ON GENERATION OF A WAVE PACKAGE IN A WAVEGUIDE, FILLED ACTIVE MEDIUM

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Excitation of electromagnetic waves in a waveguide with a medium, which is a two-level system, is considered. To describe the processes, both classical electrodynamics methods and quantum mechanics methods are used. The nature of the processes under study turns out to depend on the relationship between the Rabi frequency and the line width of the excited wave packet. It is shown that if the field energy density is high, then spatially inhomogeneous Rabi frequencies arise, which leads to oscillatory behavior of the wave field amplitudes. If the levels of the excited field are small, then the dynamics of the two-level quantum system becomes monotonic and the population inversion tends to zero.

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INTRODUCTION

Below we consider the excitation of electromagnetic waves in bounded systems that impose a choice of wavelengths on the modes of the wave packet and form the spatial structure of the field in the waveguides. As a result, a field structure is formed, which is a standing wave.

Let the waveguide be filled with a medium, which is a two-level system of dipoles. To describe such a system, we use the semiclassical model of the interaction of the field and particles (see, for example, [1]). In this case, we describe the medium by quantum mechanic, and the field in the classical representation. The important role is played here by the Rabi frequency, which determines the probabilities of induced radiation or absorption of field quanta [2, 3]. In this case, the population inversion is capable of rapidly oscillating changes (nutations). The inversion change interval is determined by the choice of initial conditions, i.e., the inversion at the initial moment can be quite small or approach its maximum value equal to the number of states at the upper level.

The nature of the process depends on the relationship between the values of the Rabi frequency $\Omega = |d_{ab}||E(t)|/\hbar$ (where d_{ab}, E – the dipole moment of the particle and the amplitude of the electric field, respectively) and the line width of the wave packet γ_{12}. Since the population inversion $\mu = \rho_a - \rho_b$ (where ρ_a, ρ_b are the number of particles at the upper and lower energy levels, respectively) due to the requirement of energy conservation is related to the number of field quanta $N = |E(t)|^2/4\pi\hbar\omega \approx \mu = \rho_a - \rho_b$, with a significant population inversion or in strong fields, the line width can be neglected $\gamma_{12} < \Omega = |d_{ab}||E(t)|/\hbar$. Obviously, the field energy density is high $|E(t)|^2/4\pi >> \gamma_{12}^2\hbar^2/|d_{ab}|^2$ (for population inversion, this condition will change $\mu >> \gamma_{12}^2 \cdot \hbar/(\omega|d_{ab}|^2)$). In this mode, noticeable nutations of population inversions with different frequencies along the waveguide length corresponding to local Rabi frequencies whose interference determines the behavior of the wave field can be expected (see, for example, [4]).

Under the condition $\gamma_{12} > \Omega$ that corresponds to low levels of electric field intensity or small values of population inversion, the behavior of a two-level system, first formulated in [5], becomes monotonic, the population inversion tends to zero, and the characteristic time of the field change increases $\tau_\gamma \approx \gamma_{12}/\Omega^2$.

The aim of the work is to determine the efficiency of oscillation generation depending on the line width and the level of energy loss to radiation.

1. MODEL OF AN ACTIVE WAVEGUIDE

We consider a one-dimensional model for perturbations of the electric field, polarization, and population inversion slowly varying with time, describing the excitation of electromagnetic waves in a two-level active medium, the equations of which can be represented in the form (see, for example, [1])

$$\frac{\partial^2 E}{\partial t^2} + \delta \frac{\partial E}{\partial t} - c^2 \frac{\partial^2 E}{\partial x^2} = -4\pi \frac{\partial^2 P}{\partial t^2}, \qquad (1)$$

$$\frac{\partial^2 P}{\partial t^2} + \gamma_{12}\frac{\partial P}{\partial t} + \omega^2 \cdot P = -\frac{2\omega|d_{ab}|^2}{\hbar}\mu E, \qquad (2)$$

$$\frac{\partial \mu}{\partial t} = \frac{2}{\hbar\omega} < E\frac{\partial P}{\partial t} >, \qquad (3)$$

where the transition frequency ω between the levels corresponds to the field frequency. We neglect the relaxation of the inversion due to external reasons.

The decrement of field absorption in the medium is δ_D.The matrix element of the dipole moment (more precisely, its projection onto the direction of the electric field) is d_{ab}. The population difference $\mu = n \cdot (\rho_a - \rho_b)$ per unit volume, and ρ_a, ρ_b the relative level populations in the absence of a field, γ_{12} – the width of the spectral line, n is the density of the dipoles of the active medium. Here, the line width is inversely proportional to the lifetime of states, which is due to relaxation processes. Fields are represented as

$$E = [E(t)\cdot\exp\{-i\omega t\} + E^*(t)\cdot\exp\{i\omega t\}],$$

$$P = [P(t)\cdot\exp\{-i\omega t\} + P^*(t)\cdot\exp\{i\omega t\}].$$

Wherein $< E^2 >= 2|E(t)|^2$. The number of field quanta is then equal $< E^2 > /4\pi\hbar\omega = 2|E|^2/4\pi\hbar\omega = N$.

For slowly varying amplitudes, use the equations

$$\frac{\partial E(t)}{\partial t} + \delta_D \cdot E(t) = 2i\pi\omega P(t), \qquad (4)$$

$$\frac{\partial P(t)}{\partial t}+\gamma_{12}P(t)=\frac{|d_{ab}|^2}{i\hbar}\mu E, \quad (5)$$

$$\frac{\partial\mu}{\partial t}=\frac{2i}{4\hbar}[E(t)P^*(t)-E^*(t)P(t)], \quad (6)$$

we use the notation $\mu/\mu_0=M$, and immediately note that changes in the population inversion will be determined by the choice μ_0 and the initial conditions, $\Omega_0=|d_{ab}|\cdot|E_0|/\hbar=|d_{ab}|\cdot[4\pi\omega\cdot\mu_0/\hbar]^{1/2}$ is the Rabi frequency corresponding to the electric field amplitude, $\mathrm{E}=\frac{E(t)}{[4\pi\hbar\omega\mu_0]^{1/2}}$, $\mathrm{P}=\frac{P(t)}{[4\pi\hbar\omega\mu_0]^{1/2}}\cdot\frac{4\pi\omega}{\Omega_0}$, $\tau=\Omega_0 t$, $\Gamma_{12}=\gamma_{12}/\Omega_0$, $\Theta=\delta_D/\Omega_0$ and rewrite equations (4) - (6) in complex form

$$\frac{\partial\mathrm{E}}{\partial\tau}+\Theta\mathrm{E}=\frac{i}{2}\mathrm{P}, \quad (7)$$

$$\frac{\partial\mathrm{P}}{\partial\tau}+\Gamma_{12}\mathrm{P}=-iM\mathrm{E}, \quad (8)$$

$$\frac{\partial\mathrm{M}}{\partial\tau}=-2i\cdot[\mathrm{E}^*\mathrm{P}-\mathrm{E}\mathrm{P}^*]. \quad (9)$$

In the case of a large absorption level, the system of the equations can be written as

$$\frac{\partial\mathrm{M}}{\partial\tau}=-2\Theta|E|^2, \quad (10)$$

$$[\frac{\partial}{\partial\tau}+\Gamma_{12}]\frac{\partial\mathrm{M}}{\partial\tau}=-\mathrm{M}|E|^2. \quad (11)$$

Here in expressions (10) and (11), a representation $\mathrm{M}=(M+M^*)/2$ is used where it is taken into account that the right-hand side of (10) is always a real value.

The generation cases described by equations (10) - (12), when the natural line width is much less than the Rabi frequency $\Gamma_{12}<<1$, under the conditions of the formation of a standing electromagnetic wave due to reflections from its boundaries, were considered in [4, 6]. If the condition $\Gamma_{12}>>1$ is satisfied, the oscillatory character (nutation) of the population inversion and, accordingly, the phase changes of the population inversion can be neglected and equations (7) - (9) should be written in the form

$$\frac{\partial\mathrm{N}}{\partial\tau}+2\Theta\mathrm{N}=\frac{\mathrm{N}\cdot M}{\Gamma_{12}}, \quad (12)$$

$$\frac{\partial M}{\partial\tau}=-2\frac{\mathrm{N}\cdot M}{\Gamma_{12}}. \quad (13)$$

Note that the transition from equation (11) to equation (12) is similar to the transition to the case of a noticeable spectral field width, when the spectral line width is greater than the inverse characteristic time of the change in the amplitude of the perturbations [7 - 11]. In this case, the line width noticeably exceeds the Rabi Ω frequency, while the characteristic time of the field change is proportional $\tau_\gamma\approx\gamma_{12}/\Omega^2$.

In dimensionless variables, this condition can be written as $\Gamma_{12}>>\partial\mathrm{P}/\mathrm{P}\cdot\partial\tau$, which corresponds to a strong inequality $\gamma_{12}>>\Omega=|d_{ab}||E(t)|/\hbar$. In other words, the line width noticeably exceeds the Rabi frequency, which is possible at sufficiently low levels of the electric field intensity or small values of the population inversion $\gamma_{12}>>[4\pi\omega]^{1/2}|d_{ab}|\mu/\hbar^{1/2}$.

It is not difficult to see that this is a somewhat simplified version of the balanced (speed) equations of a two-level system in the presence of an electric field, first formulated in [5].

To solve the problem of field interaction in a limited system (waveguide), one should use the local character of population inversion and, accordingly, polarization, determining these quantities in separate spatial sectors. The electromagnetic field in this case can be represented in the waveguide in the form of a standing wave, which is due to the partial reflection of the field from the boundaries of the system. In the case of radiation from a waveguide, when $\delta_D\neq 0$, one should choose the field dependence in each of the spatial sectors $1<j<S$ in the form of a relative number of quanta

$$|\mathrm{E}_j(\tau=0)|^2=2\frac{1}{S}\cdot|\mathrm{E}(\tau=0)|^2\cdot\mathrm{Sin}^2\{2\pi\frac{j}{S}+\alpha\}, \quad (14)$$

where α is the almost constant phase associated with δ_D. Obviously, $\sum_j 2\frac{1}{S}\mathrm{Sin}^2\{2\pi\frac{j}{S}+\alpha\}=m$ in the case considered below, when a countable number of waves fit along the waveguide $b=m\lambda$. The total (relative) number of field quanta can be written as

$$N(\tau)=2\sum_{j=1}^{S}|\mathrm{E}_j(\tau)|^2=2|\mathrm{E}(\tau)|^2. \quad (15)$$

Generally speaking, $\delta_D\approx\frac{cE^2(x=0)}{4\pi}/(\frac{<|E|^2>}{4\pi}b)$, where $b=m\lambda$ is the waveguide length. It is easy to see that $|\mathrm{E}|^2(x=0)=2<|\mathrm{E}|^2>\mathrm{Sin}^2\alpha$ and $\delta_D\approx c(\mathrm{Sin}^2\alpha)/2b$, where c is the group velocity of the wave, outside the waveguide.

The system of equations (7) - (9) in this case is transformed as follows. For local variables and, the equations

$$\frac{\partial\mathrm{P}_j}{\partial\tau}+\Gamma_{12}\mathrm{P}_j=-iM_j\mathrm{E}_j, \quad (16)$$

$$\frac{\partial M_j}{\partial\tau}=2i[\mathrm{E}_j\mathrm{P}_j^*-\mathrm{E}_j^*\mathrm{P}_j], \quad (17)$$

where $\mathrm{E}_j(\tau)=(\sqrt{\frac{2}{S}})\cdot|\mathrm{E}(\tau)|\cdot\mathrm{Sin}\{2\pi\frac{j}{S}+\alpha\}$, and $\frac{1}{2}\sum_{j=1}^{S}(M_j+M_j^*)=\mathrm{M}$.

For the number of field quanta, one can write the equation (conservation law, a consequence of equations (7) and (9))

$$\frac{\partial\mathrm{M}}{2\partial\tau}+\frac{\partial\mathrm{N}}{\partial\tau}+2\Theta\mathrm{N}=0. \quad (18)$$

The selected time scales and attenuation values in this description are related to the scales of a simple model [4] as follows $\tau\to\tau\sqrt{S}$, $\Theta\to 2\delta$. Upon transition to a new scale, all the results of this work and work [4] under neglected line width ($\Gamma_{12}=0$) are completely identical.

2. THE RESULTS OF NUMERICAL MODELING

We consider the solutions of system (16) - (18) using the following notation

$$N(\tau)=\frac{1}{S}\sum N_j(\tau)\,,\ M(\tau)=\frac{1}{S}\sum M_j(\tau)\,,$$

$$\mathrm{E}_j(\tau)=\sqrt{2N(\tau)}\cdot\mathrm{Sin}\left(2\pi\frac{j}{S}\right).$$

If $N(0) = 0.001$ $M(0) = 1$ $S = 100$, $\Gamma_{12}=0$, $\Theta=0$ the calculation results are identical [4] under conditions at the corresponding scales indicated above (Figs. 1 and 2).

An increase in the line width leads to smoothing of the field oscillations and the average population inversion in the volume of the waveguide (see Figs. 1 and 2).

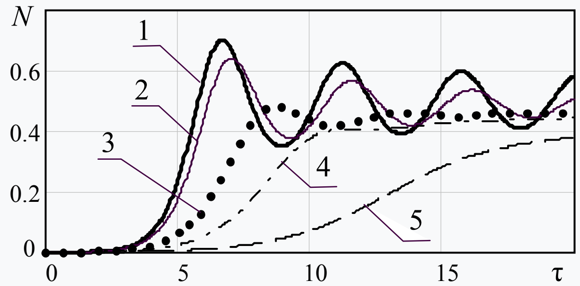


Fig. 1. Behavior of field intensity in time for values: 1 - Γ_{12}=0; 2 - Γ_{12}=0.1; 3 - Γ_{12}=0.5; 4 - Γ_{12}=0.9; 5 - Γ_{12}=1.9 in the absence of energy output (Θ=0)

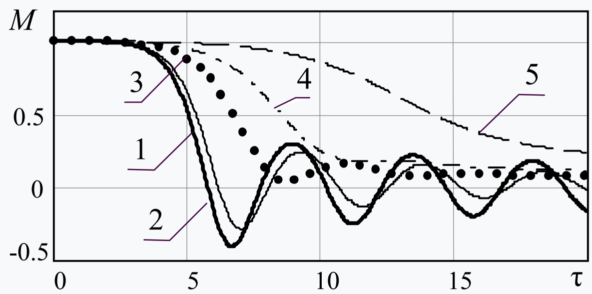


Fig. 2. The behavior of the average inversion versus time for values: 1 - Γ_{12}=0; 2 - Γ_{12}=0.1; 3 - Γ_{12}=0.5; 4 - Γ_{12}=0.9; 5 - Γ_{12}=1.9 in the absence of energy output (Θ=0)

In all cases, the inversion with time in an oscillatory manner or monotonically (with a large line width) tends to zero. The distribution of M along the length of the waveguide (by sectors j) at the moments of the first maximum and the first minimum N have the form (Figs. 3 - 5):

Γ_{12}=0.1

N_{max}=0.6424, τ=7.0 N_{min}=0.3794, τ=9.24

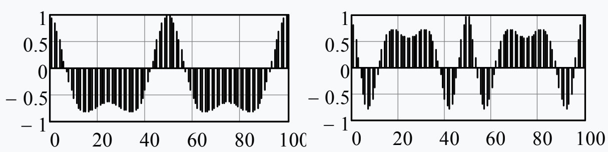


Fig. 3. The distribution of M by sectors at the moments of the first maximum and the first minimum N for line widths Γ_{12}=0.1 in the absence of energy output (Θ=0)

Γ_{12}=0.5

N_{max}=0.481, τ=8.72 N_{min}=0.4192, τ=10.88

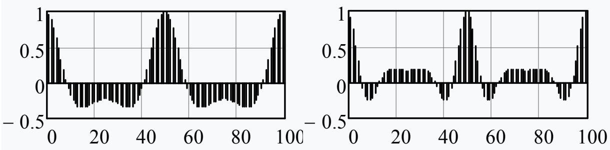


Fig. 4. The distribution of M by sectors at the moments of the first maximum and the first minimum N with line widths Γ_{12}=0.5 in the absence of energy output (Θ=0)

Γ_{12}=0.9

N_{max}=0.4096, τ=11.66 N_{min}=0.4093, τ=12.2

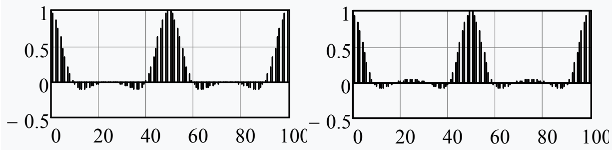


Fig. 5. The distribution of M by sectors at the moments of the first maximum and the first minimum N with line widths Γ_{12}=0.9 in the absence of energy output (Θ=0)

Let us imagine the time dependence of N for different Θ and also consider the change in the linear increment of the process $dN/Nd\tau$, the maximum relative field intensity N_{MAX}, and the maximum energy flux from the system $\Theta\cdot N_{MAX}$ as functions of Θ, which are responsible for the energy output from the waveguide (Figs. 6 - 8).

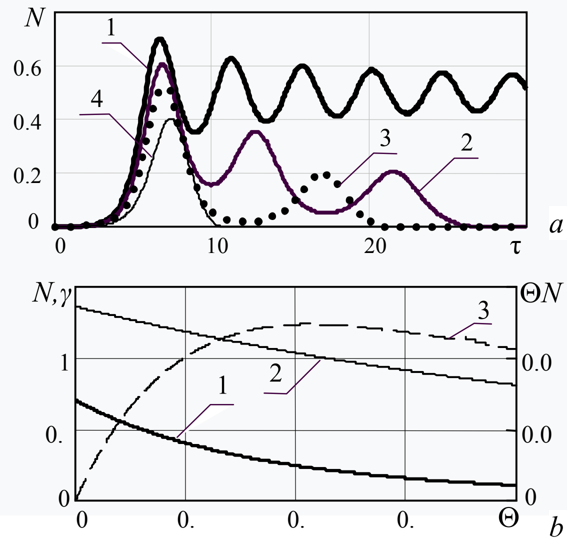
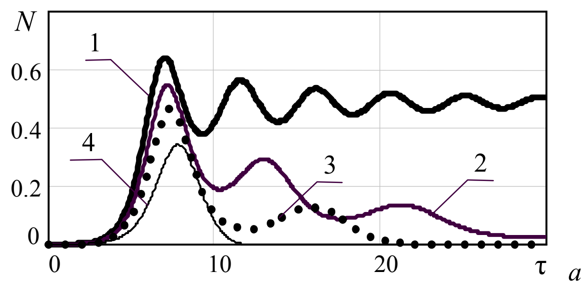


Fig. 6. Dependence of N on time for different Θ. 1 – Θ = 0; 2 – Θ = 0.05; 3 – Θ = 0.1; 4 – Θ = 0.2 (a); the linear increment of the process γ = dN/Ndτ, the maximum attainable field intensity N_{MAX}, as well as the maximum energy flux from the system $\Theta\cdot N_{MAX}$, as a function of Θ (b). 1 – N_{max}; 2 – γ_{max}; 3 – ΘN_{max}. In all cases Γ_{12}=0



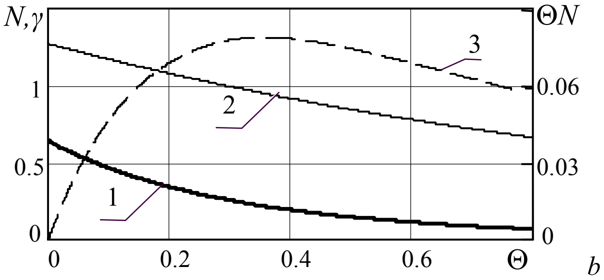


Fig. 7. Dependence of N on time for different Θ. 1 – Θ = 0; 2 – Θ = 0.05; 3 – Θ = 0.1; 4 – Θ = 0.2 (a); the linear increment of the process $\gamma = dN / Nd\tau$, the maximum attainable field intensity N_{MAX}, as well as the maximum energy flux from the system $\Theta \cdot N_{MAX}$ as a function of Θ (b). 1 – N_{max}; 2 – γ_{max}; 3 – Θ N_{max}. In all cases Γ_{12}=0.1

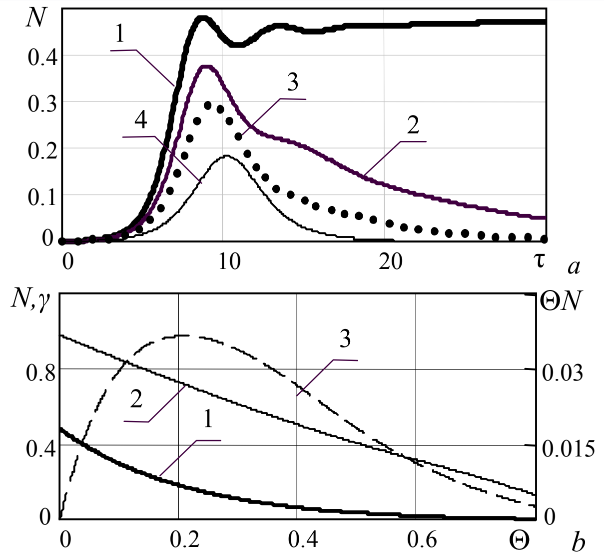


Fig. 8. Dependence of N on time for different Θ. 1 – Θ = 0; 2 – Θ = 0.05; 3 – Θ = 0.1; 4 – Θ = 0.2 (a); the linear increment of the process $\gamma = dN / Nd\tau$, the maximum attainable field intensity N_{MAX}, as well as the maximum energy flux from the system $\Theta \cdot N_{MAX}$ as a function of Θ (b). 1 – N_{max}; 2 – γ_{max}; 3 – Θ N_{max}. In all cases Γ_{12}=0.5

A feature of the solutions of system (16) - (18) for large values of Θ, when the energy is noticeably removed from the system, is the presence of only one field maximum. Apparently, this is due to the rapid decrease in the level of inversion in the waveguide [12].

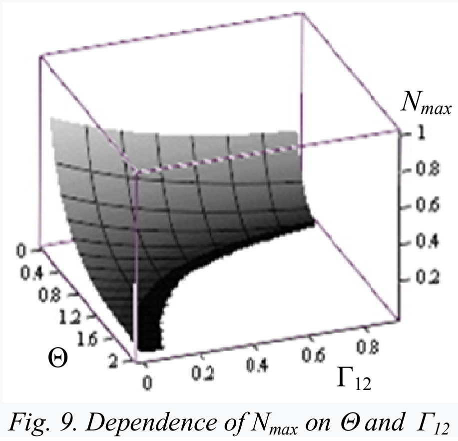


Fig. 9. Dependence of N_{max} on Θ and Γ_{12}

In Fig. 9 a diagram of the dependence of the maximum field amplitude as a function of two variables Θ and Γ_{12} is presented in Fig. 9.

CONCLUSIONS

The paper shows of the process is the excitation of electromagnetic waves in a waveguide, filled with two-level system of dipoles. The radiation generationdepends on the relationship between the values of the Rabi frequency $\Omega = |d_{ab}||E(t)|/\hbar$ (where d_{ab} and E are the dipole moment of the particle and the amplitude of the electric field, respectively) and the line width of the wave packet γ_{12}. With a significant population inversion $\mu >> \gamma_{12}^2 \cdot \hbar / (\omega |d_{ab}|^2)$ or in strong fields $\Omega = |d_{ab}||E(t)|/\hbar >> \gamma_{12}$, the line width can be neglected.

Obviously, in the latter case the field energy density is high $|E(t)|^2 / 4\pi >> \gamma_{12}^2 \hbar^2 / |d_{ab}|^2$ (for population inversion, this condition will change $\mu >> \gamma_{12}^2 \cdot \hbar / (\omega |d_{ab}|^2)$). In this mode, noticeable nutations of population inversions with different frequencies along the waveguide length (corresponding to local Rabi frequencies) are observed.

While we can observe the interference of which determines the oscillatory behavior of the wave field amplitude. Under the condition that corresponds to low levels of electric field intensity $|E(t)|^2 / 4\pi < \gamma_{12}^2 \hbar^2 / |d_{ab}|^2$ or small values of population inversion $\mu < \gamma_{12}^2 \cdot \hbar / (\omega |d_{ab}|^2)$, the behavior of a two-level system, as in [5], becomes monotonic, the population inversion tends to zero, and the characteristic time of the field change increases $\tau_\gamma \approx \gamma_{12} / \Omega^2$.

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О ГЕНЕРАЦИИ ВОЛНОВОГО ПАКЕТА В ВОЛНОВОДЕ С АКТИВНОЙ СРЕДОЙ

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Рассмотрено возбуждение электромагнитных волн в волноводе, который заполнен средой, представляющей собой двухуровневую систему. Для описания процессов используются как методы классической электродинамики, так и методы квантовой механики. Характер изучаемых процессов оказывается зависящим от соотношения между частотой Раби и шириной линии возбуждаемого волнового пакета. Показано, что если плотность энергии поля велика, то возникают пространственно-неоднородные частоты Раби, что приводит к осцилляторному поведению амплитуд поля волны. Если уровни возбуждаемого поля малы, то динамика двухуровневой квантовой системы становится монотонной, и инверсия населенностей стремится к нулю.

ПРО ГЕНЕРАЦІЮ ХВИЛЬОВОГО ПАКЕТУ У ХВИЛЬОВОДІ З АКТИВНИМ СЕРЕДОВИЩЕМ

В.В. Костенко, В.М. Куклін, Є.В. Поклонський

Розглянуто збудження електромагнітних хвиль у хвилеводі з середовищем, що представляє собою дворівневу систему. Для опису процесів використовуються як методи класичної електродинаміки, так і методи квантової механіки. Характер досліджуваних процесів є залежним від співвідношення між частотою Раби і шириною лінії збуджуваного хвильового пакета. Показано, що якщо густина енергії поля велика, то виникають просторово-неоднорідні частоти Раби, що призводить до осциляторної поведінки амплітуд поля хвилі. Якщо рівні збуджуваного поля малі, то динаміка дворівневої квантової системи стає монотонною, і інверсія населеностей прямує до нуля.