DISPERSION PROPERTIES OF THE DIELECTRIC WAVEGUIDE FOR ACCELERATION ELECTRONS BY THE WAKEFIELD OF LASER PULSE

V.A. Balakirev, I.N. Onishchenko

National Science Center "Kharkov Institute of Physics and Technology", Kharkiv, Ukraine
E-mail: onish@kipt.kharkov.ua

Dispersion properties of several dielectric waveguides are considered to provide for wakefield acceleration not only superluminal group velocity of exciting laser pulse but also high relativistic Lorentz factor of laser pulse as a driver. In such waveguides laser pulse can excite electromagnetic wakefields, in which charged particles can be accelerated to high energy before they become dephased.
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INTRODUCTION

Cherenkov electromagnetic radiation as a wakefield can be excited in a slowing down medium not only by a relativistic electron bunch but by a short laser pulse too [1]. For laser power of PW-level excited wakefields are so intensive that particle acceleration by using such dielectric wakefields is related to the advanced methods of high gradient acceleration [2-5]. In [6] the possibility to provide superluminal group velocity of the laser pulse required for wakefield excitation in dielectric waveguide has been considered taking into account the frequency dispersion of the dielectric permittivity. In this paper we consider the dispersion properties of waveguides partly filled with dielectric which are required to achieve possibly higher group velocity of the laser pulse. Phase velocity of the excited wakefield is coincided with group velocity of the laser wave packet, which is less than speed of light. Therefore for realization of effective acceleration of relativistic electrons it is necessary to provide such conditions that group velocity should be more close to speed of light. This requirement can be attained at partly filling waveguide with dielectric. In present report for realization these conditions two dielectric waveguides are considered. First dielectric waveguide is perfectly conductive tube (cylindrical mirror) in which there is thin dielectric layer near the inner wall. Second dielectric structure is dielectric coaxial line, which includes in itself same mirror and located near axis homogeneous dielectric cylinder. It is shown, that in such systems transversal dielectric inhomogeneity will only weekly changes discrete transverse wave numbers of eigen waves of the waveguide. In result phase and group velocities are weekly depend on the degree of filling of waveguide with dielectric.

DIELECTRIC WAVEGUIDE WITH PARIETAL DIELECTRIC LAYER

Let's consider tubular dielectric waveguide which metal tube of inner diameter *2b* is partially filled with a thin layer of dielectric with permittivity ε so the vacuum channel has diameter *2a*.

The dispersion equation describing the propagation of symmetric electromagnetic waves of the E-type with the field components E_z, E_r, H_φ has the view

$$\frac{\lambda J_0(\lambda)}{J_1(\lambda)} = \frac{\sigma}{\varepsilon}\frac{J_0(\sigma)N_0(\sigma\eta) - J_0(\sigma\eta)N_0(\sigma)}{J_1(\sigma)N_0(\sigma\eta) - J_0(\sigma\eta)N_1(\sigma)}, \quad (1)$$

where λ=va, $v = \sqrt{k_0^2 - k^2}$ is transverse wave number in vacuum region, σ=$k_\perp$a, $k_\perp = \sqrt{k_0^2\varepsilon - k^2}$ is transverse wave number in dielectric layer, $k_0 = \omega / c$, $\eta = b/a > 1$, ω is frequency, k is longitudinal wave number. The parameters λ and σ are related by the relation

$$\sigma^2 = \rho^2 + \lambda^2, \quad (2)$$

$\rho^2 = k_0^2 a^2 (\varepsilon - 1)$ is frequency parameter. The dispersion equation (1) together with the relation (2) has a highly universal form and determines the discrete spectrum of transverse wave $\lambda = \lambda_n(\rho, \varepsilon, \eta)$. From the last relation we find the longitudinal wave numbers of the fast electromagnetic eigen waves of the dielectric waveguide

$$k_n = \sqrt{k_0^2 - \frac{\lambda_n^2(\rho, \varepsilon, \eta)}{a^2}}.$$

In interesting for us quasi-optical frequency range

$$k_0^2 a^2 >> 1$$

the phase velocity of the eigen waves is close to the speed of light

$$k_n = k_0 - \frac{1}{2}\frac{\lambda_n^2}{k_0 a^2}, \; k_0^2 a^2 >> \lambda_n^2,$$

$$\frac{v_{ph}}{c} = 1 + \frac{1}{2}\frac{\lambda_n^2}{k_0^2 a^2}. \quad (3)$$

Accordingly, for group velocity we have expression

$$\frac{v_g}{c} = 1 - \frac{\lambda_n^2}{2k_0^2 a^2} + \frac{\lambda_n^2}{k_0^2 a^2}\psi_n(\omega), \quad \psi_n(\omega) = \frac{\omega}{\lambda_n}\frac{\partial \lambda_n}{\partial \omega}. \quad (4)$$

From this relation it follows that, when the following condition

$$\left|\frac{\lambda_n^2}{k_0^2 a^2}\psi_n(\omega)\right| << 1$$

is satisfied, the dielectric layer weakly influences on the group velocity value, which, in turn, is close to the speed of light in a vacuum.

In the quasi-optical approximation $\sigma >> 1$ we can use asymptotic representations of the Bessel and Neumann functions for large values of the argument. As a result, instead of (1), we obtain

$$\lambda\frac{J_0(\lambda)}{J_1(\lambda)} = \frac{\sigma}{\varepsilon}\mathrm{tg}(\sigma\mu),$$

$\mu=\ell/a$, $\ell=b-a$ is the thickness of the dielectric layer. For radial harmonics with numbers $n << \rho/\pi$ we obtain a simpler transcendental equation for the eigenvalues λ_n

$$\lambda\frac{J_0(\lambda)}{J_1(\lambda)} = f(\rho),\ f(\rho) = \frac{\rho}{\varepsilon}\mathrm{tg}(\rho\mu). \quad (5)$$

It is easy to verify that, under the condition $f(\rho) > 0$ the roots of this equation are in the intervals $\nu_{0(n+1)} > \lambda_n > \nu_{1n}$, and under the condition $f(\rho) < 0$ ones are in the intervals $\nu_{1(n+1)} > \lambda_n > \nu_{0n}$, where ν_{0n} and ν_{1n} are the roots of the Bessel functions $J_0(x)$ and $J_1(x)$, respectively.

To determine the group velocity (4) the function $\partial\lambda_n/\partial\omega$ easy to calculate from equation (5)

$$\frac{\omega}{\lambda_n}\frac{\partial\lambda_n}{\partial\omega} = -\frac{\rho}{\varepsilon}\frac{\mathrm{tg}(\rho\mu) + \frac{(\rho\mu)}{\cos^2(\rho\mu)}}{\left(\frac{\rho}{\varepsilon}\mathrm{tg}(\rho\mu) - 1\right)^2 + \lambda_n^2 - 1}.$$

In result for group velocity fast eigen electromagnetic wave in the high frequency limit $k_0^2 a^2 >> \lambda_n^2$ we obtain the following expression

$$\frac{v_g}{c} = 1 - \frac{\lambda_n^2}{2k_0^2 a^2} - \frac{\lambda_n^2}{2k_0^2 a^2}\frac{\rho}{\varepsilon}\frac{2\rho\mu + \sin(2\rho\mu)}{\cos^2(\rho\mu)\left[\left(\frac{\rho}{\varepsilon}\mathrm{tg}^2(\rho\mu) - 1\right)^2 + (\lambda_n^2 - 1)\right]}. \quad (6)$$

Let us investigate the expression for the group velocity (6). In the case $\mu\rho = \pi m$ (m is an integer), we obtain

$$\frac{v_g}{c} = 1 - \frac{\nu_{0n}^2}{2k_0^2 a^2} - \frac{\varepsilon - 1}{\varepsilon}\frac{\ell}{a}.$$

The additive to the group velocity does not depend on the number of the radial mode. The condition $\mu\rho = \pi m$ means that an integer number of transverse half-waves fit in the dielectric layer $\ell = m\lambda_\perp/2$, $\lambda_\perp = 2\pi/k_\perp$, $k_\perp = k_0\sqrt{\varepsilon - 1}$. In this case, on the inner surface of the layer, the longitudinal component of the electric field equals zero, and the connection between the vacuum region and the dielectric layer is realized through a magnetic field.

We now consider the case $\mu\rho = \pi(m + 1/2)$ or $\ell = m\lambda_\perp/2 + \lambda_\perp/4$. For the group velocity, we obtain

$$\frac{v_g}{c} = 1 - \frac{\nu_{1n}^2}{2k_0^2 a^2} - \frac{\nu_{1n}^2}{2k_0^2 a^2}\frac{2\ell}{a}\varepsilon.$$

In the case under consideration, the magnetic field vanishes on the inner surface of the dielectric layer, and the connection between the regions occurs through an electric field.

Thus, the analysis showed that in the quasi-optical frequency range $k_0^2 a^2 >> \lambda_n^2$ the group velocity of the fast electromagnetic eigen waves ($v_{ph} > c$) of the dielectric waveguide is close to the speed of light in a vacuum. The dielectric layer slightly slows down the wave packet so that $\gamma_g >> 1$.

DIELECTRIC COAXIAL LINE

The dielectric structure of this type is represented by a homogeneous dielectric cylinder of radius a, which is surrounded by a coaxial cylindrical mirror of larger radius $b > a$.

The dispersion equation describing symmetric electromagnetic waves of the E-type in such a dielectric structure has the form [7]

$$\lambda\frac{J_0(\lambda)N_0(\lambda\eta) - J_0(\lambda\eta)N_0(\lambda)}{J_1(\lambda)N_0(\lambda\eta) - J_0(\lambda\eta)N_1(\lambda)} = \frac{\sigma}{\varepsilon}\frac{J_0(\sigma)}{J_1(\sigma)}, \quad (7)$$

where $\lambda = va$, $\sigma = k_\perp a$, $\sigma = k_\perp a$, $\eta = b/a$.

The parameters λ and σ are related by (2). In the high-frequency limiting case $\lambda^2 \gg 1$, $\rho^2 \gg \lambda^2$, $\lambda^2/2\rho \ll 1$, the dispersion equation (7) can be simplified

$$\lambda\mathrm{tg}\lambda = \frac{\mu\rho}{\varepsilon}\frac{J_0(\rho)}{J_1(\rho)} \equiv f(\rho). \quad (8)$$

Here $\mu = \frac{b}{a} - 1$. The transverse wave number in the vacuum region λ is defined as follows

$$\lambda = v(b-a).$$

From this relation instead of formulas (3), (4) we obtain expressions for the phase and group velocities

$$\frac{v_{ph}}{c} = 1 + \frac{1}{2}\frac{\lambda_n^2}{k_0^2(b-a)^2}, \quad (9)$$

$$\frac{v_g}{c} = 1 - \frac{\lambda_n^2}{2k_0^2(b-a)^2} + \frac{\lambda_n^2}{k_0^2(b-a)^2}\psi_n(\omega), \quad (10)$$

where λ_n are roots of the transcendental equation (8). For positive values of the function $f(\rho)$, the roots of equation (8) are in intervals $\pi(n+1/2)>\lambda_n>\pi n$, n is positive integer, and for negative values $f(\rho)$ intervals are $\pi n>\lambda_n>\pi(n-1/2)$.

For a function $\psi_n(\omega)$ contained in the expression for the group velocity (10), we have.

$$\psi_n(\omega)=-\frac{\mu\rho^2}{\varepsilon\lambda_n^2}\frac{1-\frac{2}{\rho}Q(\rho)+Q^2(\rho)}{\frac{\mu^2\rho^2}{\varepsilon^2}+\frac{\mu\rho}{\varepsilon\lambda_n}Q(\rho)+Q^2(\rho)},$$

$$Q(\rho)=J_1(\rho)/J_0(\rho).$$

Let us consider some particular cases. For discrete frequencies $\omega_m=\frac{c\nu_{0m}}{a\sqrt{\varepsilon-1}},(\rho=\nu_{0m})$ from the dispersion equation (16) we find the roots $\lambda_n=\pi n$.

Correspondingly, for the function $\psi_n(\omega)$ we obtain

$$\psi_n(\omega)=-\frac{\mu\rho^2}{\varepsilon\lambda_n^2}.$$

For the phase and group velocities we have expressions

$$\frac{v_{ph}}{c}=1+\frac{1}{2}\frac{\pi^2n^2}{k_0^2(b-a)^2},$$

$$\frac{v_g}{c}=1-\frac{\pi^2n^2}{2k_0^2(b-a)^2}-\frac{\varepsilon-1}{\varepsilon}\frac{a}{b-a}.$$

For a thin dielectric rod $a\ll b$, the correction to the group velocity due to the dielectric rod is small and negative.

Let now consider the set of frequencies $\omega_m=\frac{c\nu_{1m}}{a\sqrt{\varepsilon-1}}$, $(\rho=\nu_{1n})$. Then the spectrum of transverse wave numbers is $\lambda_n=\pi(n-1/2)$. For the phase and group velocities, we obtain

$$\frac{v_{ph}}{c}=1+\frac{\pi^2(n-1/2)^2}{2k_0^2(b-a)^2},$$

$$\frac{v_g}{c}=1-\frac{\pi^2(n-1/2)^2}{2k_0^2(b-a)^2}-\frac{\varepsilon}{k_0^2(b-a)^2}\frac{a}{b-a}.$$

The addition to group velocity is also small. And, finally, far from these discrete sets of frequencies for the group velocity of fast electromagnetic waves of a dielectric coaxial line, we have expression

$$\frac{v_g}{c}=1-\frac{\lambda_n^2}{k_0^2(b-a)^2}+\frac{1}{k_0^2(b-a)^2}\frac{\varepsilon a}{b-a}\left[1+Q^2(\rho)\right].$$

Thus, in the whole quasi-optical frequency region, in the case of a thin dielectric rod $a/b\ll 1$, the group velocity of fast electromagnetic waves is close to the speed of light.

As is known, in a perfectly conducting coaxial line, there is a coaxial (TEM) electromagnetic wave with a simple dispersion law $\omega=kc$ and a phase velocity equal to the speed of light in a vacuum. An analogous quasi-coaxial wave can propagate in a dielectric coaxial line. For a theoretical analysis of this wave, we will consider the exact dispersion equation (7). We will assume that the phase velocity of the quasi-coaxial wave is close to the speed of light in a vacuum and consequently $\lambda_\eta\ll 1$. In this limiting case, the dispersion equation (7) is substantially simplified and takes the form

$$v^2a^2=\frac{1}{\varepsilon\ln(b/a)}\frac{\sigma J_0(\sigma)}{J_1(\sigma)}. \tag{11}$$

Solving equation (11) by the method of successive approximations, we find expressions for the longitudinal wave number, phase and group velocities of the quasi-coaxial wave

$$k=\frac{\omega}{c}-\frac{1}{2}\frac{\sqrt{\varepsilon-1}}{\varepsilon}\frac{1}{a\ln(b/a)}\frac{1}{Q(\rho)},$$

$$\frac{v_{ph}}{c}=1+\frac{1}{2}\frac{\sqrt{\varepsilon-1}}{\varepsilon}\frac{1}{k_0a\ln(b/a)Q(\rho)}, \tag{12}$$

$$\frac{v_g}{c}=1-\frac{\varepsilon-1}{2\varepsilon}\frac{1}{\ln(b/a)}G(\rho),$$

where $G(\rho)=-\frac{d}{d\rho}\frac{J_0(\rho)}{J_1(\rho)}\equiv 1+\frac{1}{Q^2(\rho)}-\frac{1}{\rho Q(\rho)}$.

Frequencies $\omega_n=\nu_{0n}c/(a\sqrt{\varepsilon-1})$ are called critical [7] and they separate fast and slow waves. Indeed, in the vicinity of the critical frequencies, the expression for the phase velocity (12) can be written in the form

$$\frac{v_{ph}}{c}=1-\frac{1}{2}\frac{\varepsilon-1}{\varepsilon\ln(b/a)}\frac{\omega-\omega_n}{\omega_n}.$$

When $\omega<\omega_n$ $v_{ph}>c$, and when $\omega>\omega_n$ $v_{ph}<c$.

In this case, the group velocity is equal

$$\frac{v_g}{c}=1-\frac{\varepsilon-1}{2\varepsilon}\frac{1}{\ln(b/a)}.$$

The group velocity is close to c for a large ratio of the radii of the mirror and the dielectric cylinder.

CONCLUSIONS

A necessary condition for the realization of the acceleration method of relativistic electrons (positrons) by the wake fields of a short laser pulse in dielectric structures is the possibility of propagation in them of laser wave packets with a group velocity close to the speed of light in a vacuum. This is due to the fact that the phase velocity of the Cerenkov wakefield in dielectric media coincides with the group velocity of the laser pulse. The condition $v_g\approx c$ can be realized in dielectric slowing structures, in which the dielectric only partially fills the dielectric structure in the cross section. In this case, the transverse dielectric inhomogeneity will only slightly perturb the discrete transverse wave numbers of the eigen waveguide waves. In turn, in the quasi-optical approximation $\omega a / c \gg 1$, where a is the characteristic transverse dimension of the structure, the phase and group velocities are weakly dependent on the values of the transverse wave number and, respectively, the degree dielectric filling of dielectric structure.

The theoretical analysis performed in the work on the example of two dielectric structures fully confirms the quasi-optical ideology presented above. It is shown that, indeed, in the quasi-optical approximation, the filling dielectric with a relatively small volume does not lead to a significant slowing down the group velocity.

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ДИСПЕРСИОННЫЕ СВОЙСТВА ДИЭЛЕКТРИЧЕСКИХ ВОЛНОВОДОВ ДЛЯ УСКОРЕНИЯ ЭЛЕКТРОНОВ КИЛЬВАТЕРНЫМ ПОЛЕМ ЛАЗЕРНОГО ИМПУЛЬСА

В.А. Балакирев, И.Н. Онищенко

Рассматриваются дисперсионные свойства нескольких диэлектрических волноводов, обеспечивающих для кильватерного ускорения не только сверхсветовую групповую скорость возбуждающего лазерного импульса, но и высокий релятивистский лоренц-фактор лазерного импульса как драйвера. В таких волноводах лазерный импульс может возбуждать электромагнитные кильватерные поля, в которых заряженные частицы могут быть ускорены до высокой энергии до того, как они станут дефазированными.

ДІСПЕРСІЙНІ ВЛАСТИВОСТІ ДІЕЛЕКТРИЧНИХ ХВИЛЕВОДІВ ДЛЯ ПРИСКОРЕННЯ ЕЛЕКТРОНІВ КІЛЬВАТЕРНИМ ПОЛЕМ ЛАЗЕРНОГО ІМПУЛЬСУ

В.А. Балакірев, І.М. Оніщенко

Розглядаються дисперсійні властивості декількох діелектричних хвилеводів, що забезпечують для кільватерного прискорення не тільки надсвітлову групову швидкість лазерного імпульсу, що збуджує кільватерне поле, але і високий релятивістський лоренц-фактор лазерного імпульсу як драйвера. У таких хвилеводах лазерний імпульс може збуджувати електромагнітні кільватерні поля, в яких заряджені частинки можуть бути прискорені до високої енергії до того, як вони стануть дефазованими.