatings lead to noticeable increasing of first natural oscillation frequency, but have negligible influencing on the second and higher natural oscillation frequencies as well on the natural modes of the radial oscillations of the typical cylindrical cladding of fuel rods.

The mathematical model proposed for the thin protective coatings and used for formulating the boundary conditions considering with influence of the thin coatings is not took into account the inertia of the coating, but it is took into account only the rigidness on the coatings. It is necessary to substantiate neglecting the inertia of the coatings in further researches, although it is seem that the coatings has really the significantly smaller masses comparing with the mass of the cladding. Besides, it is interesting to estimate the mechanical stresses in the cladding of fuel rods occurring due to the time harmonics of the outer pressure of the moving heat carrier with the frequencies proportional the rotation velocity of the main circulation pump.

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ВЛИЯНИЕ ТОНКИХ ЗАЩИТНЫХ ПОКРЫТИЙ НА СОБСТВЕННЫЕ ЧАСТОТЫ РАДИАЛЬНЫХ КОЛЕБАНИЙ ОБОЛОЧЕК ТВЭЛОВ ЯДЕРНЫХ РЕАКТОРОВ

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Собственные частоты и формы радиальных колебаний рассчитываются с использованием метода сеток для оболочки твэлов ядерных реакторов ВВЭР-1000, выполненной с тонкими защитными покрытиями. Получены значения более 150 кГц для первых собственных частот радиальных колебаний оболочки твэлов ядерных реакторов ВВЭР-1000. Показано, что тонкие защитные покрытия приводят к заметному увеличению первой частоты собственных колебаний, но оказывают незначительное влияние на вторую и более высокие частоты, а также на формы собственных радиальных колебаний оболочек твэлов. Увеличение собственных частот колебаний цилиндрических оболочек твэлов за счет использования тонких защитных покрытий объясняется значительным повышением радиальной жесткости оболочек благодаря наличию окружных сил в покрытиях при незначительном увеличении массы конструкции.

ВПЛИВ ТОНКИХ ЗАХИСНИХ ПОКРИТТІВ НА ВЛАСНІ ЧАСТОТИ РАДІАЛЬНИХ КОЛИВАНЬ ОБОЛОНОК ТВЕЛІВ ЯДЕРНИХ РЕАКТОРІВ

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Власні частоти і форми радіальних коливань розраховуються з використанням методу сіток для оболонки твелів ядерних реакторів ВВЕР-1000, виконаної з тонкими захисними покриттями. Отримано значення більше 150 кГц для перших власних частот радіальних коливань оболонки твелів ядерних реакторів ВВЕР-1000. Показано, що тонкі захисні покриття призводять до помітного збільшення першої частоти власних коливань, але мають незначний вплив на другу і більш високі частоти, а також на форми власних радіальних коливань оболонок твелів. Збільшення власних частот коливань циліндричних оболонок твелів за рахунок використання тонких захисних покриттів пояснюється істотним збільшенням радіальної жорсткості оболонок завдяки наявності окружних сил у покриттях при незначному збільшенні маси конструкції.