ENERGY DENSITY OF THE THERMAL LOADING AND THERMAL STABILITY OF A REFRACTORY

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Design criteria for thermal stability, which include certain physicomechanical material properties [1], do not at all satisfy practical requirements in evaluating stability and monitoring the production of refractories, especially commercial ones. The basis of experimental methods frequently ensues, means for processing experimental results are determined, and the essence of the thermal-loading process is ascertained, however, from analysis of the criteria [2-6].

The processing of experimental data on thermal stability usually proceeds from a difference in temperatures. In this case, the capacity of the material to resist nonstationary heating by a certain thermal flux, or heat exchange with a gaseous medium under prescribed conditions, is considered inadequate. These loading conditions are encountered in practice in many installations (the hood of heat exchangers and reactors, the lining of flame and electric furnaces, etc.). Here, thermal failure should be guarded against in transitional processes, and not in the stage of steady operating conditions. Therefore, a second familiar criterion R' [1] and an expression for experimental results in the form of the failing stationary thermal flux [4], which is based on this criterion, also yield a highly approximate estimate of thermal stability.

The present study is devoted to methods of evaluating the thermal stability of a material as it applies to conditions of nonstationary loading under transitional operating regimes of refractories.

Let us examine the symmetric nonstationary heating of a long strip along the narrow faces when the lateral and end surfaces are insulated. The tensile stresses on the longitudinal axis will be [7]:

$$\sigma = \alpha E (T_{av} - T_{min}), \tag{1}$$

where α is the coefficient of linear expansion, E is the elastic modulus, T_{av} is the average integral temperature, and T_{min} is the temperature on the longitudinal axis (the lowest temperature in the volume).

From the condition of heat balance

$$Q = 4dL \int_{-b}^{b} \rho c (T - T_{in})\, dy \tag{2}$$

(Q, amount of heat supplied; L, half the length of the strip; d, half the thickness of the strip; b, half the width; ρ, density; c, specific heat capacity; T, current temperature; y, transverse coordinate as read from the longitudinal axis; and T_{in}, initial temperature), we obtain an expression for the average integral temperature:

$$T_{av} = \frac{1}{2b} \int_{-b}^{b} T dy = T_{in} + \frac{Q}{v \rho c}, \tag{3}$$

where U = 8dLb is the volume of the strip.

The temperature on the longitudinal axis remains virtually at the initial temperature for a certain time. For Fo < 0.1 and a constant heating output, heating in the core of the strip does not exceed 2% of the edge heating (Fo = $a t/b^2$ is the Fourier number, a is the temperature-conductivity coefficient, and t is the time of the thermal loading).

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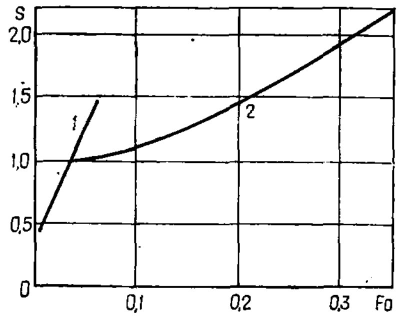

Fig. 1. Relationship between regime factor and Fourier number for one-sided (1) and two-sided (2) heating.

In the initial period ($T_{min} \approx T_{in}$), it is easy to obtain an expression for the failing quantity of heat supplied on the basis of (1) and (3):

$$Q_f = \frac{v\sigma_u \rho c}{\alpha E} \text{ for } T_{min} \approx T_{in} . \tag{4}$$

As is apparent, this quantity is proportional to the volume and to a certain factor describing only the material's properties. Let us call this factor the material's thermal-stability criterion:

$$R_q = \frac{q_u \rho c}{\alpha E} . \tag{5}$$

Three of the parameters entering into this factor are characteristic for the majority of thermal-stability criteria. The product ρc, which signifies the bulk heat capacity, is new.

The appearance of this product in the numerator is somewhat perplexing, since there are many contradictory data on the influence exerted by density on thermal stability. The reason for this should be sought, however, not in the variation of the density itself, but in the effect of the shape and quantity of pores on the strength, elasticity, thermal conductivity, surface energy, etc. For example, the density of yttrium oxide is reduced by a total of 26-30%, and its elastic modulus by a factor of 4-8, as its open porosity [3] varies from 0.1-0.3% to 28-30%. Whereas it is purely theoretical to assume the possibility of variation in the product ρc with the remaining material parameters constant, however, an increase in the bulk heat capacity diminishes the heating rate somewhat at all points in the body when boundary conditions are assigned to the thermal fluxes. In this connection, the appearance of the product ρc in the numerator can be considered completely logical, although this also contradicts the familiar criterion R'' [1].

Note that if we consider the slow and uniform cooling of an affixed rod, we obtain an expression in agreement with (4) for the amount of heat supplied to attainment of the ultimate strength.

Disregarding the quantity v in the left side of equality (4), we obtain an expression for determination of the criterion R_q during experimental specimen failure:

$$R_q = \frac{Q_f}{v} \text{ for } T_{min} \approx T_{in} . \tag{6}$$

It follows henceforth that in the physical sense, the criterion corresponds to the density of thermal energy absorbed during heating to failure; this density is the average throughout the volume of the specimen.

With a more prolonged thermal loading, T_{min} also begins to increase. For this case, we obtain

$$R_q = \frac{Q_f}{v} - \rho c (T_{min} - T_{in}) \tag{7}$$

from (1), (3), and (5). For the case in question, the criterion corresponds to the failing difference between the average energy density in the volume of the specimen and the energy density at the least heated point. The effect of the latter can be accounted for by introducing the time-dependent dimensionless regime factor S:

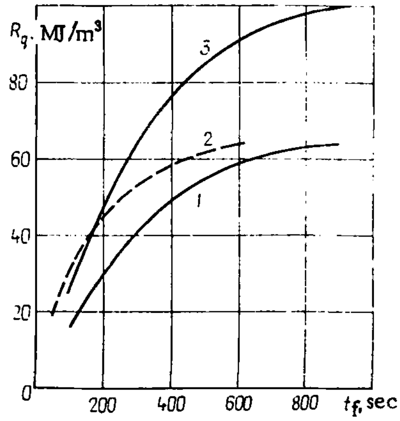

Fig. 2. Comparison of thermal failure of specimen under one-sided (curve 2 — $q = 10^4$ W/m², $T_{max} - T_{in} = 844$°K) and two-sided (curve 1 — $2q = 10^4$ W/m², $T_{max} - T_{in} = 542$°K; curve 3 — $2q = 15{,}590$ W/m², $T_{max} - T_{in} = 844$°K) heatings.

$$R_q = \frac{Q_f}{vS}. \tag{8}$$

With this approach, Eq. (8) is also suitable for specimens of another shape or another method of thermal loading.

If we conduct two-sided heating of a long strip with a constant density of the thermal flux q, we obtain

$$R_q = \frac{qt_f}{bS}, \tag{9}$$

considering $Q_f = 8qt_fLd$, where t_f is the time required for thermal failure.

We analyzed the variation of S with time (Fig. 1) and derived an approximate equation for its calculation with an error of up to 1% for $0.04 \leqslant \text{Fo} \leqslant 0.35$:

$$S = 3.027 - 2.04 \exp(-9.26\,\text{Fo}^{2.25}). \tag{10}$$

These values of the Fourier number are selected proceeding from methodological expediency. The lower limit is obtained from the condition that the compressive stresses at the moment of failure did not exceed five times the tensile stresses. When $\text{Fo} > 0.35$, further heating only negligibly increases the temperature stresses (by not more than 5% when $\text{Fo} \to \infty$).

Up till now, we have examined only two-sided heating. One-sided heating is more convenient, however, in methodological respects, although the maximum value of the thermal stresses is somewhat smaller here. Equation (7) is unsuitable in this case; use of the regime factor (Fig. 1), however, dictates the applicability of Eq. (8); in this case, it is also possible to use the approximate equation proposed below (an error of up to 1% when $0.005 \leqslant \text{Fo} \leqslant 0.06$):

$$S = 0.342 + 18.87\,\text{Fo}. \tag{11}$$

For the case in question, the Fourier number differs from that used earlier, since the entire width of the specimen, and not half the width, is designated by the symbol b during one-sided heating. For the same reasons as in the case of two-sided heating, it is methodologically expedient to select that heating intensity for which thermal failure sets in at $0.005 \leqslant \text{Fo} \leqslant 0.06$.

Comparison of one- and two-sided heating in real time (Fig. 2) suggests high savings generated by the former at a given output. Under the condition of a restriction placed on the maximum temperature T_{max} without restrictions placed on the output of two-sided heating, it is possible to ensure the failure of more thermally stable materials than with one-sided heating (curves 1 and 3). Computations were performed for a lightweight material with the parameters $a = 4 \cdot 10^{-7}$ m²/sec and $\rho c = 5 \cdot 10^5$ J/(m³·°K).

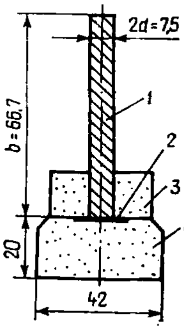

Fig. 3. Diagram showing heating of specimen and heater insulation.

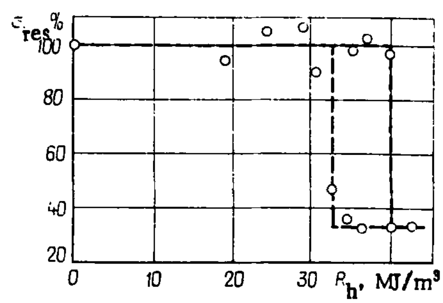

Fig. 4. Dependence of residual strength of ultraporous fireclay specimens on magnitude of preliminary thermal loading.

Equation (1) is also valid for the maximum tensile tangential stresses on heating inside a thin ring, if the temperature of the external surface is adopted as T_{min}.

Omitting the proof, let us present only the final expression for the criterion R_q, which can be computed in processing the results of determination of the thermal stability of annular specimens with an insulated outer surface:

$$R_q = \frac{Pt_f}{\pi (r_o^2 - r_i^2)} - \rho c (T_{min} - T_{in}), \tag{12}$$

where P is the output of the heater per unit of length, and r_o and r_i are the outside and inside radii of the specimen.

Methodologically, it is simpler to determine this criterion than, for example, the failing temperature difference, since temperature measurement of the heated inner surface of annular specimens is obviated. The temperature of the outer surface, however, remains at the initial level for a certain time, and then rises at a low rate to a low level with a small gradient. Regulation of heater output is obviated; this is done with the desire to maintain a given heating rate [3]. It is sufficient to measure the current and voltage drop across an effective section of the heater, and record the time of failure and the temperature of the outer surface at that time. If the heater output varies significantly for any reasons during heating, it is sufficient to substitute the energy consumption per unit of heater length, which can be determined using, for example, an electric metering device, for the product Pt_f. Use of ballast rings and shields between specimens virtually excludes longitudinal heat loss. Virtually all energy liberated by the heater will therefore move in the radial direction.

The matter is more complex with the heating of a strip or plate. Here, only a portion of the energy goes into heating the specimen, while a significant amount of it penetrates through the insulation into the surrounding medium. In this case, however, it is also possible to use an equation similar to (9):

$$R_q = \frac{q_h t_f}{b S_*}, \tag{13}$$

where q_h is the average density of the thermal flux on the surface of the heater, and S_* is an empirical regime factor that takes into account the quality of the heater's insulation, the extent to which the coldest zone is heated, the ratio of plate length to width, and the form of heating (one-sided or two-sided).

The time dependence of S_* can be obtained on one specimen, and used later on for an entire series of tests of materials that are similar in nature. Remember that the finiteness of the length can be accounted for using the relationship [6]

$$S_* = \frac{S_l}{1 - \exp(-l^{2.46}/1.44)}, \tag{14}$$

where S_l is the regime factor for a long strip, and l is the ratio of the plate's length to its width.

As an example, let us examine the results of determination of the thermal stability of the ultraporous fireclay refractory ShTL with a density of 526 kg/m³. Specimens in the form of a rectangular plate with the dimensions 7.5 × 66.7 × 116 mm were cut from bricks. The initial bending strength was determined on three specimens. Subsequent specimens were subjected to thermal loading for different periods t_h, the magnitude of the thermal load R_h computed from Eq. (13) by substituting t_h for t_f, and the residual bending strength then determined.

One-sided heating of specimen 1 (Fig. 3) was carried out by passing a 48-A current through 0.2 × 17.8-mm Nichrome strip 2, which was protected from the medium by insulated blocks 3 and base 4, which were fashioned from the same refractory as the specimen. At this current, q_h = 19,775 W/m².

Analysis of the temperature field in the specimen indicated that the computed density of the thermal flux on the surface of the specimen was 2.7% lower than the average density of the thermal flux on the surface of the heater. This coincidence can be explained by the fact that the specimen material was used as heater insulation. The t_h = 7449Fo relationship was established empirically. Considering that $q_h = 1.027q$, the relationship between the empirical regime factor and the loading time is obtained from Eqs. (11) and (14):

$$S_* = \frac{0.351 + t_h/86200b^2}{1 - \exp(-l^{2.46}/1.44)}. \tag{15}$$

It was established as a result of the tests that the material virtually does not soften for R_h to 32.8 MJ/m³ (Fig. 4). A thermal crack causing a strength reduction of up to 34% of the initial strength on the average appeared in the specimen in the stress range from 32.8 to 39.6 MJ/m³. Some of the specimens retained the initial strength in this stress range owing to neutral variation of properties. Let us adopt the average value for the range — 36.2 MJ/m³ — as the criterion R_q. An estimate of the coefficient of variation — 7.5% — can be obtained on the basis of the range of variation.

For the example in question, all specimens had the same dimensions and were subjected to heating with the same output. Equations (13) and (15) could also be used, however, in cases where the dimensions of the specimens and the current in the heater are varied in each individual test. This also results in superiority of the proposed criterion.

Since density is one of the most widely used characteristics of a refractory, and the specific heat capacity of the materials is slightly dependent on the structure and porosity [8], the criterion R_c [6], which corresponds to the failing temperature difference, where temperature deformations are restrained along one axis, is readily computed on the basis of results of these tests:

$$R_c = \frac{R_q}{\rho c}. \tag{16}$$

For the example in question, we obtain R_c = 72.5°K using c = 950 J/(kg·°K) [8].

Thus, it is shown that the failing energy density of a thermal loading, which is averaged throughout the volume of a solid, depends only on the properties of the material in certain special cases. A new criterion for the thermal stability of a material is obtained, and an appropriate method of testing refractories for thermal stability during the nonstationary heating of specimens in the form of plates or rings is also proposed on this basis. The method consists in determination of the amount of heat absorbed by the specimen and its division by the volume of the specimen and the regime factor.

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STRESS—STRAIN AND LIMITING STATE OF SPOKED FLYWHEELS FORMED FROM COMPOSITE MATERIALS.

REPORT 1. COMPUTATIONAL RELATIONSHIPS

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Computation of the stress-strain state of a flywheel built from composite materials in the form of an energy-storing element — a rim connected with spokes using a chorded star-shaped polygon (Fig. 1) — is performed in a rather complete statement with allowance for the interaction between spokes at the points of their intersection. In addition to this, the effect of this interaction on the adequacy of the solution, which has been found to be rather cumbersome, has not been evaluated.

The purpose of the present study is to obtain a rather simple solution for this problem which is convenient for practical engineering computations, and to evaluate its discrepancy with the accurate [1] solution, and also its correspondence with experimental data on the limiting bearing capacity for two flywheel models of various composite materials.

We computed the stress—strain state of a flywheel with consideration given to the joint deformation of the rim and spokes. In this stage of the investigation, it was proposed that stress and strains are caused by short-term loading, and the behavior of the material is elastic, and conforms to Hooke's law. The shape that the rim assumes and other parameters are determined by the plane axisymmetric character of the loading, and also by the axisymmetric anisotropy, or quasi-isotropy of the material used. It was therefore assumed that the flywheel has a plane median surface to which the load is applied. In this case, the stressed state was considered plane. The loading consisted of a mass force caused by rotation, and uniformly distributed radial boundary forces.

1. The equation of equilibrium for an elementary volume bounded by two radial sections and two concentric circles placed at a distance r from the disk's axis of rotation assumes the form (Fig. 2)

$$\frac{d}{dr}(r\sigma_r) - \sigma_\varphi + P_r = 0, \tag{1}$$

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