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Gentle m-Calabi–Yau tilted algebras

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Abstract. We prove that all gentle 2-Calabi–Yau tilted algebras are Jacobian, moreover their bound quiver can be obtained via block decomposition. For two related families, the m-cluster-tilted algebras of type $\mathbb{A}$ and $\tilde{\mathbb{A}}$, we prove that a module M is stable Cohen-Macaulay if and only if $\Omega^{m+1}\tau M \simeq M$.

Introduction

Gentle algebras are a class of finite dimensional algebras whose module (and derived) category is well understood. These algebras have good properties, they are Gorenstein [18], tame, and their module category is described via strings and parametrized bands [11]. On the other hand, 2-Calabi–Yau (2-CY for short) tilted algebras are generalization of the concept of cluster-tilted algebras. A cluster-tilted algebra is the endomorphism algebra of a cluster-tilting object in the cluster category of a hereditary algebra, 2-CY tilted algebras are obtained by replacing the cluster category by a 2-CY triangulated category. These algebras are Goresntein of dimension at most one [21]. Cluster categories and cluster-tilted algebras were introduced in [9, 10, 12]. Jacobian algebras were defined in [14], these algebras are defined by a quiver with potential (Q, W). In [1] were introduced 2-CY categories $\mathcal{C}_{(Q,W)}$ associated to quivers with potential,

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in such way that Jacobian algebras are obtained as 2-CY tilted algebras arising from $\mathcal{C}_{(Q,W)}$.

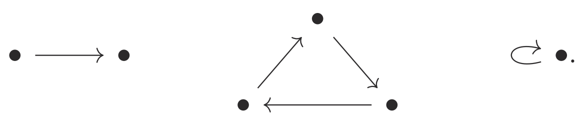
A well known class of gentle 2-CY tilted algebras is the Jacobian algebras arising from unpunctured surfaces defined in [3]. This family includes the cluster-tilted algebras of type $\mathbb{A}$ and $\tilde{\mathbb{A}}$.

Two related families of gentle algebras appeared recently in the literature:

1) Jacobian algebras from triangulation of a polygon with an orbifold [25].
2) m-cluster-tilted algebras of types $\mathbb{A}$ and $\tilde{\mathbb{A}}$ [5, 19, 27]. These algebras have a geometric realization via partitions of unpunctured surfaces.

In Section 2, we characterize gentle 2-CY tilted k-algebras in the case $\operatorname{char} k \neq 3$.

Theorem 1. Let $\Lambda = kQ/I$ be a gentle algebra of Gorenstein dimension at most one and such that $\Omega^2 \tau M \simeq M$ for all $M \in \underline{\mathrm{CM}}(\Lambda)$. Then (Q, I) is obtained via block decomposition, matching blocks of type I, II and loop:



Immediately, we obtain the next result.

Corollary 1. Let k be an algebracailly closed field and $\operatorname{char} k \neq 3$. If kQ/I is a gentle 2-CY tilted algebra then kQ/I is Jacobian.

These algebras include those arising from triangulations of a polygon with an orbifold.

In Section 3, we prove a result generalizing the formula that characterizes the modules in the singularity category of 2-CY tilted algebras, in the context of gentle m-cluster tilted algebras.

Theorem 2. Assume that Λ is an m-cluster tilted algebra of one of the types $\mathbb{A}$ and $\tilde{\mathbb{A}}$.

1) Λ is Gorenstein of dimension $d \leqslant m$.
2) $N \in \underline{\mathrm{CM}}(\Lambda)$ if and only if $\Omega^{m+1} \tau N = N$.

The paper is organized as follows. In Section 1, we recall basic facts about gentle algebras, 2-CY tilted algebras and m-cluster-tilted algebras. Section 2 is devoted to our characterization of gentle 2-Calabi–Yau tilted algebras. The study of the modules in singularity categories over m-cluster-tilted algebras is given in Section 3.

1. Preliminaries

Throughout these notes, let k be an algebraically closed field and let $Q = (Q_0, Q_1)$ be a finite quiver, where Q_0 is the set of vertices and Q_1 the set of arrows. Let $s, t\colon Q_1 \to Q_0$ be the functions that indicate the source and the target of each arrow, respectively. We will only consider *finite-dimensional basic k-algebras*. Every finite- dimensional basic k-algebra is isomorphic to a quotient kQ/I, where I is an admissible ideal. The pair (Q, I) is called a *bound quiver*. For more details, see [4, Chapter III].

1.1. Gentle algebras

We recall the definition of gentle algebra and results due to Geiss and Reiten [18], and Kalck [20]. See also [4, Section IX.6].

Definition 1.1. A k-algebra $\Lambda = kQ/I$ is gentle if

(G1) For each vertex $x_0 \in Q_0$ there are at most two arrows such that x_0 is their source, and at most two arrows such that x_0 is their target.

(G2) The ideal I is generated by paths of length 2.

(G3) For each arrow β there is at most one arrow α and at most one arrow γ such that $\alpha\beta \in I$ and $\beta\gamma \in I$.

(G4) For each arrow β there is at most one arrow α and at most one arrow γ such that $\alpha\beta \notin I$ and $\beta\gamma \notin I$.

We will often refer to the generators in I as *zero-relations*.

Definition 1.2. Let $\Lambda = kQ/I$ be a gentle algebra.

(a) A cycle $x_1 \xrightarrow{\alpha_1} \cdots \to x_n \xrightarrow{\alpha_n} x_1$ is saturated if $\alpha_i\alpha_{i+1} \in I$, for i an integer modulo n. In particular, a saturated loop is an arrow δ such that $s(\delta) = t(\delta)$ and $\delta^2 \in I$.

(b) An arrow β is gentle if there is no other arrow α such that $\alpha\beta \in I$.

(c) A path $\alpha_1 \dots \alpha_n$ is formed by consecutive relations if $\alpha_i\alpha_{i+1} \in I$ for $1 \leqslant i < n$.

(d) A path $\alpha_1 \dots \alpha_n$ is critical if it is formed by consecutive relations and α_1 is a gentle arrow.

When there is no gentle arrow, we set $n(\Lambda) = 0$. When there is a gentle arrow, let $n(\Lambda)$ be the maximal length computed over all critical paths. This number is bounded, since Q is finite.

Let Ω be the usual syzygy operator, τ the Auslander-Reiten (AR) translation [4, Section IV.2], and $D = \operatorname{Hom}_k(-, k)$.

Definition 1.3. A k-algebra Λ is Gorenstein if

$$\text{inj.dim}\,\Lambda = \text{proj.dim}\,D(\Lambda^{\text{op}}) = d$$

for some non-negative integer d. In this case we say that Λ is Gorenstein of dimension d.

Theorem 1.4. [18] Let $\Lambda = kQ/I$ be a gentle algebra with $n(\Lambda)$ the maximum length of critical paths. Then $\text{inj.dim}\,\Lambda = n(\Lambda) = \text{proj.dim}\,D(\Lambda^{\text{op}})$ if $n(\Lambda) > 0$, and $\text{inj.dim}\,\Lambda = \text{proj.dim}\,D(\Lambda^{\text{op}}) \leqslant 1$ if $n(\Lambda) = 0$. In particular, Λ is Gorenstein.

An algebra $\Lambda = kQ/I$ where I is generated by paths and (Q, I) satisfies the two conditions (G1) and (G4) is called a *string algebra*, thus every gentle algebra is a string algebra. A *string* in Λ is by definition a reduced walk w in Q avoiding the zero-relations, thus w is a sequence $x_1 \overset{\alpha_1}{\longleftrightarrow} x_2 \overset{\alpha_2}{\longleftrightarrow} \cdots \overset{\alpha_n}{\longleftrightarrow} x_{n+1}$ where the x_i are vertices of Q and each α_i is an arrow between the vertices x_i and x_{i+1} in either direction such that there is no $\overset{\beta}{\longrightarrow}\overset{\beta}{\longleftarrow}$, and no $\overset{\beta_1}{\longleftarrow} \cdots \overset{\beta_t}{\longleftarrow}$ or $\overset{\beta_1}{\longrightarrow} \cdots \overset{\beta_t}{\longrightarrow}$ with $\beta_1 \dots \beta_t \in I$. If the first and the last vertex of w coincide, then the string is cyclic. A *band* is a cyclic string b such that each power b^n is a cyclic string but b is not a power of some string. The classification of indecomposable modules over a string algebra $\Lambda = kQ/I$ is given by Butler and Ringel in terms of strings and bands in (Q, I). Each string w defines an indecomposable module $M(w)$, called a *string module*, and each band b defines a family of indecomposable modules $M(b, \lambda, n)$, called *band modules*, with parameters $\lambda \in k$ and $n \in \mathbb{N}$. We refer to [11] for the definition of string and band modules.

Consider the subcategory of maximal Cohen-Macaulay modules (also called Gorenstein projective modules) defined by

$$\text{CM}(\Lambda) = \{M \colon \text{Ext}^i_\Lambda(M, \Lambda) = 0 \text{ for all } i > 0\}.$$

The stable category $\underline{\text{CM}}(\Lambda) = \text{CM}(\Lambda)/(\mathcal{P})$, where $(\mathcal{P})$ denotes the ideal of morphisms factoring through a projective Λ-module, is the *singularity category* of Λ. Let $x_1 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_{n-1}} x_n \xrightarrow{\alpha_n} x_1$ be a saturated cycle, the indecomposable proyective module $P(x_i)$ and the indecomposable injective module $I(x_i)$ are string modules given by $P(x_i) = M(u_i^{-1}\alpha_i u_{i+1})$ and $I(x_i) = M(v_{i-1}\alpha_{i-1}v_i^{-1})$ (see Figure 1).

Remark 1.5. For a Gorenstein algebra Λ of dimension d, a Λ-module M is Cohen-Macaulay if and only if M is a d-th syzygy, see [6, Proposition

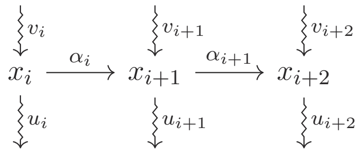


FIGURE 1. Local situation for a saturated cycle. The path u_i is the maximal path starting at the vertex x_i and the path v_i is the maximal path ending at x_i.

6.20]. In this case each Λ-module either has infinite projective dimension or has projective dimension at most d.

We are interested in computing projective resolutions and AR translations of the modules in $\underline{\mathrm{CM}}(\Lambda)$. For that reason, we need the next result.

Theorem 1.6. [20, Theorem 2.5] Let $\Lambda = kQ/I$ be a gentle algebra. Let $x_1 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_{n-1}} x_n \xrightarrow{\alpha_n} x_1$ be a saturated cycle. The string module $M(u_i)$, where u_i is the string starting at x_i as in Figure 1, is Cohen-Macaulay. Moreover, all indecomposable modules in $\underline{\mathrm{CM}}(\Lambda)$ are obtained in such manner.

1.2. 2-CY tilted algebras and Jacobian algebras

A triangulated k-category $\mathcal{C}$, Hom-finite with split idempotents, is *d-Calabi–Yau* (d-CY for short) if there is a bifunctorial isomorphism

$$\mathrm{Hom}_{\mathcal{C}}(X, Y) \simeq D\mathrm{Hom}_{\mathcal{C}}(Y, X[d]), \ \textit{for all } X, Y \in \mathcal{C}.$$

Let $\mathcal{C}$ be a 2-CY category, an object T is *cluster-tilting* if it is basic and

$$\mathrm{add}T = \{X \in \mathcal{C} \colon \mathrm{Hom}_{\mathcal{C}}(X, T[1]) = 0\}.$$

Definition 1.7. The endomorphism algebra of a cluster-tilting object, $\mathrm{End}_{\mathcal{C}}(T)$, is called a 2-CY tilted algebra.

Examples of 2-CY tilted algebras are the *cluster-tilted* algebras defined in [10]. We will need a result due to Keller and Reiten.

Proposition 1.8. [21] Let Λ be a 2-CY tilted algebra, Λ is Gorenstein of dimension less than or equal to one.

We will also need the next result.

Theorem 1.9. [17] Let Λ be a 2-CY tilted algebra. Then, $M \in \underline{\mathrm{CM}}(\Lambda)$ if and only if $\Omega^2\tau M \simeq M$.

Quivers with potential were introduced in [14]. A potential W is a (possibly infinite) linear combination of cycles in Q, up to cyclic equivalence. Given an arrow α and a cycle $\alpha_1 \dots \alpha_l$, the cyclic derivative ∂_α is defined by

$$\partial_\alpha(\alpha_1 \dots \alpha_l) = \sum_{k+1}^{l} \delta_{\alpha\alpha_k} \alpha_{k+1} \dots \alpha_l \alpha_1 \dots \alpha_{k-1},$$

where $\delta_{\alpha\alpha_k}$ is the Kronecker delta, and ∂_α extends by linearity. Notice that the cycle $\alpha_1 \dots \alpha_l$ may have repetitions. Let $R\langle\langle Q\rangle\rangle$ be the complete path algebra consisting of all (possibly infinite) linear combinations of paths in Q. Let (Q,W) be a quiver with potential, the *Jacobian algebra* is defined to be $Jac(Q,W) = R\langle\langle Q\rangle\rangle/\langle\partial_\alpha W, \alpha \in Q_1\rangle$.

Amiot [1, Sec. 3] showed that Jacobian algebras are 2-CY tilted constructing a 2-CY category $\mathcal{C}_{(Q,W)}$, the result just asks $Jac(Q,W)$ to be Jacobi-finite, this means that $Jac(Q,W)$ is finite-dimensional as a k-algebra. In [2], Amiot asked whether all 2-CY tilted algebras are Jacobian algebras. In section 2 we study gentle algebras and prove that the answer is affirmative when $\mathrm{char}\, k \neq 3$.

1.3. *m*-cluster categories and *m*-cluster tilted algebras

The *cluster category* $\mathcal{C}_Q$ associated to Q was introduced in [9] as the quotient category $\mathcal{D}^b(\mathrm{mod}\, kQ)$ over the functor $F = \tau^{-1}[1]$. The *m-cluster category* associated to Q, that we denote by $\mathcal{C}_Q^m$, was defined in [27] as the quotient category $\mathcal{D}^b(\mathrm{mod}\, kQ)$ over the functor $F_m = \tau^{-1}[m]$. The category $\mathcal{C}_Q^m$ is triangulated.

A basic object T in $\mathcal{C}_Q^m$ is called *m-cluster tilting* if

* $\mathrm{Ext}^i_{\mathcal{C}_Q^m}(T,T') = 0$ for all $T, T' \in \mathrm{add}\, T$, for $i = 1, \dots, m$
* if $X \in \mathcal{C}_Q^m$ is such that $\mathrm{Ext}^i_{\mathcal{C}_Q^m}(X,T) = 0$ for all $T \in \mathrm{add}\, T$, and for $i = 1, \dots, m$, then $X \in \mathrm{add}\, T$.

Remark 1.10. We follow the notation in [27]. It is important to recall that $\mathcal{C}_Q^m$ is an example of $(m+1)$-CY category and the category $\mathrm{add}\, T$, were T is an m-cluster tilting object, is an example of $(m+1)$-cluster tilting subcategory in the sense of [7, 21, 22].

Any m-cluster-tilting object has $|Q_0|$ summands, [27, Theorem 2]. The endomorphism algebra $\Lambda = \mathrm{End}_{\mathcal{C}_Q^m}(T)$ is called an *m-cluster tilted* *algebra*, and it is a case of $(m+1)$-Calabi–Yau tilted algebra. If Q is such that its underlying graph Δ_Q is a Dynkin or euclidean graph, we say that $\Lambda = \mathrm{End}_{\mathcal{C}_Q^m}(T)$ is an m-cluster tilted algebra of type Δ_Q. When Δ_Q is of type $\mathbb{A}$ or $\tilde{\mathbb{A}}$, the m-cluster categories and their corresponding m-cluster tilting objects were realized geometrically and studied in [5, 26], and [19, 28], respectively. These geometric realizations generalize those from [3, 8, 12] in the case of the unpunctured disc and the annulus.

The case $\mathbb{A}$: Let Π be a disk with $nm+2$ marked points (or equivalently a $nm+2$-gon). A *m-diagonal* is a diagonal dividing Π into an $(mj+2)$-gon and an $(m(n-j)+2)$-gon for some $1 \leqslant j \leqslant n-1/2$. A $(m+2)$-angulation is a collection of non-intersecting m-diagonals that form a partition of Π into $(m+2)$-gons. There are bijections:

m-diagonals in Π	$\leftrightarrow$	indecomposable objects in $\mathcal{C}_{\mathbb{A}}^m$
$(m+2)$-angulations of Π	$\leftrightarrow$	m-cluster tilting objects in $\mathcal{C}_{\mathbb{A}}^m$

The case $\tilde{\mathbb{A}}$: While in [19,28] the authors use an annulus with marked points, we will use the universal cover given by the strip Σ, having copies of the mp, and mq, marked points on the boundary components that we denote $\mathcal{B}_p$, and $\mathcal{B}_q$, respectively. The points having the same label in a fixed boundary are considered up to equivalence given by congruence modulo mp and mq. There are (isotopy classes of) arcs on Σ, called *m-diagonals*. Each m-diagonal belongs to a family:

* Transjective: an arc α having an endpoint x in $\mathcal{B}_p$ and the other endpoint y in $\mathcal{B}_q$. The labels in x and y are congruent modulo m. (See Figure 2 - right)
* Regular on a p-tube: an arc α having both endpoints over $\mathcal{B}_p$, starting at u going in positive direction counting $u+km+1$ steps, with $k \geqslant 1$.
* Regular on a q-tube: analogous to the previous case.

As in the previous case, there are bijections

m-diagonals in Σ	$\leftrightarrow$	indecomposable rigid objects in $\mathcal{C}_{\tilde{\mathbb{A}}}^m$
$(m+2)$-angulations of Σ	$\leftrightarrow$	m-cluster tilting objects in $\mathcal{C}_{\tilde{\mathbb{A}}}^m$

Given a $(m+2)$-angulation $\mathcal{T}$ of Π or Σ, the bound quiver $(Q_{\mathcal{T}}, I_{\mathcal{T}})$ of the m-cluster tilted algebra $\Lambda_{\mathcal{T}}$ defined by the associated m-cluster tilting object is obtained form the geometric configuration, see [19, 26]. We recall this bound quiver construction in the following example.

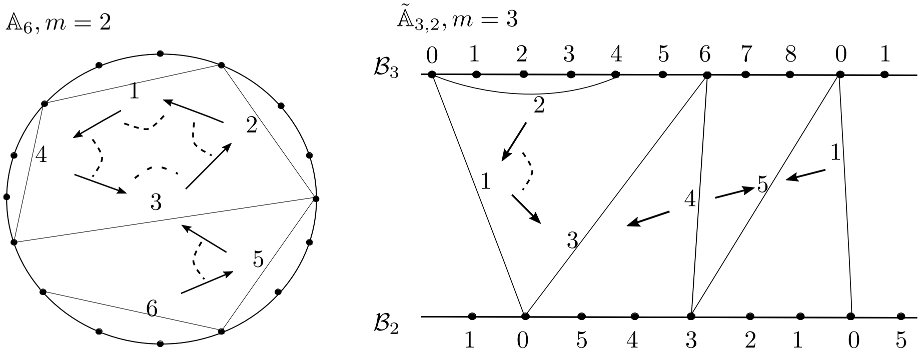


FIGURE 2. 4-angulation of Π (left) and 5-angulation of Σ (right), and bound quivers defined by them. The labels at the endpoints of transjective arcs are congruent modulo 3. The arrows defining $I_{\mathcal{T}}$ are connected by a dotted arc.

Example 1.11. In Figure 2, we show the bound quiver defined by a 4-angulation of Π (left) that corresponds to a 2-cluster tilting object in $\mathcal{C}^m_{\mathbb{A}_6}$, and a 5-angulation of Σ (right) that corresponds to a 3-cluster tilting object in $\mathcal{C}^m_{\tilde{\mathbb{A}}}$ where $p = 3$ and $q = 2$. In both cases, the vertices in $Q_{\mathcal{T}}$ are in one-to-one correspondence with the elements in $\mathcal{T}$. For any two vertices $i, j \in Q_{\mathcal{T}}$, there is an arrow $i \to j$ when the corresponding m-diagonals x_i and x_j share a vertex, they are edges of the same $(m+2)$-gon and x_i follows x_j clockwise. Given consecutive arrows $i \xrightarrow{\alpha} j \xrightarrow{\beta} k$, then $\alpha\beta \in I_{\mathcal{T}}$ if and only if x_i, x_j and x_k are edges in the same $(m+2)$-gon.

2. Gentle 2-CY tilted algebras

Throughout this section, we work with gentle algebras $\Lambda = kQ/I$. First we gather information about the zero-relations and saturated cycles in (Q, I) in order to find all the possible configurations for a 2-CY tilted algebra. After that, we will consider blocks, as it was done in [15], to describe all the possible bound quivers that can define a 2-CY tilted algebra.

Lemma 2.1. Let $\Lambda = kQ/I$ be a gentle algebra of Gorenstein dimension at most one, and such that $\Omega^2\tau M \simeq M$ for all M in $\underline{\mathrm{CM}}(\Lambda)$. Then,

1) each zero-relation lies in a saturated cycle.
2) all saturated cycles have length three or are saturated loops.

Proof. (1) Let uv be a zero-relation that is not part of a saturated cycle. If u is a gentle arrow, then uv is a critical path of length two, so $n(\Lambda) \geqslant 2$

and Λ is Gorenstein of dimension at least two. Absurd, by Proposition 1.8. If the arrow u is not gentle, the there is an arrow u_1 such that u_1u and uv are zero-relations. Since uv is not part of a saturated cycle there is a maximal critical path $u_t \dots u_1$ such that $u_t \dots u_1uv$ is a critical path, because Q is finite and none of these arrows can be in a saturated cycle. Then we have $n(\Lambda) \geqslant 2$, again it is not possible by Proposition 1.8. Thus uv must lay in a saturated cycle.
(2) Let $x_1 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_{n-1}} x_n \xrightarrow{\alpha_n} x_1$ be a saturated cycle. The non-zero paths starting and ending at the vertices x_i determine the indecomposable projective and injective modules $P(x_i) = M(u_i^{-1}\alpha_i u_{i+1})$ and $I(x_i) = M(v_{i-1}\alpha_{i-1}v_i^{-1})$. See Figure 1. By Theorem 1.6, $M(u_i)$ is in $\underline{\mathrm{CM}}$. By Theorem 1.9, we have an isomorphism $\Omega^2\tau M(u_i) = M(u_i)$. We start computing $\tau M(u_i)$, first we need a minimal projective presentation.

$$M(u_{i+1}^{-1}\alpha_{i+1}u_{i+2}) \xrightarrow{\quad f \quad} M(u_i^{-1}\alpha_i u_{i+1}) \to M(u_i) \to 0.$$
$$M(u_{i+2}) \hookrightarrow M(u_{i+1}^{-1}\alpha_{i+1}u_{i+2}) \twoheadrightarrow M(u_{i+1}) \hookrightarrow M(u_i^{-1}\alpha_i u_{i+1})$$

We apply the Nakayama functor $\nu = D\mathrm{Hom}_\Lambda(-, \Lambda)$,

$$0 \to M(v_{i+1}) \hookrightarrow M(v_i\alpha_i v_{i+1}^{-1}) \xrightarrow{\quad \nu f \quad} M(v_{i-1}\alpha_{i-1}v_i^{-1}) \twoheadrightarrow M(v_{i-1})$$
$$M(v_i\alpha_i v_{i+1}^{-1}) \to M(v_i) \to M(v_{i-1}\alpha_{i-1}v_i^{-1})$$

the kernel of νf is $\tau M(u_i) = M(v_{i+1})$.

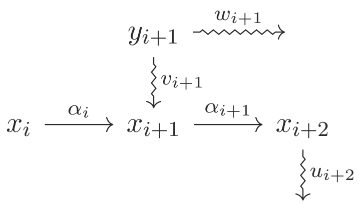
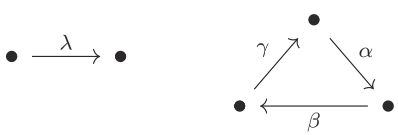


FIGURE 3. Local information needed to compute the projective cover of $M(v_{i+1})$

Now we compute $\Omega M(v_{i+1})$. Let $S(y_{i+1}) = \mathrm{top}\, M(v_{i+1})$. The projective cover of $M(v_{i+1})$ is $P(y_{i+1}) = M(w_{i+1}^{-1}v_{i+1}\alpha_{i+2}u_{i+2})$. Then, $\Omega M(v_{i+1}) = M(w'_{i+1}) \oplus M(u_{i+2})$, where $M(w'_{i+1})$ is the maximal sumbmodule of the uniserial $M(w_{i+1})$ and is at the same time a direct summand of $\mathrm{rad}\, P(y_{i+1})$.

Therefore $\Omega^2\tau M(u_i) = \Omega(M(w'_{i+1}) \oplus M(u_{i+2})) = \Omega M(w'_{i+1}) \oplus \Omega M(u_{i+2})$. It is easy to see that $\Omega M(u_{i+2}) = M(u_{i+3}) \neq 0$. Therefore, by hypothesis, it must be $\Omega M(w'_{i+1}) = 0$ and $M(u_{i+3}) = M(u_i)$, thus $\operatorname{top} M(u_{i+3}) = S(x_{i+3}) = \operatorname{top} M(u_i) = S(x_i)$, for all $i \in \{1, \ldots, n\}$. The only possible long saturated cycles satisfying this condition are of length three $x_1 \xrightarrow{\alpha_1} x_2 \xrightarrow{\alpha_2} x_3 \xrightarrow{\alpha_3} x_1$ where x_1, x_2, x_3 are different vertices. Indeed, if there were only two different vertices, let $\alpha_1 \circlearrowright \bullet$ be a saturated cycle such that $\alpha_i\alpha_{i+1} \in I$ for i index modulo 3, then by the gentleness $\alpha_1^2 \notin I$ so this will define an infinite dimensional algebra. If the saturated cycle is $\bullet \underset{\alpha_1}{\overset{\alpha_2}{\rightleftarrows}} \bullet$, then the condition $\operatorname{top} M(u_i) = \operatorname{top} M(u_{i+3})$ does not hold. The last possibility is considering loops. Notice that two different loops attached to a vertex $\delta_1 \circlearrowright \bullet \circlearrowleft \delta_2$ would define an infinite dimensional algebra. The last option is using a single loop $\delta \circlearrowright \bullet$ such that $\delta^2 \in I$, besides the 3-cycle with three vertices, this is the only configuration allowing both conditions: $M(u_{i+3}) = M(u_i)$ and Λ is finite dimensional. □

The algebras arising from surface triangulations $\Lambda_{\mathcal{T}} = kQ_{\mathcal{T}}/I_{\mathcal{T}}$ ($m = 1$) were defined in [3]. Following [15, Sec. 13], the quiver $Q_{\mathcal{T}}$ can be constructed matching directed graphs, or *blocks*, of type I (a single arrow), and type II (3-cycle).



In view of Lemma 2.1, in the following subsection we will add a new block $\delta \circlearrowright \bullet$. We call this block *type loop*.

2.1. Block decomposition

Consider the blocks type I, II and loop mentioned above. The blocks contain also the information
(R1) For a block of type II, $\alpha\beta = \beta\gamma = \gamma\alpha = 0$.
(R2) For a block of type loop, $\delta^2 = 0$.
All the vertices in the blocks are *outlet vertices.* A bound quiver (Q, I) is *gentle-block-decomposable* if it can be obtained from a collection of disjoint blocks by the following procedure. Take a partial matching of the combined set of outlets.

1) Matching an outlet to itself or to another outlet from the same block is not allowed.

2) Matching two outlets corresponding to different blocks type loop is not allowed.

Identify (or *glue*) the vertices within each pair of the matching. After the gluing , having a pair of arrows connecting the same pair of vertices but going in opposite directions is not allowed. The next is the main result of this section.

Theorem 2.2. Let $\Lambda = kQ/I$ be a gentle algebra of Gorenstein dimension at most one and such that $\Omega^2\tau M \simeq M$ for all $M \in \underline{\mathrm{CM}}(\Lambda)$. Then, the bound quiver (Q, I) is gentle-block-decomposable.

Proof. By Lemma 2.1, the only zero-relations allowed are $\delta^2 = 0$, when δ is a loop, and those in a saturated 3-cycle. All gentle bound quivers satisfying these conditions can be built matching blocks of type I, II and loop. □

Remark 2.3. The block decomposition for Q may include blocks of type loop, in order to interpret (Q, I) as a Jacobian algebras we also need that $\mathrm{char}\, k \neq 3$. The reason is that we use the quiver $\delta \circlearrowright \bullet$ with potential $W = \delta^3$ and, in this case, the ideal for the Jacobian algebra is generated by $\partial_\delta(\delta^3) = \delta\delta + \delta\delta + \delta\delta = 3\delta^2$.

Corollary 2.4. Let k be algebraically closed and $\mathrm{char}\, k \neq 3$. If kQ/I is a gentle 2-CY tilted algebra, then kQ/I is Jacobian.

Proof. By Proposition 1.8 and Theorem 1.9, kQ/I is Gorenstein dimension at most one and $\Omega^2\tau M \simeq M$ for all $M \in \underline{\mathrm{CM}}(\Lambda)$, so we are under the hypothesis of Theorem 2.2. Then (Q, I) is gentle-block-decomposable. We can express kQ/I as a Jacobian algebra $Jac(Q, W)$. The potential is the sum

$$W = \sum_i \alpha_i\beta_i\gamma_i + \sum_j \delta_j^3,$$

where the index i runs over all the 3-cycles $\alpha_i\beta_i\gamma_i$ and the index j runs over all the loops. Since $\mathrm{char}\, k \neq 3$ the zero-relation $3\delta^2 = 0$ remains, and is equivalent to $\delta^2 = 0$. □

The previous result is written with the restriction k is not of characteristic 3. A hyperpotential on a quiver Q is a collection of elements $(\rho_\alpha)_{\alpha\in Q_1}$ over the complete algebra $R\langle\langle Q\rangle\rangle$ such that for $\alpha\colon i \to j$, ρ_α is a (possibly infinite) linear combination of paths $j \rightsquigarrow i$ and $\sum_{\alpha\in Q_1}[\alpha, \rho_\alpha] = 0$. The *Jacobian algebra of a hyperpotential* is the quotient $R\langle\langle Q\rangle\rangle/\overline{\langle\rho_\alpha\rangle}$, see [23, Proposition 1]. In view of this, if we admit hyperpotentials, the last result extends to algebraically closed fields of any positive characteristic.

Example 2.5. Let Q be the quiver in Figure 4, and consider the potential $W = \delta_1^3 + \sum_{i=1}^{3} \alpha_i \beta_i \gamma_i$. Then $Jac(Q, W)$ is a gentle 2-CY tilted algebra and Q is a matching of two blocks of type I, three blocks of type II and a block of type loop.

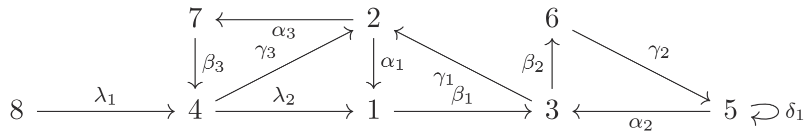


Figure 4. Gentle bound quiver (Q, I), Example 2.5

Corollary 2.6. Let (Q, I) be a bound quiver such that kQ/I is a gentle 2-CY tilted algebra and Q has no loops. Then kQ/I is a Jacobian algebra arising from an unpunctured surface in the sense of [3].

3. Gentle m-cluster tilted algebras

In this section we study gentle algebras $kQ_{\mathcal{T}}/I_{\mathcal{T}}$ arising from $(m+2)$-angulations, $m \geqslant 1$. One can generalize the definition of $(Q_{\mathcal{T}}, I_{\mathcal{T}})$, given in Example 1.11, to $(m+2)$-angulations of unpunctured Riemann surfaces.

The following properties were observed in [26, Rem. 2.18] and [19, Sec. 7] in the case of the disc and annulus, and they can be easily proved in the context of gentle algebras arising from $(m+2)$-angulations.

Proposition 3.1. Let $(Q_{\mathcal{T}}, I_{\mathcal{T}})$ be a bound quiver arising from a $(m+2)$-angulation.

1) $\Lambda_{\mathcal{T}} = kQ_{\mathcal{T}}/I_{\mathcal{T}}$ is a gentle algebra.
2) The only possible saturated cycles in $(Q_{\mathcal{T}}, I_{\mathcal{T}})$ are $(m+2)$-cycles.
3) There can be at most $m-1$ consecutive zero-relations not lying in a saturated cycle.

Immediately, we have the following observation.

Lemma 3.2. Let $\Lambda_{\mathcal{T}} = kQ_{\mathcal{T}}/I_{\mathcal{T}}$ be an algebra arising from a $(m+2)$-angulation. Then, $\Lambda_{\mathcal{T}}$ is Gorenstein of dimension $d \leqslant m$.

Proof. The case $m = 1$ follows from Proposition 1.8, and also from [3, Lemma 2.6]. Let $m \geqslant 2$. Since $\Lambda_{\mathcal{T}}$ is gentle, we can apply Theorem 1.4. First assume that there is no gentle arrow in $(Q_{\mathcal{T}}, I_{\mathcal{T}})$, then $n(\Lambda_{\mathcal{T}}) = 0$, so d is zero or one and $d \leqslant m$. The statement follows.
Now, assume there are gentle arrows in $(Q_{\mathcal{T}}, I_{\mathcal{T}})$, and let α_1 be one of

them. It follows that α_1 is not part of a saturated cycle. Let $\alpha_1 \dots \alpha_r$ be a critical path, since α_1 is not part of a saturated cycle, then none of the arrows α_i for $1 \leqslant i \leqslant r$ is part of a saturated cycle. By Proposition 3.1 (3), the maximal number of consecutive zero-relations outside of a saturated cycle is $m-1$. Therefore, $r \leqslant m$, and by Theorem 1.4, $\Lambda_{\mathcal{T}}$ is Gorenstein of dimension $d \leqslant m$. □

Most of the arguments in the following lemma can be found also in [20, Section 4].

Lemma 3.3. Let $\Lambda = kQ/I$ be a gentle algebra of Gorenstein dimension $d \geqslant 1$. Let $x \in Q_0$, and let N be an indecomposable direct summand of $\operatorname{rad} P(x)$. Then,

(a) $N \in \underline{\mathrm{CM}}(\Lambda)$, or
(b) $\operatorname{proj.dim} N \leqslant d-1$.

Proof. If N is projective, we are in case (b). Let N be non projective. Let $P(x) = M(u^{-1}\alpha^{-1}\beta w)$ be the indecomposable projective and $N = M(u)$ so that $S(t(\alpha)) = \operatorname{top} M(u)$. We study the cases:

(i) α is part of a saturated cycle $x_1 \to \cdots \to x_i \xrightarrow{\alpha} x_{i+1} \cdots \to x_1$.
(ii) α is not part of a saturated cycle.

(i) Let $x_i \xrightarrow{\alpha} x_{i+1}$, then $M(u)$ is a direct summand of $\operatorname{rad} P(x_i)$. By Theorem 1.6, $M(u) \in \underline{\mathrm{CM}}(\Lambda)$.
(ii) Since $N = M(u)$ is not projective, there exists an arrow δ_1 such that $\alpha\delta_1 \in I$. Also δ_1 is not part of a saturated cycle, if it were the case also α would be part of the saturated cycle. Let $P(t(\alpha)) = M(c^{-1}\delta_1^{-1}u)$, then there is an exact sequence

$$0 \to M(c) \to P(t(\alpha)) \to M(u) \to 0. \tag{1}$$

If the string module $M(c)$ is not projective, then it satisfies the same conditions as $M(u)$, so we can construct a new exact sequence

$$0 \to M(c_1) \to P(t(\delta_1)) \to M(c) \to 0. \tag{2}$$

Recursively, we obtain a path $\alpha\delta_1 \cdots \delta_n$ such that each quadratic factor belongs to I. This process has to finish after a finite number of steps, being the direct summand $M(c_n)$ of $P(t(\delta_{n-1}))$ a projective module. If there were not finite steps and $M(c_n)$ was not projective, we would find new arrows $\delta_{n+1}, \dots$ and form a path $\alpha\delta_1 \cdots \delta_n \cdots$ such that each quadratic factor is in I. The quiver Q is finite, so the only way to construct an infinite path $\alpha\delta_1 \cdots \delta_n \cdots$ is reaching a saturated cycle. By the gentleness, if one of the arrows δ_i is in a saturated cycle, then all $\alpha, \delta_1, \ldots, \delta_n$ are in the saturated cycle, this contradicts the condition imposed on α. Therefore the procedure to find the short exact sequences in Equations (1), (2), stops. The short exact sequences are the steps needed to find a minimal projective resolution for $M(u)$, that is finite, so $\text{proj.dim}\, M(u) < \infty$. By Remark 1.5, we have $\text{proj.dim}\, M(u) \leqslant d$. Now, we can also express $M(u)$ as $M(u) = \Omega M(\beta w)$. If we had $\text{proj.dim}\, M(u) = d$, then we would have $\text{proj.dim}\, M(\beta w) = d + 1$ and this is impossible by Remark 1.5. Thus, $\text{proj.dim}\, M(u) \leqslant d - 1$. □

To complete the previous lemma, observe that if Λ is selfinjective (that is Λ is Gorenstein of dimension zero) then every indecomposable module is either projective or $\underline{\text{CM}}$.

The next theorem is the main result of this section.

Theorem 3.4. Let $\Lambda_{\mathcal{T}} = Q_{\mathcal{T}}/I_{\mathcal{T}}$ be an algebra arising from a $(m+2)$-angulation and let N be a $\Lambda_{\mathcal{T}}$-module. Then, $N \in \underline{\text{CM}}(\Lambda_{\mathcal{T}})$ if and only if $\Omega^{m+1}\tau N \simeq N$.

Proof. Let M be an indecomposable module in $\underline{\text{CM}}(\Lambda_{\mathcal{T}})$, by Theorem 1.6, $M = M(u_i)$ where u_i is the maximal non-zero path starting at x_i.

$$\begin{array}{ccccc}
\downarrow v_i & & \downarrow v_{i+1} & & \downarrow v_{i+2} \\
x_i & \xrightarrow{\alpha_i} & x_{i+1} & \xrightarrow{\alpha_{i+1}} & x_{i+2} \\
\downarrow u_i & & \downarrow u_{i+1} & & \downarrow u_{i+2}
\end{array}$$

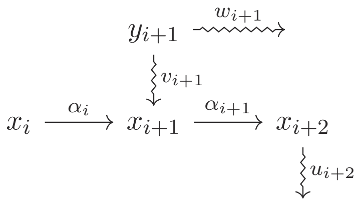
We compute a minimal projective presentation of $M(u_i)$, as we did in Lemma 2.1.

$$\begin{array}{ccccccc}
& M(u_{i+1}^{-1}\alpha_{i+1}u_{i+2}) & \xrightarrow{p_1} & M(u_i^{-1}\alpha_i u_{i+1}) & \to & M(u_i) & \to 0. \\
\nearrow & & \searrow & \nearrow & & & \\
M(u_{i+2}) & & & M(u_{i+1}) & & &
\end{array}$$

Observe that $\Omega^t M(u_i) = M(u_{i+t})$, where t is an integer considered modulo $m+2$. Applying Nakayama functor we get

$$\begin{array}{ccccccc}
0 \to M(v_{i+1}) & \hookrightarrow & M(v_i\alpha_i v_{i+1}^{-1}) & \xrightarrow{\nu p_1} & M(v_{i-1}\alpha_{i-1}v_i^{-1}) & \twoheadrightarrow & M(v_{i-1}). \\
& & \searrow & & \nearrow & & \\
& & & M(v_i) & & &
\end{array}$$

Then, $\tau M(u_i) = \ker \nu p_1 = M(v_{i+1})$. As in Lemma 2.1, let $S(y_{i+1}) = \operatorname{top} M(v_{i+1})$. Let $P(y_{i+1}) = M(w_{i+1}^{-1} v_{i+1} \alpha_{i+1} u_{i+2})$ be the projective cover of $M(v_{i+1})$.



Therefore, $\Omega M(v_{i+1}) = M(w'_{i+1}) \oplus M(u_{i+2})$, where $M(w'_{i+1})$ is the maximal submodule of $M(w_{i+1})$. The syzygy functor is additive, then

$$\Omega^{m+1} \tau M(u_i) = \Omega^{m+1} M(v_{i+1}) = \Omega^m M(w'_{i+1}) \oplus \Omega^m M(u_{i+2}).$$

Since $\Omega^t M(u_i) = M(u_{i+t})$, we have

$$\Omega^m M(w'_{i+1}) \oplus \Omega^m M(u_{i+2}) = \Omega^m M(w'_{i+1}) \oplus M(u_i).$$

Now, we only need to prove that $\Omega^m M(w'_{i+1}) = 0$. Observe that $M(w'_{i+1})$ is a direct summand of $\operatorname{rad} P(y_{i+1})$.

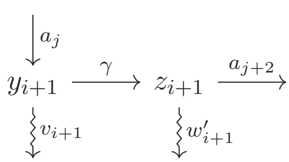
We know, by Lemma 3.2 that $\Lambda_{\mathcal{T}}$ is Gorenstein of dimension $d \leqslant m$. By Lemma 3.3 one of the following holds:

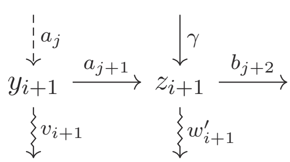
1) $\operatorname{proj.dim} M(w'_{i+1}) \leqslant m-1$, or
2) $M(w'_{i+1}) \in \underline{\mathrm{CM}}(\Lambda_{\mathcal{T}})$.

If (1) holds, then $\Omega^m M(w'_{i+1}) = 0$ and we are done.

We assume (2) holds, so $M(w'_{i+1}) \in \underline{\mathrm{CM}}(\Lambda_{\mathcal{T}})$ and prove that this leads to a contradiction. Let z_{i+1} be the vertex such that $\operatorname{top} M(w'_{i+1}) = S(z_{i+1})$. By the description in Theorem 1.6, the vertex z_{i+1} is a target of an arrow γ in a saturated $(m+2)$-cycle and $\gamma w'_{i+1} \neq 0$.

(2a) If the arrow γ is $y_{i+1} \xrightarrow{\gamma} z_{i+1}$, see the figure bellow (left), then there is an arrow a_j in the saturated $(m+2)$-cycle, such that $a_j \gamma \in I_{\mathcal{T}}$. Then, $a_j v_{i+1} \neq 0$ and this contradicts that $I(x_{i+1}) = M(v_i \alpha_i v_{i+1}^{-1})$ is the indecomposable injective associated to x_i. Absurd.





(2b) If the arrow γ in a saturated cycle is such that $s(\gamma) \neq y_{i+1}$, see figure above (right), there is an arrow b_{j+2} following the saturated cycle such that $\gamma b_{j+2} \in I_{\mathcal{T}}$. Thus, we have $\gamma w'_{i+1} \neq 0$ and by gentleness, $a_{j+1}b_{j+2} \notin I_{\mathcal{T}}$. But recall that w'_{i+1} is a submodule of $\operatorname{rad} P(y_{i+1})$, so b_{j+2} has to be the first arrow in the string w'_{i+1} such that $M(w'_{i+1}) \in \underline{\mathrm{CM}}(\Lambda_{\mathcal{T}})$. Absurd.

Thus, $M(w'_{i+1}) \notin \underline{\mathrm{CM}}(\Lambda_{\mathcal{T}})$ and case (1) is the only possibility so $\Omega^{m+1}\tau M(u_i) = M(u_i)$. As Ω and τ are additive functors, if $N \in \underline{\mathrm{CM}}(\Lambda_{\mathcal{T}})$, then $\Omega^{m+1}\tau N = N$.

The converse affirmation can be proved easily. If $N = \Omega^{m+1}\tau N$, and $N \neq 0$ is not a projective $\Lambda_{\mathcal{T}}$-module, then N is a m-th syzygy. By Remark 1.5 this module is in $\underline{\mathrm{CM}}(\Lambda_{\mathcal{T}})$. □

As a corollary, we obtain the next result that generalizes the properties known for cluster-tilted algebras: Proposition 1.8 and Theorem 1.9.

Theorem 3.5. Let Λ be an m-cluster tilted algebra of type $\mathbb{A}$ or $\tilde{\mathbb{A}}$. Then,

1) Λ is Gorenstein of dimension $d \leqslant m$.
2) $N \in \underline{\mathrm{CM}}(\Lambda)$ if and only if $\Omega^{m+1}\tau N = N$.

Proof. Part (1) follows from Lemma 3.2. Part (2) follows from Theorem 3.4. □

3.1. On the Gorenstein property

It is known that Theorem 3.5 does not hold in general for d-CY tilted algebras. In [21, Section 5.3] there is an example (due to Iyama) of a d-CY tilted algebra that is not Gorenstein. Moreover, a recent preprint [24] says that all finite dimensional k-algebras are d-CY tilted for some $d > 2$.

Still, there are results in this subject due to Keller and Reiten [22, Section 4.6], and Beligiannis [7, Theorem 6.4]. Both results ask $\operatorname{add} T$ to be *corigid* in some degree, that is there exist a non negative integer u such that $\operatorname{Hom}_{\mathcal{C}}(\operatorname{add} T, \operatorname{add} T[-t]) = 0$ for all $1 \leqslant t \leqslant u$, to conclude that $\operatorname{End}_{\mathcal{C}}(T)$ is Gorenstein.

The m-cluster categories $\mathcal{C}^m_Q$ of types $\mathbb{A}$ and $\tilde{\mathbb{A}}$ are special cases of triangulated $(m+1)$-CY categories and the subcategories $\operatorname{add} T$ are $(m+1)$-cluster tilting subcategories, as we pointed out in Remark 1.10. It is easy to see that the subcategory $\operatorname{add} T$ might not be corigid already in some examples of Dynkin type, hence this result is independent of the mentioned above.

Example 3.6. Let $\mathcal{C}_Q^2$ be the 2-cluster category, where Q is of type $\mathbb{A}_4$ and T is in Figure 5.

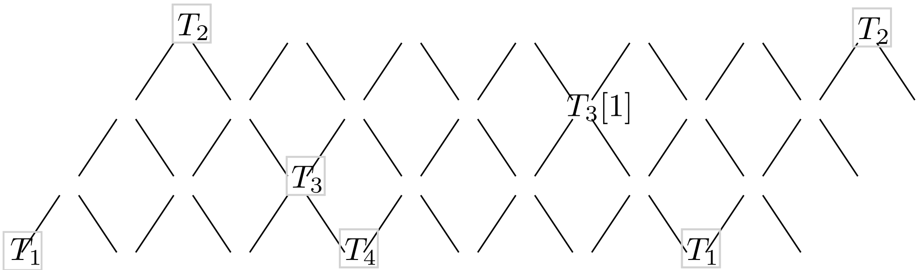


FIGURE 5. 2-cluster tilting object in $\mathcal{C}_Q^2$.

The subcategory add$T$ in Example 3.6 is not corigid since $\mathrm{Hom}(T_3, T_1[-1]) \simeq \mathrm{Hom}(T_3[1], T_1) \neq 0$. By Theorem 3.5 the 2-cluster tilted algebra $\mathrm{End}(T)$ is Gorenstein of dimension at most two. In fact, in this example the algebra is of global dimension two, given by the quiver bellow and bounded by $\beta\alpha = 0$.

$$1 \xleftarrow[\alpha]{} 2 \xleftarrow[\beta]{} 3 \longleftarrow 4$$

Remark 3.7. In a forthcoming paper [16] the Corollary 1 of Section 1 is generalized to monomial algebras, a family that contains gentle algebras, but using more recent results. Moreover, the computations in the last section are revisited and a new approach is proposed in [16, Appendix].

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