

CHERENKOV RADIATION OF THE ELECTRON BUNCH IN DIELECTRIC MEDIA WITH FREQUENCY DISPERSION

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Excitation of the Cherenkov electromagnetic radiation by a relativistic electron bunch in a dielectric waveguide is considered taking into account the frequency dispersion of the dielectric permittivity. Electric polarization in an isotropic dielectric medium and, accordingly, polarization charges and currents induced by the Coulomb electric field of a relativistic electron bunch are determined. The spatial-temporal structure of the excited wakefield in a dielectric waveguide is obtained and investigated. It is shown that the excited field consists of a potential polarization electric field and a set of eigen electromagnetic waves of the dielectric waveguide.

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INTRODUCTION

Electric charge (electron bunch), moving in a dielectric medium with a super light speed, radiates wake electromagnetic waves – Cherenkov radiation [1, 2]. Since the wakefields in the dielectric are slow ($v_{ph} < c, v_{ph}$ – phase velocity of the wave), they can be used for acceleration charged particles. The method of acceleration of charged particles by the wakefields excited in a dielectric medium is very prospective and presently much give one's attention to it. In theoretical works devoted to this topic [3-6], as a rule, the frequency dispersion of the dielectric permittivity of the medium is not taken into account. In the framework of this approximation, the permittivity is independent of frequency and is a constant. Meanwhile, accounting of frequency dispersion leads to a number of qualitative features in the picture of excitation of wakefields in dielectric media. First of all, the permittivity can be equal to zero $\epsilon(\omega_g) = 0$. The frequency $\omega = \omega_g$ corresponds to the longitudinal (potential) polarization oscillations of the dielectric medium. In addition, the condition for the appearance of Cherenkov radiation, its frequency spectrum and, in general, the wakefield field picture in a dielectric medium change.

In this paper the process of excitation of Cherenkov radiation of electromagnetic waves by a relativistic electron bunch in a condensed medium, taking into account the frequency dispersion of the dielectric constant is investigated. For the case of a dielectric waveguide, expressions for the wakefield fields are obtained and studied.

FORMULATION OF THE PROBLEM. BASIC EQUATIONS

We consider a dielectric waveguide made in the form of a homogeneous dielectric cylinder whose lateral surface is covered by an ideally conducting film. A monoenergetic relativistic bunch of charged particles propagates uniformly and rectilinearly along the waveguide. The initial system of equations contains the Maxwell's equations

$$\begin{aligned} \text{rot}\vec{E} &= -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}, \quad \text{rot}\vec{H} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}_{\text{ext}}, \\ \text{div}\vec{D} &= 4\pi\rho_{\text{ext}}, \quad \text{div}\vec{H} = 0, \end{aligned} \quad (1)$$

$\rho_{\text{ext}}, \vec{j}_{\text{ext}}$ are densities of external charges and currents, in our case of an electron bunch, $\vec{D} = \vec{E} + 4\pi\vec{P}$ is electric induction, \vec{P} is vector of electrical polarization of the dielectric medium. The Maxwell equation must be supplemented by a material equation for the electrical polarization of the \vec{P} .

In a condensed medium, each atom is in a local electric field \vec{E}_{loc} , which can be very different from the macroscopic field \vec{E} , which is described by Maxwell's equations (1). The local electric field \vec{E}_{loc} includes both the external field \vec{E} and the total electric field of the induced dipoles surrounding the given atom. In a crystalline medium with a cubic crystal lattice, the local electric field is described by the Lorentz formula [7]

$$\vec{E}_{\text{loc}} = \vec{E} + \frac{4\pi}{3} \vec{P}. \quad (2)$$

Taking into account the Lorentz formula (2), the material equation (3) has the form

$$\frac{\partial^2 \vec{P}}{\partial t^2} + \omega_d^2 \vec{P} = \frac{Ze^2 N}{m} \vec{E}, \quad (3)$$

where $\omega_d^2 = \omega^2 - \omega^2/3$, $\frac{4\pi Z^2 e^2}{m}$ is plasma frequency, ω_d is the frequency of the dipole oscillations of the dielectric condensed medium, which is lower than the frequency of the individual atomic dipole oscillator ω_0 .

We shall solve the problem of exciting the wakefield by an axisymmetric electron bunch in a dielectric waveguide as follows. Let us first determine the field (Green's function) of a moving charge, which has the form of an infinitely thin ring with a charge density

$$d\rho = -dQ \frac{1}{v_0} \frac{\delta(r-r_0)}{2\pi r_0} \delta(t - \frac{z}{v_0} - t_0), \quad (4)$$

where r_0 is ring radius, t_0 is the time of flight of an elementary ring bunch into a waveguide, v_0 is bunch velocity, $dQ(r_0, t_0)$ is the charge of particles in the ring connected with the current density of the bunch at the entrance to the dielectric waveguide ($z=0$) $j_0(t_0, r_0)$ by the relation $dQ = j_0(t_0, r_0)2\pi r_0 dr_0 dt_0$. The current density of an elementary ring charge is related to the charges (4) by the relation $d\vec{j} = v_0 d\rho \vec{e}_z$, \vec{e}_z is unit vector in the longitudinal direction. We expand the values in the Fourier integrals in the Maxwell's equations (1) and also in the material equation (3)

$$(\vec{E}_G, \vec{H}_G) = \int_{-\infty}^{\infty} (\vec{E}_\omega, \vec{H}_\omega) e^{-i\omega\tau} d\omega, \quad \vec{P} = \int_{-\infty}^{\infty} \vec{P}_\omega e^{-i\omega\tau} d\omega, \quad (5)$$

$\tau = t - z/v_0 - t_0$, \vec{E}_G, \vec{H}_G is electromagnetic field (Green's function) of elementary charge (4) and current.

From the material equation (3) we find the expression for the Fourier component of the polarization

$$\vec{P}_\omega = \frac{\varepsilon(\omega) - 1}{4\pi} \vec{E}_\omega, \quad (6)$$

where
$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_d^2}. \quad (7)$$

The system of Maxwell's equations (1), taking into account the relations (7), is conveniently reduced to equations for the longitudinal Fourier component of the electric field

$$\frac{1}{r} \frac{d}{dr} r \frac{dE_{z\omega}}{dr} + k_\perp^2 E_{z\omega} = \frac{i}{\pi} \frac{k_\perp^2}{\omega \varepsilon(\omega)} dQ \frac{\delta(r - r_0)}{r_0}, \quad (8)$$

$k_\perp^2 = k_0^2 \varepsilon(\omega) - k^2$, $k = \omega/v_0$, $k_0 = \omega/c$. On the ideally conducting side surface of the dielectric waveguide $r = b$, the longitudinal component of the electric field is equal zero

$$E_{z\omega}(r = b) = 0. \quad (9)$$

On the surface $r = r_0$ along which the ring electron bunch propagates, the longitudinal Fourier component of the electric field satisfies the boundary conditions

$$E_{z\omega}(r = r_0 + 0) - E_{z\omega}(r = r_0 - 0), \quad \frac{dE_{z\omega}}{dr} \Big|_{r=r_0+0} - \frac{dE_{z\omega}}{dr} \Big|_{r=r_0-0} = \frac{i}{\pi r_0} \frac{k_\perp^2(\omega)}{\omega \varepsilon(\omega)} dQ. \quad (10)$$

The solution of equation (8) taking into account the boundary conditions (9)-(10) has the form

$$E_{Gz}(r, \tau) = -\frac{i}{2} dQ \int_{-\infty}^{\infty} d\omega e^{-i\omega\tau} \frac{k_\perp^2(\omega) G_0(k_\perp r, k_\perp r_0)}{\omega \varepsilon(\omega) J_0(k_\perp b)}, \quad (11)$$

where

$$G_0(k_\perp r, k_\perp r_0) = \begin{cases} J_0(k_\perp r) \Delta_0(k_\perp r_0, k_\perp b), & r \leq r_0, \\ J_0(k_\perp r_0) \Delta_0(k_\perp r, k_\perp b), & r \geq r_0, \end{cases}$$

$$\Delta_0(k_\perp r, k_\perp b) = J_0(k_\perp r) N_0(k_\perp b) - J_0(k_\perp b) N_0(k_\perp r).$$

The integrand in (11) has simple poles $\omega = \pm\omega_g - i0$ ($\omega_g = \sqrt{\omega_p^2 + \omega_d^2}$), located in the lower half-plane of the complex variable ω and being zeros of the dielectric constant $\varepsilon(\omega) = 0$. As noted above, these poles correspond to the excitation of longitudinal polarization oscillations of the dielectric medium. Calculating the

residues at these poles, we find the expression for the polarization wakefield. excited by a ring electron bunch

$$E_{Gz}^{(pol)}(r, \tau) = 3dQ \frac{\omega_p^2}{v_0^2} \chi(\tau) \cos \omega_g \tau \Gamma_0(k_p r, k_p r_0),$$

$$\Gamma_0(k_p r, k_p r_0) = \frac{1}{I_0(k_p b)} \begin{cases} I_0(k_p r) F_0(k_p r_0, k_p b), & r < r_0, \\ I_0(k_p r_0) F_0(k_p r, k_p b), & b > r > r_0, \end{cases}$$

$$F_0(k_p r, k_p b) = K_0(k_p r) I_0(k_p b) - K_0(k_p b) I_0(k_p r),$$

where $k_p = \omega_g / v_0$, $\chi(\tau)$ is unit function.

The integrand in (11) has also simple poles, which are determined from the equations

$$D_n(\omega) \equiv \frac{\omega^2}{c^2} \varepsilon(\omega) - \frac{\omega^2}{v_0^2} - \frac{\lambda_n^2}{b^2} = 0. \quad (12)$$

These roots determine the frequency spectrum of the Cerenkov electromagnetic waves excited by an electron ring bunch in a dielectric waveguide. In the spectral equation (12), λ_n are the roots of the Bessel function $J_0(x)$. The spectral equation (12), taking into account the explicit expression for the dielectric constant (7), can be given a form more convenient for analysis

$$D_n(\omega) = \frac{1}{v_0^2 \gamma_0^2} \frac{1}{\omega^2 - \omega_d^2} (\omega^2 - \omega_{chn}^2)(\omega^2 + v_{sm}^2) = 0,$$

where

$$\gamma_0^2 = (1 - \beta_0^2)^{-1}, \quad \beta_0 = v_0 / c, \quad \omega_{chn} = \omega_d x_{chn}, \quad v_{sm} = \omega_d x_{sm},$$

$$x_{chn}^2 = -\frac{1}{2}(b_0 + y_n^2) + \sqrt{\frac{1}{4}(b_0 + y_n^2)^2 + y_n^2}, \quad (13)$$

$$x_{sm}^2 = \frac{1}{2}(b_0 + y_n^2) + \sqrt{\frac{1}{4}(b_0 + y_n^2)^2 + y_n^2}, \quad (14)$$

$$b_0 = \gamma_g^2 (\beta_g^2 \varepsilon_0 - 1), \quad y_n = \frac{\lambda_n c}{\omega_d b} \beta_g \gamma_g,$$

$\varepsilon_0 = 1 + \omega_p^2 / \omega_d^2$ is static permittivity of the dielectric medium $\varepsilon_0 = \varepsilon(\omega \rightarrow 0)$. The poles $\omega = \pm\omega_{chn} - i0$ are also located in the lower half-plane of the complex variable ω nearly the real axis. The frequencies $\omega = \omega_{chn}$ are the frequencies of the electromagnetic waves of the dielectric waveguide, which are in the Cerenkov synchronism $\beta_0^2 \varepsilon(\omega_{chn}) = 1$ with the electron bunch. Since these frequencies are always real, the Cerenkov radiation of electromagnetic waves excited by an electron bunch in a dielectric waveguide takes place for all values of the electron bunch velocity and parameters of the dielectric waveguide (in our case, the values of the static dielectric constant ε_0 and the radius of the waveguide b).

In addition to the real Cerenkov poles, there are also a pair of complex conjugate poles $\omega = \pm i v_{sm}$ in the integrand (11) located on the imaginary axis. These poles correspond to a quasi-static electromagnetic field localized in the region of the electron bunch. Calculating the residues at all these poles of the integrand (11), we find the expression for the Cerenkov electromagnetic field excited by a thin ring electron bunch

$$E_{Gz}^{(em)}(r, \tau) = \frac{2dQ}{b^2} \sum_{n=1}^{\infty} \mu_n \Pi_n(r, r_0) \left[\sigma_{sm} \text{sign}\tau e^{-v_{sm}|\tau|} + 2\sigma_{sm} \chi(\tau) \cos \omega_{chn} \tau \right], \quad (15)$$

where

$$\mu_n = \frac{y_n^2 \omega_d^2}{\omega_d^2 + v_{sm}^2}, \quad \sigma_{chn} = \frac{\omega_d^2 - \omega_{chn}^2}{\omega_{chn}^2 \varepsilon_{chn}}, \quad \sigma_{sm} = \frac{\omega_d^2 - v_{sm}^2}{v_{chn}^2 \varepsilon_{sm}},$$

$$\varepsilon_{chn} = \frac{\omega_{chn}^2 - \omega_d^2 \varepsilon_0}{\omega_{chn}^2 - \omega_d^2}, \quad \varepsilon_{sm} = \frac{v_{sm}^2 + \omega_d^2 \varepsilon_0}{v_{sm}^2 + \omega_d^2},$$

$$\Pi_n(r, r_0) = \frac{J_0(\lambda_n r / b) J_0(\lambda_n r_0 / b)}{J_1^2(\lambda_n)}.$$

The Cerenkov electromagnetic field includes a bipolar field pulse which propagates in the waveguide with the bunch velocity and localizes in the vicinity of the ring electron bunch, as well as a set of eigen monochromatic waves of the dielectric waveguide propagating behind the bunch.

Let us investigate the expression for the electromagnetic field (15) in the quasistatic approximation $\omega_d^2 \gg \omega^2$. In this approximation, the permittivity is independent of frequency $\varepsilon_{ch} = \varepsilon_0$. Consider the most interesting case $b_0 > 0$ or $v_0 > c / \sqrt{\varepsilon_0}$, when the Cerenkov radiation condition is satisfied in the static approximation. In this case, from expressions (13), (14) we find

$$\omega_{chn} = \frac{\omega_d}{\sqrt{b_0}} = \frac{\lambda_n y_g}{b \sqrt{\beta_g^2 \varepsilon_0 - 1}},$$

$$v_{sm} = \omega_d \sqrt{b_0} = \omega_d \gamma_g \sqrt{\beta_g^2 \varepsilon_0 - 1} \equiv v_{st}.$$

Accordingly, for the expression of the Cerenkov electric field, we obtain

$$E_{Cz}^{(em)}(r, \tau) = -\frac{4dQ}{b^2 \varepsilon_0} \sum_{n=1}^{\infty} \Pi_n(r, r_0) \left[\chi(\tau) \cos \omega_{chn} \tau + \frac{1}{2} \beta_0^2 (\varepsilon_0 - 1) \varepsilon_0 \frac{\omega_{chn}^2}{\omega_d^2} \frac{\omega_{chn}^2 b^2}{c^2 \lambda_n^2} \text{sign}\tau e^{-v_{sm}|\tau|} \right].$$

The amplitude and width of the quasi-static electromagnetic pulse in the considered limiting case are small. In the case $b_0 < 0$ or $v_0 < c / \sqrt{\varepsilon_0}$, when the Cerenkov radiation condition is not satisfied in the quasistatic approximation we have

$$\omega_{chn} = \omega_d \sqrt{-b_0} = \omega_d \gamma_g \sqrt{1 - \beta_g^2 \varepsilon_0},$$

$$v_{sm} = \frac{\omega_d y_n}{\sqrt{-b_0}} = \frac{\lambda_n v_g}{b \sqrt{1 - \beta_g^2 \varepsilon_0}} \equiv v_{st}.$$

Then for the longitudinal component of the electric field we obtain the following expression

$$E_{Cz}^{(em)}(r, \tau) = \frac{4dQ}{b^2 \varepsilon_0} \sum_{n=1}^{\infty} \Pi_n(r, r_0) \left[\frac{1}{2} \text{sign}\tau e^{-v_{sm}|\tau|} + \beta_0^2 (\varepsilon_0 - 1) \varepsilon_0 \frac{v_{chn}^2 b^2}{\omega_d^2 c^2 \lambda_n^2} \frac{v_{sm}^2}{\omega_d^2} \chi(\tau) \cos \omega_{chn} \tau \right].$$

The electromagnetic field contains a set of bipolar pulses with a characteristic width $1/v_{sm}$ and a monochromatic high-frequency wakefield with a frequency ω_{ch} and a small amplitude.

Consider an electron bunch with a current density

$$j_0(r_0, t_0) = j_0 R(r_0 / r_b) T(t_0 / t_b), \quad (16)$$

where the function $R(r_0 / r_b)$ describes the dependence of the bunch density on the radius (transverse profile), r_b is the characteristic transverse dimension, and the function $T(t_0 / t_b)$ describes the longitudinal profile of bunch density, t_b is the characteristic duration of the bunch. The value j_0 is related to the total charge Q by the relation $j_0 = Q / (s_* t_*)$, where s_* , t_* are the effective area of the bunch transverse section and bunch duration.

The resulting electromagnetic field of the electron bunch of the axisymmetric form (16) is found by summing fields of the elementary ring charges (4)

$$E_z(r, \tau) = E_z^{(pol)}(r, \tau) + E_z^{(em)}(r, \tau),$$

$$E_z^{(pol)}(r, \tau) = 2E_0 \frac{\varepsilon_0 - 1}{\varepsilon_0} k_p^2 b^2 \frac{1}{s_* t_*} P_{pol}(r) Z_{pol}(\tau),$$

$$E_0 = 4\pi \frac{Q}{b^2}, \quad P_{pol}(r) = \int_0^b G(k_p r, k_p r_0) R(r_0) r_0 dr_0.$$

$$Z_{pol}(\tau) = \int_{-\infty}^{\tau} T(t_0 / t_b) \cos \omega_g (t_0 - \tau) dt_0,$$

$$E_z^{(em)}(r, \tau) = E_0 \frac{1}{s_* t_*} \sum_{n=1}^{\infty} \mu_n \frac{J_0(\lambda_n r / b)}{J_1^2(\lambda_n)} P_n \left[2\sigma_{chn} Z_{chn}(\tau) + \sigma_{sm} Z_{sm}(\tau) \right], \quad P_n = \int_0^b R(r / r_b) J_0(\lambda_n r / r_b) r dr, \quad (17)$$

$$Z_{chn}(\tau) = \int_{-\infty}^{\tau} T(t_0 / t_b) \cos \omega_{chn} (t_0 - \tau) dt_0,$$

$$Z_{sm}(\tau) = \int_{-\infty}^{\tau} T(t_0 / t_b) \text{sign}(\tau - t_0) e^{-v_{sm}|\tau - t_0|} dt_0.$$

The electric field $E_z^{(pol)}$ describes the potential polarization field excited in the dielectric waveguide by an electron bunch, $E_z^{(em)}$ takes into account the contribution to full field of the electromagnetic (vortex) component of field, which is the sum of the eigen radial harmonics of the dielectric waveguide.

Let us investigate the nature of the distribution of the electric field in the dielectric waveguide, depending on the shape of the bunch. First of all we consider a symmetric bunch in the longitudinal direction, whose density has a maximum at the entry time $t_0 = 0$ and decreases to zero at $t_0 \rightarrow \pm\infty$, $T(t_0) = T(-t_0)$. In the wave zone $\tau \gg t_b$, the wakefield has the form of a set of monochromatic waves

$$E_z(r, \tau) = 2E_0 \frac{1}{s_* t_*} \left[\frac{\varepsilon_0 - 1}{\varepsilon_0} k_p^2 b^2 P_{pol}(r) \hat{T}(\omega_g) \cos \omega_g \tau + \sum_{n=1}^{\infty} \mu_n \frac{J_0(\lambda_n r / b)}{J_1^2(\lambda_n)} \sigma_{chn} P_n \hat{T}(\omega_{chn}) \cos \omega_{chn} \tau \right],$$

where

$$\hat{T}(\omega_\alpha) = 2 \int_0^{\infty} T(t_0 / t_b) \cos \omega_\alpha t_0 dt_0$$

are amplitudes of the Fourier component of the function $T(t_0/t_b)$ on eigen frequencies $\omega_\alpha = \omega_g, \omega_{chn}$.

To determine the field in the volume of the waveguide, including the region of the bunch, it is necessary to specify specific longitudinal and transverse density profiles. As an example, for a bunch with the model profile $T(t_0/t_b) = \exp(-|t_0|/t_b)$ we have

$$Z_{pol}(\tau) = \frac{t_b}{\omega_g^2 t_b^2 + 1} \left[2\chi(\tau) \cos \omega_g \tau - \text{sign} \tau \exp(-|\tau|/t_b) \right],$$

$$Z_{chn}(\tau) = \frac{t_b}{\omega_{chn}^2 t_b^2 + 1} \left[2\chi(\tau) \cos \omega_{chn} \tau - \text{sign} \tau \exp(-|\tau|/t_b) \right],$$

$t_* = 2t_b$. As follows from these expressions the electric field of potential polarization oscillations, as well as the fields of all radial harmonics, have the same structure. Behind a bipolar solitary impulse, a monochromatic wake eigen wave of a dielectric waveguide propagates. Amplitudes of the wakefield waves essentially depend on the duration of the bunch. For a fixed number of particles in the bunch, the amplitudes of wake waves are the larger, the shorter the bunch. If more realistic conditions $\omega_g t_b \gg 1$, $\omega_{chn} t_b \ll 1$ are fulfilled, then the polarization field of the electron bunch excites noncoherence and its level is small. On the other hand, for an electromagnetic wakefield, the electron bunch is a completely coherent source.

The wakefield field (17) consists of a set of electromagnetic eigen modes of the dielectric waveguide. For Gaussian transverse profile model of the laser pulse intensity $R(r/r_b) = \exp(-r^2/r_b^2)$ we obtained $P_n = (r_b^2/4) \exp(-\lambda_n^2 r_b^2/4b^2)$. So, a finite number of radial harmonics are efficiently excited, if $\lambda_n r_b/2b \ll 1$, and $\omega_{chn} t_b \ll 1$.

ЧЕРЕНКОВСКОЕ ИЗЛУЧЕНИЕ ЭЛЕКТРОННОГО СГУСТКА В ДИЭЛЕКТРИЧЕСКОЙ СРЕДЕ С ЧАСТОТНОЙ ДИСПЕРСИЕЙ

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Рассматривается возбуждение черенковского электромагнитного излучения релятивистским электронным пучком в диэлектрическом волноводе с учетом частотной дисперсии диэлектрической проницаемости. Определены электрическая поляризация в изотропной диэлектрической среде и, соответственно, поляризационные заряды и токи, индуцированные кулоновским электрическим полем релятивистского электронного пучка. Получена и исследована пространственно-временная структура возбужденного волнового поля в диэлектрическом волноводе. Показано, что возбужденное поле состоит из электрического поля потенциальной поляризации и множества собственных электромагнитных волн диэлектрического волновода.

ЧЕРЕНКІВСЬКЕ ВИПРОМІНЮВАННЯ ЕЛЕКТРОННОГО ЗГУСТКА В ДІЕЛЕКТРИЧНОМУ СЕРЕДОВИЩІ З ЧАСТОТНОЮ ДИСПЕРСІЄЮ

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Розглядається збудження черенківського електромагнітного випромінювання релятивістським електронним згустком у діелектричному хвилеводі з урахуванням частотної дисперсії діелектричної проникності. Визначено електричну поляризацію в ізотропному діелектричному середовищі і, відповідно, поляризаційні заряди та струми, індуковані кулонівським електричним полем релятивістського електронного пучка. Отримано та досліджено просторово-часову структуру збудженого кильватерного поля в діелектричному хвилеводі. Показано, що збуджене поле складається з потенційного поляризаційного електричного поля та набору власних електромагнітних хвиль діелектричного хвилеводу.

CONCLUSIONS

The process of excitation of the wake Cherenkov radiation by an electron bunch in a dielectric waveguide is investigated. The polarization of the dielectric medium induced by the electric field of the REB is determined. It is shown that the excited electric field consists of a potential field of polarization oscillations and a set of eigen electromagnetic waves of a dielectric waveguide.

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