RESEARCH ARTICLE

Semisymmetric Z_p-covers of the $C20$ graph

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ABSTRACT. A graph X is said to be G-semisymmetric if it is regular and there exists a subgroup G of $A := \mathrm{Aut}(X)$ acting transitively on its edge set but not on its vertex set. In the case of $G = A$, we call X a semisymmetric graph. Finding elementary abelian covering projections can be grasped combinatorially via a linear representation of automorphisms acting on the first homology group of the graph. The method essentially reduces to finding invariant subspaces of matrix groups over prime fields. In this study, by applying concept linear algebra, we classify the connected semisymmetric z_p-covers of the $C20$ graph.

Introduction

In this study, all graphs considered are assumed to be undirected, finite, simple, and connected, unless stated otherwise. For a graph X, $V(X)$, $E(X)$, $Arc(X)$, and $\mathrm{Aut}(X)$ denote its vertex set, edge set, arc set, and full automorphism group, respectively. Let G be a subgroup of $\mathrm{Aut}(X)$. For $u, v \in V(X)$, uv denotes the edge incident to u and v in X, and $N_X(u)$ denotes the neighborhood of u in X, that is, the set of vertices adjacent to u in X.

A graph $\widetilde{X}$ is called a covering of a graph X with projection $p \colon \widetilde{X} \to X$ if there is a surjection $p \colon V(\widetilde{X}) \to V(X)$ such that $p|_{N_{\widetilde{X}}(\widetilde{v})} \colon N_{\widetilde{X}}(\widetilde{v}) \to N_X(v)$ is a bijection for any vertex $v \in V(X)$ and $\widetilde{v} \in p^{-1}(v)$.

A permutation group G on a set Ω is said to be semiregular if the stabilizer G_v of v in G is trivial for each $v \in \Omega$, and is regular if G is transitive, and semiregular.

Let K be a subgroup of $\mathrm{Aut}(X)$ such that K is intransitive on $V(X)$. The quotient graph X/K induced by K is defined as the graph such that the set Ω of K-orbits in $V(X)$ is the vertex set of X/K and $B, C \in \Omega$ are adjacent if and only if there exist a $u \in B$ and $v \in C$ such that $\{u, v\} \in E(X)$.

A covering $\widetilde{X}$ of X with a projection p is said to be regular (or N-covering) if there is a semiregular subgroup N of the automorphism group $\mathrm{Aut}(\widetilde{X})$ such that graph X is isomorphic to the quotient graph $\widetilde{X}/N$, say by h, and the quotient map $\widetilde{X} \to \widetilde{X}/N$ is the composition ph of p and h (in this paper, all functions are composed from left to right). If N is a cyclic or an elementary Abelian, then, $\widetilde{X}$ is called a cyclic or an elementary Abelian covering of X, and if $\widetilde{X}$ is connected, N becomes the covering transformation group.

An s-arc in a graph X is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \leqslant i \leqslant s$, and $v_{i-1} \neq v_{i+1}$ for $1 \leqslant i < s$; in other words, a directed walk of length s that never includes a backtracking. For a graph X and a subgroup G of $\mathrm{Aut}(X)$, X is said to be G-vertex-transitive, G-edge-transitive, or G-s-arc-transitive if G is transitive on the sets of vertices, edges, or s-arcs of X, respectively, and G-s-regular if G acts regularly on the set of s-arcs of X. Similarly, a regular graph is G-semisymmetric if it is G-edge-transitive but not G-vertex-transitive. A graph X is said to be vertex-transitive, edge-transitive, s-arc-transitive, or s-regular if X is $\mathrm{Aut}(X)$-vertex-transitive, $\mathrm{Aut}(X)$-edge-transitive, $\mathrm{Aut}(X)$-s-arc-transitive, or $\mathrm{Aut}(X)$-s-regular, respectively. In particular, 1-arc-transitive means arc-transitive or symmetric. It can be shown that a G-edge-transitive but not G-vertex-transitive graph is necessarily bipartite, where the two partite parts of the graph are orbits of G. Moreover, if X is regular these two partite sets have equal cardinality.

Covering techniques have long been known as a powerful tool in topology, and graph theory. Regular covering of a graph is currently an active topic in algebraic graph theory. The class of semisymmetric graphs was first introduced by Folkman [7], where several infinite families of such graphs were constructed, and eight open problems were posed that spurred the interest in this topic, Subsequently, Bouwer [2,3], Klin [12], Iofinova and Ivanov [10], Ivanov [11], Du and Xu [5], and others did significant work on semisymmetric graphs. They presented new constructions of such graphs by combinatorial, and group-theoretical methods. The answers

to most of Folkman's open problems are now known. A census of all semisymmetric cubic graphs up to 768 vertices has been obtained by Conder et al. [4].

A good result on the automorphism groups of cubic semisymmetric graphs of twice odd order was presented by Parker [20]. Marušič [19] constructed the first infinite family of cubic semisymmetric graphs as one of the first applications of covering techniques.

Note that a semisymmetric graph cannot be a covering of the complete graph K_4 of the order 4 because K_4 is not bipartite. A simple observation then, shows that there are no connected cubic semisymmetric graphs of order $4p$, $4p^2$, or $4p^3$. Lu et al. [13] classified connected cubic semisymmetric graphs of order $6p^2$. Malnič et al.[18] classified cubic semisymmetric graphs of order $2p^3$ for a prime p, while Folkman [7] proved that there are no cubic semisymmetric graphs of order $2p$ or $2p^2$. Some general methods of an elementary Abelian coverings were developed in [15, 16]. The elementary Abelian coverings of the three-dimensional hypercube Q_3 were classified in [6]. The semisymmetric elementary abelian covers of the Möbius-Kantor graph were considered by Malnič et al in [17]. Furthermore, Wang and Chen [25] classified semisymmetric cyclic or elementary abelian covers of the complete bipartite graph $K_{3,3}$, when the fibre-preserving group is edge- but not vertex-transitive. Talebi investigated semisymmetric and s-regular graphs by employing the covering technique, group-theoretical construction and concept linear algebra [9, 23, 24].

In this paper, by applying concept linear algebra, we classify the connected semisymmetric z_p-covers of the $C20$ graph.

1.   Preliminaries related to covering, Voltage graphs, lifting problems and the first homology group

Let X be a graph and K be a finite group. By a^{-1} we mean the reverse arc to an arc a. A voltage assignment (or K-voltage assignment) of X is a function $\xi\colon A(X) \to K$ with the property that $\xi(a^{-1}) = \xi(a)^{-1}$ for each arc $a \in A(X)$. The values of ξ are called voltages, and K is the voltage group. The graph $X \times_\xi K$ ($Cov(X, \xi)$) derived from a voltage assignment $\xi : A(X) \to K$ has vertex set $V(X) \times K$ and edge set $E(X) \times K$, so that an edge (e, g) of $X \times$ K joins a vertex (u, g) to $(v, \xi(a)g)$ for $a = (u, v) \in A(X)$ and $g \in K$, where $e = \{u, v\}$. [22] The voltage assignment ξ on arcs extends to a voltage assignment on walks in a natural way, that is, the voltage on a walk W, say with consecutive incident arcs $a_1, a_2, \ldots, a_n$, is $\xi(a_1)\xi(a_2)\ldots\xi(a_n)$.

Clearly, the derived graph $X \times_\xi K$ is a covering of X with the first co-ordinate projection $p\colon X \times_\xi K \to X$, which is called the natural projection. By defining $(u, g')^g = (u, g'g)$ for any $g \in K$ and $(u, g') \in V(X \times_\xi K)$, K becomes a subgroup of $\mathrm{Aut}(X \times_\xi K)$ which acts semiregularly on $V(X \times_\xi K)$. Therefore, $X \times_\xi K$ can be viewed as a K-covering. For each $u \in V(X)$ and $\{u, v\} \in E(X)$, the vertex set $\{(u, g) | g \in K\}$ is the fibre of u and the edge set $\{(u, g)(v, \xi(a)g) | \in K\}$ is the fibre of $\{u, v\}$, where $a = (u, v)$. Conversely, each regular covering $\widetilde{X}$ of X with a covering transformation group K can be derived from a K-voltage assignment. Given a spanning tree T of the graph X, a voltage assignment ξ is said to be T-reduced if the voltages on the tree arcs are the identity. Gross and Tucker [8] showed that every regular covering $\widetilde{X}$ of a graph X can be derived from a T-reduced voltage assignment $\widetilde{X}$ with respect to an arbitrary fixed spanning tree T of X.

Let $\widetilde{X}$ be a K-covering of X with a projection p. If $\alpha \in \mathrm{Aut}(X)$ and $\widetilde{\alpha} \in \mathrm{Aut}(\widetilde{X})$ satisfy $\widetilde{\alpha}p = p\alpha$, we call $\widetilde{\alpha}$ a lift of α, and α the projection of $\widetilde{\alpha}$. Concepts such as a lift of a subgroup of $\mathrm{Aut}(X)$ and the projection of a subgroup of $\widetilde{X}$ are self-explanatory [15]. The lifts and projections of such subgroups are of course subgroups in $\mathrm{Aut}(\widetilde{X})$ and $\mathrm{Aut}(X)$, respectively. In particular, if the covering graph $\widetilde{X}$ is connected, then the covering transformation group K is the lift of the trivial group, that is,

$$\mathrm{K} = \{\widetilde{\alpha} \in \mathrm{Aut}(\widetilde{X})\colon p = \widetilde{\alpha}p\}.$$

Let T be a spanning tree of a graph X. A closed walk W that contains only one cotree arc is called a fundamental closed walk. Similarly, a cycle W that contains only one cotree arc is called a fundamental cycle. Observe that a voltage assignment on arcs extends to a voltage assignment on walks in a natural way. Given $\alpha \in \mathrm{Aut}(X)$, we define a function $\bar{\alpha}$ from the set of voltages on fundamental closed walks based at a fixed vertex $v \in V(X)$ to the voltage group K by

$$(\xi(C))^{\bar{\alpha}} = \xi(C^\alpha),$$

where C ranges over all fundamental closed walks at v, and $\xi(C)$ and $\xi(C^\alpha)$ are the voltages on C and C^α, respectively. Note that if K is abelian, $\bar{\alpha}$ does not depend on the choice of the base vertex, and the fundamental closed walks at v can be substituted by the fundamental cycles generated by the cotree arcs of X.

Two coverings $\widetilde{X}_1$ and $\widetilde{X}_2$ of X with projection p_1 and p_2, respectively, are said to be isomorphic if there exist an automorphism $\alpha \in \mathrm{Aut}(X)$ and

an isomorphism $\tilde{\alpha} : \widetilde{X}_1 \to \widetilde{X}_2$ such that $\tilde{\alpha} p_2 = p_1 \alpha$. In particular, if α is the identity automorphism of X, then we say $\widetilde{X}_1$ and $\widetilde{X}_2$ are equivalent.

For a graph X, $D(X)$ is a set of darts, which is required to be disjoint from $V(X)$, I is a mapping of $D(X)$ onto $V(X)$, called the incidence function, and λ is an involutory permutation of $D(X)$, called the dart-reversing involution. For convenience or if λ is not explicitly specified we sometimes write x^{-1} instead of λx. Intuitively, the mapping I assigns to each dart its initial vertex, and the permutation λ interchanges a dart and its reverse. The terminal vertex of a dart x is the initial vertex of λx. The set of all darts initiated at a given vertex u is denoted by D_u, called the neighborhood of u. The cardinality $|D_u|$ of D_u is the valency of the vertex u. The orbits of λ are called edges; thus each dart determines uniquely its underlying edge. An edge is called a semiedge if $\lambda x = x$, a loop if $\lambda x \neq x$ and $I\lambda x = Ix$, and it is called a link otherwise. A walk of length $n \geqslant 1$ is a sequence of n darts $W = x_1 x_2 \ldots x_n$ such that, for each index $1 \leqslant k \leqslant n - 1$, the terminal vertex of x_k coincides with the initial vertex of x_{k+1}. Moreover, we define each vertex to be a trivial walk of length 0. The initial vertex of W is the initial vertex of x_1, and the terminal vertex of W is the terminal vertex of x_n. The walk is closed if the initial and the terminal vertex coincide. In this case we say that the walk is based at that vertex. If W has initial vertex u and terminal vertex v, then we usually write $W : u \to v$. Let W_1 and W_2 be two walks such that the terminal vertex of W_1 coincides with the initial vertex of W_2. We define the product $W_1 W_2$ as the juxtaposition of the two sequences. A walk W is reduced if it contains no subsequence of the form xx^{-1}.

By $\pi(X)$ we denote the fundamental groupoid of a graph X, that is, the set of all reduced walks equipped with the product $W_1 W_2$. The group $\pi(X, u)$ is called the fundamental group of X at u. The fundamental group is not a free group in general. Consequently, the first homology group $H_1(X)$, obtained by abelianizing $\pi(X, u)$, is not necessarily a free Z-module. Namely, let $r_e + r_s$ be the minimal number of generators of $\pi(X, u)$, where r_s is the number of semiedges and r_e is the number of cotree loops and links relative to some spanning tree. Then $H_1(X) \cong Z^{r_e} \times Z_2^{r_s}$. [16] The first homology group $H_1(X, Z_p) \cong H_1(X)/pH_1(X)$ with Z_p as the coefficient ring can be considered as a vector space over the field Z_p. Observe that

$$H_1(X, Z_p) \cong \left\{ \begin{array}{ll} Z_p^{r_e + r_s} & p = 2 \\ Z_p^{r_e} & p \geqslant 3. \end{array} \right.$$

Let us call a covering projection $p\colon \overline{X} \to X$ vertex-transitive (edge-transitive), if a vertex-transitive (edge-transitive) subgroup of $\mathrm{Aut}(X)$ lifts along p. If p is edge- but not vertex-transitive, then p is called semisymmetric. Observe that the derived graph of a semisymmetric covering projection is a good candidate for a semisymmetric graph.

Assume that a connected graph X and a subgroup $G \leqslant \mathrm{Aut}(X)$ are given. Choose a spanning tree T of X and a set of arcs $\{x_1, \ldots, x_r\} \subseteq A(X)$ containing exactly one arc from each edge in $E(X \setminus T)$. Let B_T be the corresponding basis of the first homology group $H_1(X, Z_p)$ determined by $\{x_1, \ldots, x_r\}$. Further, denote by $G^{*h} = \{\alpha^{*h} | \alpha \in G\} \leqslant GL(H_1(X, Z_p))$ the induced action of G on $H_1(X, Z_p)$, and let $M_G \leqslant Z_p^{r \times r}$ be the matrix representation of G^{*h} with respect to the basis B_T. By M_G^t we denote the dual group consisting of all transposes of matrices in M_G.

The following proposition is necessary to classify semisymmetric z_p-covers of the $C20$ graph. This proposition is a special case of [16, Proposition 6.3, Corollary 6.5].

Proposition 1. *Let T be a spanning tree of a connected graph X and let the set $\{x_1, x_2, \ldots, x_r\} \subseteq A(X)$ contain exactly one arc from each cotree edge. Let $\xi \colon A(X) \to Z_p^{d \times 1}$ be a voltage assignment on X which is trivial on T, and let $Z(\xi) = [\xi(x_1), \xi(x_2), \ldots, \xi(x_r)]^t$. Then the following holds.*

(a) A group $G \leqslant \mathrm{Aut}(X)$ lifts along $p_\xi \colon Cov(X, \xi) \to X$ if and only if the induced subspace $\langle Z(\xi) \rangle$ is an M_G^t-invariant d-dimensional subspace.

(b) If $\xi' \colon A(X) \to Z_p^{d \times 1}$ is another voltage assignment satisfying (a), then $Cov(X, \xi')$ is equivalent to $Cov(X, \xi)$ if and only if $\langle Z(\xi) \rangle = \langle Z(\xi') \rangle$, as subspaces. Moreover, $Cov(X, \xi')$ is isomorphic to $Cov(X, \xi)$ if and only if there exists an automorphism $\alpha \in \mathrm{Aut}(X)$ such that the matrix M_α^t maps $\langle Z(\xi') \rangle$ onto $\langle Z(\xi) \rangle$.

To find all semisymmetric G-admissible Z_p-covering projections of $C20$, we have to find, by Proposition 1, all invariant 1-dimensional subspaces of the transpose of the matrix M_G.

2.   Finding invariant subspaces

The problem of finding all elementary abelian regular covering projections of a given connected graph, admissible for a given group of automorphisms, is reduced to finding all invariant subspaces of an associated (finite) matrix group over a prime field. In this context we recall Masche's theorem which states that if the characteristic Char F of the field does not divide the order of the group, then the representation is completely

reducible. In this case one essentially needs to find just the minimal common invariant subspaces of the generators of the group in question, for the non-minimal subspaces can be expressed as direct sums of some of the minimal ones. (Still, this may involve knowing all invariant subspaces of the generators, in view of the fact that a minimal invariant subspace for the whole group need not be minimal for neither of the individual generators—although invariant subspaces of a generator are direct sums of the minimal ones for that generator. Additional information about the relations between generators coming from the presentation of the group is beneficial; this is the point where ad hoc techniques are most helpful.) The remaining cases where Char F divides the order of the given group could be, technically, more difficult to analyze.

Let $A \in F^{n,n}$ be an $n \times n$ invertible matrix over a field F, acting as a linear transformation $\underline{x} \to A\underline{x}$ on the column vector space $F^{n,1}$. Let $A(x) = f_1(x)^{n_1} f_2(x)^{n_2} \ldots f_k(x)^{n_k}$ be the characteristic polynomial and $m_A(x) = f_1(x)^{s_1} f_2(x)^{s_2} \ldots f_k(x)^{s_k}$ the minimal polynomial of A where $f_j(x), j = 1, \ldots, k$, are pairwise distinct irreducible factors over F. Then $F^{n,1}$ can be written as a direct sum of the A-invariant subspaces $F^{n,1} = \operatorname{Ker} f_1(A)^{s_1} \oplus \operatorname{Ker} f_2(A)^{s_2} \oplus \cdots \oplus \operatorname{Ker} f_k(A)^{s_k}$: Moreover, all A-invariant subspaces can be found by first considering the invariant subspaces of $\operatorname{Ker} f_j(A)^{s_j}, j = 1, \ldots, k$, and then taking direct sums of some of these. In particular, the minimal ones are just the minimal A-invariant subspaces of $\operatorname{Ker} f_j(A)s_j, j = 1, \ldots, k$. Now the subspace $\operatorname{Ker} f_j(A)^{s_j}$ has dimension $d_j n_j$, where $d_j = degf_j(x)$ is the degree of the polynomial $f_j(x)$. Its minimal A-invariant subspaces are cyclic of the form $< v, Av, \ldots, A^{d_j-1}v >$, where $v \in \operatorname{Ker} f_j(A)$, and each such defines an increasing sequence of length at most s_j of nested invariant subspaces (at least one is precisely of length s_j). If $n_j > s_j$, then a variety of pairwise disjoint minimal cyclic subspaces exist in $\operatorname{Ker} f_j(A)^{s_j}$, and a unique one if $n_j = s_j$. In particular, if $n_j = s_j = 1$, then $\operatorname{Ker} f_j(A)$ itself is the only A-invariant subspace contained in $\operatorname{Ker} f_j(A)$ and hence minimal. Consequently, if $A(x) = m_A(x)$ with all $n_j = s_j = 1$, then $\operatorname{Ker} f_j(A), j = 1, \ldots, k$, are the only minimal A-invariant subspaces, and all others are direct sums of these [17].

3. $C20$ graph

In the mathematical field of graph theory, the $C20$ graph [21] is a symmetric bipartite tetravalent graph with 20 vertices and 40 edges

$$V(X) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

and

$$E(X) = \{\{1,2\}, \{1,10\}, \{1,11\}, \{1,17\}, \{2,3\}, \{2,12\}, \{2,18\},$$
$$\{3,4\}, \{3,13\}, \{3,19\}, \{4,5\}, \{4,14\}, \{4,20\}, \{5,6\},$$
$$\{5,11\}, \{5,15\}, \{6,7\}, \{6,12\}, \{6,16\}, \{7,8\}, \{7,13\},$$
$$\{19,20\}, \{7,17\}, \{8,9\}, \{8,14\}, \{8,18\}, \{9,10\}, \{9,15\},$$
$$\{9,19\}, \{10,16\}, \{10,20\}, \{17,18\}, \{11,12\}, \{11,20\},$$
$$\{12,13\}, \{13,14\}, \{14,15\}, \{15,16\}, \{16,17\}, \{18,19\}\}.$$

We choose

$$\alpha = (2,10)(3,9)(4,8)(5,7)(11,17)(12,16)(13,15)(18,20),$$
$$\beta = (2,17,10,11)(3,7,9,5)(4,13,8,15)(6,19)(12,18,16,20),$$
$$\gamma = (1,2,3,4,5,6,7,8,9,10)(11,12,13,14,15,16,17,18,19,20).$$

as automorphisms of $C20$ graph. Then $\text{Aut}(C20) = \langle \alpha, \beta, \gamma \rangle$. The auto-
morphism group of the $C20$ graph is a group of order 80. It acts transitively
on the vertices, on the edges, and on the arcs of the graph. Therefore the
$C20$ graph is a symmetric graph. It has automorphisms that take any
vertex to any other vertex and any edge to any other edge. By [1], the
automorphism group of the $C20$ graph has one semisymmetric subgroup
$G := \langle (\beta\gamma)^2, \beta \rangle$
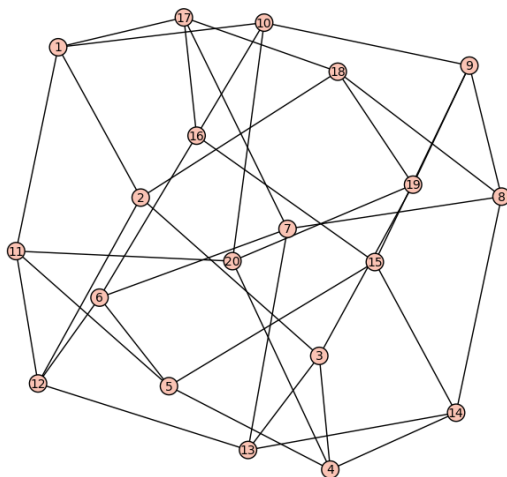
Figure 1. $C20$ graph.

We choose a spanning tree T of $C20$ graph consisting of the edges

$$\{1,2\}, \quad \{1,10\}, \quad \{1,11\}, \quad \{1,17\},$$
$$\{2,3\}, \quad \{2,12\}, \quad \{2,18\}, \quad \{4,3\},$$
$$\{3,13\}, \quad \{3,19\}, \quad \{4,5\}, \quad \{4,14\},$$
$$\{4,20\}, \quad \{5,6\}, \quad \{5,15\}, \quad \{6,7\},$$
$$\{16,6\}, \quad \{7,8\}, \quad \{8,9\}.$$

By choosing T, we can define a T-reduced voltage assignment. We show the cotree arcs by setting

$$x_0 = (19,20), \quad x_1 = (18,19), \quad x_2 = (17,18),$$
$$x_3 = (16,17), \quad x_4 = (15,16), \quad x_5 = (14,15),$$
$$x_6 = (13,14), \quad x_7 = (12,13), \quad x_8 = (11,12),$$
$$x_9 = (9,10), \quad x_{10} = (5,11), \quad x_{11} = (6,12),$$
$$x_{12} = (7,13), \quad x_{13} = (8,14), \quad x_{14} = (9,19),$$
$$x_{15} = (10,19), \quad x_{16} = (10,20), \quad x_{17} = (11,20),$$
$$x_{18} = (8,18), \quad x_{19} = (7,17), \quad x_{20} = (9,15).$$

4.   Semisymmetric z_p-covers of the $C20$ graph

Semisymmetric graphs (regular edge- but not vertex-transitive graphs), have recently received a wide attention. Regular covers, and elementary abelian in particular, have proved to be very useful in this context. In this section, we compute all those (connected) semisymmetric p-elementary abelian regular covering projections $p\colon \overline{X} \to C20$.

Now, we express the following lemma.

Lemma 1. *Let B and C be the transposes of the matrices which represent the linear transformations β^{*h}, and γ^{*h} relative to $B_T = \{C_{x_i} | 0 \leqslant i \leqslant 20\}$; the standard ordered basis of $H1(C20, Z_p)$ associated with the spanning tree T and the arcs $x_i (i = 0, \ldots, 20)$, respectively. Then*

$$B = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$C = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

Proof. The rows of these matrices are obtained by letting the automorphisms β, γ act on B_T. For example, the permutation γ maps the cycle

$$[19, 20, 4, 3, 19]$$

corresponding to x_0, to the cycle

$$[20, 11, 5, 4, 20].$$

Since the latter is the sum of the base cycles corresponding to x_{10}^{-1} and x_{17}^{-1}, the first row of C is

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0).$$

By similar computations we can get the matrices B and C.   □

By [1] we have the following lemma.

Lemma 2. *The minimal polynomials of B and $H = (B \cdot C)^2$ are $m_B(x) = (x-1)(x+1)(x^2+1)$, and $m_H(x) = x^2 - 1$, respectively.*

Suppose that p be a prime. A polynomial $x^2 + 1$ has distinct solutions in Z_p if and only if $p \equiv 1 \bmod 4$. In order to find $\langle B, H \rangle$-invariant subspaces over Z_p, it is useful to consider B and H as matrices over the splitting field $Z_p(i)$ where i is a zero of the polynomial $x^2 + 1$. By a straightforward calculation, lemma 5.1 and lemma 5.2, we have

$$\ker(B - I) = \langle u_0, u_1, u_2, u_3 \rangle,$$
$$\ker(B + I) = \langle u_4, u_5, u_6, u_7, u_8 \rangle,$$
$$\ker(B + iI) = \langle u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14} \rangle,$$
$$\ker(B - iI) = \langle u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20} \rangle,$$

where

$$u_0 = [1, -1, 0, 0, -1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, -1, 0, -1]^t,$$
$$u_1 = [0, 0, 1, 0, -1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, -1, 1, -1]^t,$$
$$u_2 = [0, 0, 0, 1, 1, 0, 0, -1, -1, -1, -1, 0, 0, 0, 0, 0, -1, 0, 1, -1, 1]^t,$$
$$u_3 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, -1]^t,$$
$$u_4 = [1, -1, 0, 0, 1, 0, 0, -1, 0, 0, 0, -1, 1, 0, 1, 0, 0, 0, -1, 0, 0]^t,$$
$$u_5 = [0, 0, 1, 0, 1, 0, 0, -1, 0, 0, 0, 0, \frac{1}{2}, 0, 0, -1, 0, 1, -1, 0, \frac{1}{2}]^t,$$
$$u_6 = [0, 0, 0, 1, 1, 0, 0, -1, -1, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 1, 0, -1, 0, \frac{1}{2}]^t,$$

$$u_7 = [0, 0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0, -\frac{1}{2}, 1, 0, 0, 0, 0, 0, 0, -\frac{1}{2}]^t,$$

$$u_8 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, -1, -\frac{1}{2}]^t,$$

$$u_9 = [1, 1, 0, 0, i, 0, 0, i, 0, 0, 0, i, 0, 0, -i, 0, 0, 0, 1, 0, i]^t,$$

$$u_{10} = [0, 0, 1, 0, -i, 0, 0, -i, 0, 0, 0, 0, 0, 0, i, i, 0, -1, -1, 0, -i]^t,$$

$$u_{11} = [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, -1, 0, -i, 0, -i, 0, 1]^t,$$

$$u_{12} = [0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, i, 1, 0, 0, 0, 0, 0, -1]^t,$$

$$u_{13} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, i, 0, 0, 0, -1, 0, 0, 0, 0, -i, 0]^t,$$

$$u_{14} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, -i - 1, 0, 0, 0, 0, 0, 1]^t,$$

$$u_{15} = [1, 1, 0, 0, -i, 0, 0, -i, 0, 0, 0, -i, 0, 0, i, 0, 0, 0, 1, 0, -i]^t,$$

$$u_{16} = [0, 0, 1, 0, i, 0, 0, i, 0, 0, 0, 0, 0, 0, -i, -i, 0, -1, -1, 0, i]^t,$$

$$u_{17} = [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, -1, 0, i, 0, i, 0, 1]^t,$$

$$u_{18} = [0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, -i, 1, 0, 0, 0, 0, 0, -1]^t,$$

$$u_{19} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -i, 0, 0, 0, -1, 0, 0, 0, 0, i, 0]^t,$$

$$u_{20} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, i - 1, 0, 0, 0, 0, 0, 1]^t$$

and

$$\ker(H + I) = \langle v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9 \rangle,$$
$$\ker(H - I) = \langle v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20} \rangle,$$

where

$$v_0 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, -1, -1, -1, 0]^t,$$

$$v_1 = [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, -1, 1, 1, 0, 0]^t,$$

$$v_2 = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, 0, 1, -1, 1, -1, -1, 1, 0]^t,$$

$$v_3 = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, -1, 1, 1, 0, 0]^t,$$

$$v_4 = [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, -1, -1, -1, 1]^t,$$

$$v_5 = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, -1, 0, 1, -1, 1, 1, 1, 1, -2]^t,$$

$$v_6 = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, -1, -1, -1, 1]^t,$$

$$v_7 = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, -1, 1, -1, 1, 1, 1, 0]^t,$$

$$v_8 = [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, -1, -1, 0, 1, -1, 1, -1, 0, 0, 0]^t,$$

$$v_9 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, -1, 1, -1, 0, -1, -1, 1]^t,$$

$$v_{10} = [1, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 1, 0, -1, 0, 0, 0, 0]^t,$$

$$v_{11} = [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 1, -1, 1, 0, 0]^t,$$

$$v_{12} = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^t,$$
$$v_{13} = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, -1, 1, -1, 0, 0]^t,$$
$$v_{14} = [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, 1, 0, 0, 0, 1]^t,$$
$$v_{15} = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^t,$$
$$v_{16} = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, -1, 0, 0, 0, -1]^t,$$
$$v_{17} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, -1, -1, 1, -1, 1, -1, 0]^t,$$
$$v_{18} = [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0]^t,$$
$$v_{19} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, -1, 0, 0, 0, 1, 0, 0, 1]^t,$$
$$v_{20} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1, 0, 1, 1, -1, 1, -1, 1, 0]^t.$$

Now, we have over the field $Z_p(i)$

$$\mathrm{Ker}(B - I) \cap \mathrm{ker}(H - I) = 0,$$
$$\mathrm{Ker}(B - I) \cap \mathrm{ker}(H + I) = 0,$$
$$\mathrm{Ker}(B + iI) \cap \mathrm{ker}(H + I) = \langle K_0 \rangle,$$
$$\mathrm{Ker}(B - iI) \cap \mathrm{ker}(H + I) = \langle K_1 \rangle,$$
$$\mathrm{Ker}(B + iI) \cap \mathrm{ker}(H - I) = \langle K_2 \rangle,$$
$$\mathrm{Ker}(B - iI) \cap \mathrm{ker}(H - I) = \langle K_3 \rangle,$$
$$\mathrm{Ker}(B + I) \cap \mathrm{ker}(H - I) = \langle K_4 \rangle,$$

and

$$\mathrm{Ker}(B + I) \cap \mathrm{ker}(H + I) = 0,$$

where $\langle K_0 \rangle$, $\langle K_1 \rangle$, $\langle K_2 \rangle$, $\langle K_3 \rangle$ and $\langle K_4 \rangle$ are invariant 1-dimensional subspaces (see Table 1).

If $p = 2$, then minimal polynomials are $m_B(x) = (x-1)^4$, and $m_H(x) = (x - 1)^2$. By the same argument as above, we may prove that $\langle K_5 \rangle$ is invariant 1-dimensional subspace (see Table 1).

If $p \equiv 3 \bmod 4$, then minimal polynomials are $m_B(x) = (x - 1)(x + 1)(x^2 + 1)$, and $m_H(x) = (x-1)(x+1)$. There is a 1-dimensional subspace $\langle K_4 \rangle$.

Due to the above description, we have the following result.

Theorem 1. *Let p be a prime. Let $\overline{X}$ be a semisymmetric Z_p-cover of the $C20$ graph, along which the group G lifts. Then, the connected semisymmetric elementary abelian regular Z_p-covers of $C20$ graph are given in the following table:*

inv.sub	$\xi(x_0)$	$\xi(x_1)$	$\xi(x_2)$	$\xi(x_3)$	$\xi(x_4)$	$\xi(x_5)$	condition
$\langle K_0 \rangle$	(1)	(1)	(1)	(1)	(1)	(1)	$p \equiv 1 \mod 4$
$\langle K_1 \rangle$	(1)	(1)	(1)	(1)	(1)	(1)	$p \equiv 1 \mod 4$
$\langle K_2 \rangle$	(1)	(1)	(1)	(1)	(1)	(1)	$p \equiv 1 \mod 4$
$\langle K_3 \rangle$	(1)	(1)	(1)	(1)	(1)	(1)	$p \equiv 1 \mod 4$
$\langle K_4 \rangle$	(1)	(-1)	(1)	(-1)	(1)	(-1)	$p \equiv 1 \; or \; 3 \mod 4$
$\langle K_5 \rangle$	(1)	(1)	(1)	(1)	(1)	(1)	$p = 2$

inv.sub	$\xi(x_6)$	$\xi(x_7)$	$\xi(x_8)$	$\xi(x_9)$	$\xi(x_{10})$	condition
$\langle K_0 \rangle$	(1)	(1)	(1)	(1)	(i)	$p \equiv 1 \mod 4$
$\langle K_1 \rangle$	(1)	(1)	(1)	(1)	$(-i)$	$p \equiv 1 \mod 4$
$\langle K_2 \rangle$	(1)	(1)	(1)	(-1)	$(-i)$	$p \equiv 1 \mod 4$
$\langle K_3 \rangle$	(1)	(1)	(1)	(-1)	(i)	$p \equiv 1 \mod 4$
$\langle K_4 \rangle$	(1)	(-1)	(1)	(-1)	(1)	$p \equiv 1 \; or \; 3 \mod 4$
$\langle K_5 \rangle$	(1)	(1)	(1)	(1)	(1)	$p = 2$

inv.sub	$\xi(x_{11})$	$\xi(x_{12})$	$\xi(x_{13})$	$\xi(x_{14})$	$\xi(x_{15})$	condition
$\langle K_0 \rangle$	(i)	(i)	(i)	$(-i)$	(i)	$p \equiv 1 \mod 4$
$\langle K_1 \rangle$	$(-i)$	$(-i)$	$(-i)$	(i)	$(-i)$	$p \equiv 1 \mod 4$
$\langle K_2 \rangle$	(i)	$(-i)$	(i)	(i)	(i)	$p \equiv 1 \mod 4$
$\langle K_3 \rangle$	(i)	$(-i)$	(i)	(i)	(i)	$p \equiv 1 \mod 4$
$\langle K_4 \rangle$	(-1)	(1)	(-1)	(1)	(-1)	$p \equiv 1 \; or \; 3 \mod 4$
$\langle K_5 \rangle$	(1)	(1)	(1)	(1)	(1)	$p = 2$

inv.sub	$\xi(x_{16})$	$\xi(x_{17})$	$\xi(x_{18})$	$\xi(x_{19})$	$\xi(x_{20})$	condition
$\langle K_0 \rangle$	$(-i)$	(-1)	$(-i)$	$(-i)$	(i)	$p \equiv 1 \mod 4$
$\langle K_1 \rangle$	(i)	(-1)	(i)	(i)	$(-i)$	$p \equiv 1 \mod 4$
$\langle K_2 \rangle$	$(-i)$	(-1)	$(-i)$	(i)	$(-i)$	$p \equiv 1 \mod 4$
$\langle K_3 \rangle$	$(-i)$	(-1)	(i)	$(-i)$	(i)	$p \equiv 1 \mod 4$
$\langle K_4 \rangle$	(-1)	(1)	(-1)	(1)	(1)	$p \equiv 1 \; or \; 3 \mod 4$
$\langle K_5 \rangle$	(1)	(1)	(1)	(1)	(1)	$p = 2$

Table 1. Semisymmetric elementary abelian regular Z_p-covers of the $C20$ graph.

5. Conclusion

Covering techniques have long been known as a powerful tool in topology, and graph theory. Regular covering of a graph is currently an active topic in algebraic graph theory. In this study, by applying concept linear algebra, we classify the connected semisymmetric elementary abelian Z_p-covers of the $C20$ graph.

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