SLOW WAVE PROPAGATION IN PLASMA WITH NON-UNIFORMITY NOT PERPENDICULAR TO THE MAGNETIC FIELD

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Slow wave propagation in 1D non-uniform plasma with tilted magnetic field with respect of direction of non-uniformity is considered. The second order differential equation describing the slow wave is derived from the Maxwell's equations. The analysis of this equation reveals a singular point for the solutions, which could be associated with the Lower Hybrid Resonance. The condition of the resonance is found to be dependent on the tilting angle. Among two WKB solutions only one is singular. The wave vector behaves as $1/x$ in LHR point for the singular solution. The amplitude diverges only for *x*-component of the electric field. The solution describes propagating wave both to the left and to the right of the LHR point. The analytical solution obtained in the vicinity of the LHR has a special feature of having drop of its amplitude in the LHR point because of residual damping of the wave inside the LHR location. The energy flux also makes drop down there.

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INTRODUCTION

The slow wave (SW) plays an important role in certain scenarios of plasma heating and current drive, and also in wall conditioning discharge sustaining. Its field structure is studied well within one-dimensional model including the zone of lower hybrid resonance (LHR) [1-3]. The LHR phenomenon is a base for the lower hybrid heating and current drive concepts. The mode conversion scenario of the minority heating also includes the LHR mechanism for the wave absorption. In a standard minority heating scenario the LHR appears at the plasma periphery, and its role in wave propagation and power balance is not yet studied sufficiently.

In hot plasma in a LHR zone, the slow wave converts into the ion Bernstein wave. In cases of radio-frequency discharge start-up or a wall conditioning discharge the ions are cold and the wavelength of ion Bernstein wave becomes extremely short. Under such conditions, it is expedient to treat LHR without account of wave conversion.

The previous theoretical considerations it was assumed that plasma gradients are oriented perpendicular to the steady magnetic field. This is almost true for fusion machines because the plasma density is approximately constant at the magnetic surface. However, the magnetic field module has some variations, and the plasma dielectric tensor follows them. For such reason it is of interest to consider a case when the magnetic field is not perpendicular to plasma gradients.

In this paper, a 1D non-uniform plasma with a tilted magnetic field is considered. The second order differential equation describing the slow wave is derived from the Maxwell's equations. The analysis of this equation reveals a singular point for the solutions. However, the point located aside of the lower hybrid resonance found using earlier theoretical results. The solutions obtained are also different. These solutions and location of the singular point are discussed in this paper.

SLOW WAVE EQUATION AT LHR VICINITY

The problem is considered in slab geometry with non-uniformity of plasma along the *x* coordinate. Time-harmonic Maxwells equations read:

$$\nabla\times\nabla\times\mathbf{E}-k_0^2\mathbf{D}=0\,. \quad (1)$$

The electric displacement field in cold plasma is

$$\mathbf{D}=\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{E}=\varepsilon_\perp\mathbf{E}-ig\mathbf{h}\times\mathbf{E}+(\varepsilon_\parallel-\varepsilon_\perp)\mathbf{hh}\cdot\mathbf{E}\,. \quad (2)$$

Where **h** is the unitary vector along the magnetic field:

$$\mathbf{h}=(\sin\alpha,0,\cos\alpha)\,. \quad (3)$$

Uniformity of plasma and magnetic field in *y* and *z* directions allows one to represent the electric field through Fourier harmonics

$$\mathbf{E}(x,y,z)=\mathbf{E}(x)\exp(ik_y y+ik_z z)\,. \quad (4)$$

Closeness to the LHR means that $\dfrac{d}{dx}\gg k_y,\,k_z$. Using this and formulas (1-4), one can obtain the following equation for the slow wave:

$$\frac{d}{dx}a\frac{d}{dx}E_z+b\frac{d}{dx}E_z+cE_z=0\,. \quad (5)$$

Here

$$a=k_0^2\varepsilon^*/d_0\,,$$
$$b=2ik_z\sin\alpha\cos\alpha k_0^2(\varepsilon_\parallel-\varepsilon_\perp)/d_0\,, \quad (6)$$
$$c=\sin^2\alpha\cos^2\alpha k_0^4(\varepsilon_\parallel-\varepsilon_\perp)^2/d_0-k_0^2(\varepsilon_\perp\sin^2\alpha+\varepsilon_\parallel\cos^2\alpha)+\frac{ik_z}{d_0}\frac{d}{dx}[k_0^2\sin\alpha\cos\alpha(\varepsilon_\parallel-\varepsilon_\perp)],$$

and

$$d_0=k_z^2-k_0^2\varepsilon^*\,,$$
$$\varepsilon^*=\varepsilon_\perp\cos^2\alpha+\varepsilon_\parallel\sin^2\alpha\,,$$
$$k_0=\omega/c.$$

WKB ANALYSIS

In the WKB analysis $\frac{d}{dx} = ik_x$, and one can obtain the dispersion equation for the equation (5). As it is expected, it could be written in the form standard for the slow wave:

$$k_\perp^2 = -\frac{\varepsilon_\parallel}{\varepsilon_\perp}(k_\parallel^2 - k_0^2\varepsilon_\perp). \quad (7)$$

In the particular case under consideration

$$k_\perp^2 = k_x^2 \cos^2\alpha + k_y^2 + k_z^2 \sin^2\alpha, \quad (8)$$

$$k_\parallel^2 = k_x^2 \sin^2\alpha + k_z^2 \cos^2\alpha. \quad (9)$$

In our problem only k_x is allowed to vary, and if the LHR resonance point is reached, then $k_x^2 \to \infty$. In this case both $k_\perp$ and $k_\parallel$ diverge. The dispersion equation permits this if

$$k_\perp^2 / k_\parallel^2 = -\varepsilon_\parallel / \varepsilon_\perp. \quad (10)$$

When $k_x^2 \to \infty$, from formulas (8), (9) one can obtain

$$k_\perp^2 / k_\parallel^2 = \tan^{-2}\alpha \quad (11)$$

and with account of this, the condition (10) could be written as

$$\varepsilon^* = 0, \quad (12)$$

if ε^* is real. If it is complex, the condition is

$$\mathrm{Re}\,\varepsilon^* = 0. \quad (13)$$

It is interesting to note that

$$\varepsilon^* = \mathbf{e}_x \cdot \hat{\boldsymbol{\varepsilon}} \cdot \mathbf{e}_x. \quad (14)$$

This indicates that the LHR point occurs when the diagonal component of the dielectric tensor in the direction of plasma non-uniformity nullifies.

The singular solution has the wave vector component

$$k_x = ib/a \quad (15)$$

and the regular one has

$$k_x = ic/b. \quad (16)$$

Note here that the quantity a nullifies if the LHR condition (12) is met.

In WKB approximation the singular solution is

$$E_z = \exp\left(\int \frac{b'-c}{b-a'}dx\right)\exp\left(\int ik_x dx\right). \quad (17)$$

Here the derivative over x is denoted by prime. The phase of the solution logarithmically increases on approach to the singular point. The amplitude remains regular. The E_x component of the field

$$E_x = [ik_z \frac{d}{dx}E_z - k_0^2 \sin\alpha\cos\alpha(\varepsilon_\parallel - \varepsilon_\perp)E_z]/d_0 \quad (18)$$

has the singularity both in the phase and amplitude.

ANALYTICAL SOLUTIONS OF SLOW WAVE EQUATION AT LHR VICINITY

Near the LHR point two solutions of equation (5) could be found analytically using smallness of the coefficient before the leading derivative, $a<<bL$, where L is the characteristic spatial scale of the non-uniformity of plasma. The approximate solution of the differential equation (5) can be obtained neglecting its last term. The solution reads:

$$E_z = \int \exp\left(-\int b\frac{dx}{a}\right)\frac{dx}{a}. \quad (19)$$

Keeping lowest terms in Taylor series, $a \approx a_1' x$ and $b \approx b_0$ the integrals above can be taken using analytical continuation around point x=0.

$$E_z = E_0 \begin{cases} \exp(ib_0/|a_1|\pi - b_0/a_1 \ln x), & for\ x>0 \\ \exp[-b_0/a_1 \ln(-x)], & for\ x<0. \end{cases} \quad (20)$$

The solutions (20) fit well to the WKB solution (17) even in the vicinity of the LHR point since the first exponent in (17) does not vary rapidly there.

There is a drop in the amplitude of solutions (20). The tunneling factor is

$$S = \exp(-|\mathrm{Im} b_0|/|a_1|\pi). \quad (21)$$

The drop in amplitude indicates the residual damping of the wave in the LHR point. The x component of the Pointing vector is

$$\Pi_x^- = \frac{3c^2}{16\pi\omega}\mathrm{Im} b_0 |E_0|^2 \quad (22)$$

for negative x, and for positive x the energy flux density is smaller by the square of the tunneling factor

$$\Pi_x^+ = S^2 \Pi_x^-. \quad (23)$$

This is so if $\mathrm{Im} b_0 > 0$. In the opposite case, the picture reverses.

Note here that the tunneling factor decreases with $|k_z|$, $\sin 2\alpha$ and the non-uniformity space scale.

CONCLUSIONS

Slow wave propagation in 1D non-uniform plasma with tilted magnetic field with respect of direction of non-uniformity is considered. The second order differential equation describing the slow wave is derived from the Maxwell's equations. The analysis of this equation reveals a singular point for the solutions, which could be associated with the Lower Hybrid Resonance. The condition of the resonance can be written as $\mathrm{Re}(\mathbf{e}_x \cdot \hat{\boldsymbol{\varepsilon}} \cdot \mathbf{e}_x) = 0$ ($\mathbf{e}_x$ here is the direction of plasma non-uniformity). This condition gives the conventional LHR condition when the non-uniformity direction is perpendicular to the magnetic field. When the magnetic field is tilted, the condition reveals the dependence on tilting angle.

Among two WKB solutions only one is singular. The wave vector behaves as $1/x$ in LHR point for the singular solution. The amplitude diverges only for x-component of the electric field. The solution describes propagating wave both to the left and to the right of the LHR point.

The analytical solution obtained in the vicinity of the LHR is written in terms of the exponential functions and fits well to the WKB solution. The special feature of it is dropping of its amplitude in the LHR point because of residual damping of the wave inside the LHR location. The energy flux also makes droping there.

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РАСПРОСТРАНЕНИЕ МЕДЛЕННЫХ ВОЛН В ПЛАЗМЕ С НЕОДНОРОДНОСТЬЮ, НЕ ПЕРПЕНДИКУЛЯРНОЙ МАГНИТНОМУ ПОЛЮ

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Рассмотрено распространение медленных волн в одномерной неоднородной плазме с наклонным магнитным полем относительно направления неоднородности. Дифференциальное уравнение второго порядка, описывающее медленную волну, выводится из уравнений Максвелла. Анализ этого уравнения выявляет особую точку для решений, которая может быть связана с нижним гибридным резонансом (LHR). Обнаружено, что условие резонанса зависит от угла наклона. Среди двух решений ВКБ только одно является сингулярным. Волновой вектор ведет себя как $1/x$ в точке LHR для сингулярного решения. Амплитуда расходится только для х-составляющей электрического поля. Решение описывает бегущую волну как слева, так и справа от точки LHR. Аналитическое решение, полученное в окрестности LHR, имеет особенность, заключающуюся в падении его амплитуды в точке LHR из-за остаточного затухания волны внутри местоположения LHR. Поток энергии также падает в этой зоне.

ПОШИРЕННЯ ПОВІЛЬНИХ ХВИЛЬ У ПЛАЗМІ З НЕОДНОРІДНІСТЮ, НЕ ПЕРПЕНДИКУЛЯРНОЮ ДО МАГНІТНОГО ПОЛЯ

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Розглянуто поширення повільних хвиль в одновимірній неоднорідній плазмі з похилим магнітним полем щодо направлення неоднорідності. Диференціальне рівняння другого порядку, що описує повільну хвилю, виводиться з рівнянь Максвелла. Аналіз цього рівняння виявляє особливу точку для рішень, яка може бути пов'язана з нижнім гібридним резонансом (LHR). Виявлено, що умова резонансу залежить від кута нахилу. Серед двох рішень ВКБ тільки одне є сингулярним. Хвильовий вектор поводиться як $1/x$ в точці LHR для сингулярного рішення. Амплітуда розходиться тільки для х-компоненти електричного поля. Рішення описує хвилю, що біжить як зліва, так і праворуч від точки LHR. Аналітичне рішення, отримане в околиці LHR, має особливість, яка полягає в падінні його амплітуди в точці LHR за рахунок залишкового загасання хвилі усередині розташування LHR. Потік енергії також падає в цій зоні.