TRANSITION RADIATION OF THE CHARGED PARTICLE IN THE INHOMOGENEOUS PLASMA WITH THE LONGITUDINAL MAGNETIC FIELD

I.O. Anisimov[1], Yu.N. Borokh[2]

Taras Shevchenko National University of Kiev, Radio Physics Faculty, Kiev, Ukraine
E-mail: [1]ioa@univ.kiev.ua; [2]y.borokh@gmail.com

Regions of the most effective transformation of current wave into electromagnetic wave for the charged particle moving along the density gradient in magnetic field, which is parallel to the plasma density gradient, are found. Magnitude of transition radiation of the extraordinary wave is calculated.

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1. INTRODUCTION

Transition radiation in plasma attracts interest because of its possible applications (usage of modulated electron beams as radioemitters in ionosphere [1], transillumination of the dense plasma barriers via electron beams [2], diagnostics of the inhomogeneous plasma using transition radiation of electron bunches [3] etc). But this fundamental problem was not yet solved even for the simplest model of cold planarly-stratified plasma with magnetic field parallel to its density gradient [4]. In this work linear transformation of the given current waves into electromagnetic waves for such model was studied.

2. MODEL DESCRIPTION AND WAVE EQUATION FOR VECTOR-POTENTIAL

Cold collisionless plasma is considered. Its density gradient is constant and parallel to the z-axis. Charged particle is moving along the density gradient. Magnetic field parallel to the density gradient is applied to the system, so plasma permittivity tensor has a form [5]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp(z) & -i\alpha(z) & 0 \\ i\alpha(z) & \varepsilon_\perp(z) & 0 \\ 0 & 0 & \varepsilon_{//}(z) \end{pmatrix}, \tag{1}$$

where

$$\begin{aligned} \varepsilon_\perp(z) &= 1 - \frac{\Omega_p{}^2}{1^2 - \Omega_c{}^2}; \\ \varepsilon_{II}(z) &= 1 - \Omega_p{}^2; \\ \alpha(z) &= \frac{\Omega_c \Omega_p{}^2}{1 - \Omega_c{}^2}. \end{aligned} \tag{2}$$

Charged particle moves along z-axis with the velocity v_0. Ampere's circuital law with Maxwell's correction is taken as a basic equation to find the field caused by this particle:

$$rot\vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{j}, \tag{3}$$

here j is a current density

$$\vec{j} = \vec{e}_z e v_0 \delta(x)\delta(y)\delta(z - v_0 t), \tag{4}$$

and e is the particle charge.

Electric and magnetic field intensities can be expressed in the terms of scalar and vector potentials

$$\begin{aligned} \vec{H} &= rot\vec{A}, \\ \vec{E} &= -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla\phi. \end{aligned} \tag{5}$$

Scalar potential vanishing calibration is used, so the following equation for vector-potential is obtained:

$$rotrot\vec{A} + \frac{\hat{\varepsilon}}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{4\pi}{c}\vec{j}. \tag{6}$$

3. SCALAR EQUATION FOR VECTOR-POTENTIAL COMPONENT

Equation (6) should be expanded in a Fourier integral by time and transversal coordinates for the further solving. Current density can be written as a $j_m\exp[i(\omega t - \kappa_\perp y - \kappa_{//} z)]$, $\kappa_{//} = \omega/v_0$. Solution should be searched in a form of

$$\vec{A}(\vec{r},t) = \vec{A}(z)\exp\left[i(\omega t - \kappa_\perp y)\right]. \tag{7}$$

After such substitutions equation (7) can simply be rewritten as a set of scalar equations. Since there are three scalar equations and three unknown functions, equation for one of the vector potential components is obtained

$$k_4'(z)\frac{d^4 A_x}{dz^4} + k_2'(z)\frac{d^2 A_x}{dz^2} + k_0'(z)A_x = k_l'(z), \tag{8}$$

where

$$\begin{aligned} &k_4'(z) = \varepsilon_{//}(z); \\ &k_2'(z) = 2k_0{}^2\varepsilon_\perp(z)\varepsilon_{//}(z) - \kappa_\perp{}^2\left[\varepsilon_{//}(z) + \varepsilon_\perp(z)\right]; \\ &k_0'(z) = \left[\kappa_\perp{}^2 - k_0{}^2\varepsilon_{//}(z)\right]\times \\ &\times\left[k_0{}^2\alpha^2(z) - \kappa_\perp{}^2\varepsilon_\perp{}^2(z) + \varepsilon_\perp(z)\kappa_\perp{}^2\right]; \\ &k_l'(z) = i\frac{4\pi}{c}\kappa_{//}\kappa_\perp\alpha(z) j_m \exp(-i\kappa_{//} z). \end{aligned} \tag{9}$$

4. METHOD OF GEOMETRICAL OPTICS

Equation (8) can be solved using method proposed in [6] for analysis of the distributed reflection. Characteristic length of the plasma inhomogeneity is the large parameter. Then solution of (8) can be given in a form

$$A_x(z) = A(z)\exp[i\varphi(z)], \tag{10}$$

where A is an amplitude, and φ is an eikonal function. I-th derivative of $A(z)$ has i-th order of vanishing and i-th derivative of $\varphi(z)$ has $(i-1)$-th order of vanishing.

After substitution of (10) to (8) and neglecting of the higher orders of vanishing summands general solution of the homogeneous equation (8) may be found:

$$\begin{aligned} A(z) &= \exp[I_1]\left(A_1^+ \exp\left[i\int_{z_0}^{z} \sqrt{K_1(z')}dz' \right] + \right. \\ &\left. + A_1^- \exp\left[-i\int_{z_0}^{z} \sqrt{K_1(z')}dz' \right] \right) + \\ &+ \exp[I_2]\left(A_2^+ \exp\left[i\int_{z_0}^{z} \sqrt{K_2(z')}dz' \right] + \right. \\ &\left. + A_2^- \exp\left[-i\int_{z_0}^{z} \sqrt{K_2(z')}dz' \right] \right), \end{aligned} \tag{11}$$

where

$$\begin{aligned} K_{1,2}(z) &= \frac{k_2'(z) \pm \sqrt{k_2'(z)^2 - 4k_0'(z)k_4'(z)}}{2k_4'(z)}; \\ I_{1,2} &= \int \frac{\left(6k_4'(z)K_{1,2}(z) - k_2'(z)\right)}{\left(2k_2'(z) - 4k_4'(z)K_{1,2}(z)\right)} \frac{K_{1,2}{}'(z)}{K_{1,2}(z)}dz. \end{aligned} \tag{12}$$

5. METHOD OF THE CONSTANTS VARIATION AND MUTUAL TRANSFORMATION OF WAVES

To obtain the general solution of inhomogeneous equation (8) dependencies on z-coordinate should be implemented for amplitudes $A_{1,2}^{\pm}$ and then constants' variation method should be used. As a result we obtain a set of four equations for $A_{1,2}^{\pm}$ amplitudes and their derivatives. From this set derivatives of amplitudes can be expressed. Expressions for the derivatives contain summands with $\exp\left[i\left(\pm\tilde{K}_1 \pm \tilde{K}_2\right)\right]$, where

$$\tilde{K}_{1,2}(z) = \int_{z_0}^{z} \sqrt{K_{1,2}(z')}dz', \tag{13}$$

that describe mutual transformation of electromagnetic waves, and summands with $\exp\left[i\left(\pm\tilde{K}_1 - \kappa_z z\right)\right]$, that describe transformation of the current wave into electromagnetic waves, i.e. transition radiation. To estimate the amplitude of this radiation, these summands should be integrated. Integrals have a form

$$A_{1,2}^{\pm} = \mp\int_{-\infty}^{+\infty} \exp\left[-i\left(\kappa_z z \pm \tilde{K}_{1,2}(z)\right) - I_{1,2}\right] \frac{2\pi e v_0 \kappa_z \kappa_y \alpha(z)}{c k_4'(z)\sqrt{K_{1,2}(z)}}dz. \tag{14}$$

6. INTEGRATING

Integrals (14) can be taken using residue method for poles' vicinities and stationary phase method for the vicinities of Cherenkov resonant points. Analysis of (14) shows that only one $\Omega_p^2=1$ pole is of a mathematical interest. For this pole two qualitatively different situations exists: $\Omega_c^2<1$ and $\Omega_c^2>1$.

1) $\Omega_c^2>1$:

For A_2^+ amplitude only $\Omega_p^2=1$ pole makes contribution to the integral:

$$\begin{aligned} A_2^+ &= -\frac{\pi e v_0 \kappa_z \kappa_\perp}{c} \frac{d^2}{d\left(\Omega_p^2\right)^2} \left\{ \frac{\alpha\left(\Omega_p^2\right)}{\sqrt{K_2\left(\Omega_p^2\right)}} \times \right. \\ &\left. \times \exp\left[-i\left(\kappa_z z + \tilde{K}_2\left(\Omega_p^2\right)\right)\right] \right\}\Bigg|_{\Omega_p^2=1}. \end{aligned} \tag{15}$$

For A_2^- amplitude also vicinities of the Cherenkov resonant point make contributions to the integral. Cherenkov resonant points correspond to the roots of equation

$$a\Omega_p^6 + b\Omega_p^4 + c\Omega_p^2 + d = 0, \tag{16}$$

where

$$\begin{aligned} a &= \frac{k_0^4}{\Omega_c^2 - 1}; \\ b &= k_0^2 \frac{k_0^2 - 2\kappa_y^2 - 2\kappa_z^2}{1 - \Omega_c^2}; \\ c &= \Omega_p^2 \kappa_z^2 \left(-\kappa_z^2 + \left(2k_0^2 + \kappa_y^2\right) \frac{2 - \Omega_c^2}{1 - \Omega_c^2} \right) + \\ &+ k_0^2 \left(\kappa_y^2 - k_0^2 + \frac{2\Omega_c^2 k_0^2}{1 - \Omega_c^2} \right); \\ d &= \kappa_y^4 - 2\kappa_y^2\left(k_0^2 + \kappa_y^2\right) - \left(k_0^2 - \kappa_y^2\right)\left(\kappa_y^2 + \frac{2\Omega_c^2 k_0^2}{1 - \Omega_c^2} - k_0^2 \right). \end{aligned} \tag{17}$$

After substitution $W_p=\Omega_p^2+b/3a$ roots of the equation (16) can be found:

$$\left[\begin{aligned} W_{p1} &= \alpha + \beta; \\ W_{p2,3} &= -\frac{\alpha + \beta}{2} \pm i\frac{\alpha - \beta}{2}\sqrt{3}, \end{aligned}\right. \tag{18}$$

where

$$\begin{aligned} \alpha &= \sqrt[3]{-\frac{q}{2} + \sqrt{Q}}, \\ \beta &= \sqrt[3]{-\frac{q}{2} - \sqrt{Q}}, \\ Q &= \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2. \end{aligned} \tag{19}$$

We are interested only in the points that satisfy the following conditions:

$$\begin{cases} \Omega_p^2 \in \mathbb{R}; \\ 2k_4'\left(\Omega_p^2\right)\kappa_z^2 - k_2'\left(\Omega_p^2\right) \geq 0. \end{cases} \tag{20}$$

Then amplitude can be written as

$$\begin{aligned} A_2^- &= \frac{\pi e v_0 \kappa_z \kappa_\perp}{c} \frac{d^2}{d\left(\Omega_p^2\right)^2} \left\{ \frac{\alpha\left(\Omega_p^2\right)}{\sqrt{K_2\left(\Omega_p^2\right)}} \times \right. \\ &\left. \times \exp\left[-i\left(\kappa_z z - \tilde{K}_2\left(\Omega_p^2\right)\right)\right] \right\}\Bigg|_{\Omega_p^2=1} + \\ &+ \sum_l \exp\left[iS_2\left(\Omega_{pl}^2\right)\right] \exp\left[i(\pi/4)\delta_{2l}\right] \times \\ &\times \sqrt{\frac{2\pi}{\left|S_2''\left(\Omega_{pl}^2\right)\right|}} f_2\left(\Omega_{pl}^2\right), \end{aligned} \tag{21}$$

where

$$S_2(\Omega_p) = -(\kappa_z z - \tilde{K}_2(z));$$
$$\delta_2 = \operatorname{sgn} S''(\Omega_p); \quad (22)$$
$$f_2 = \frac{2\pi e v_0 \kappa_z \kappa_y \alpha(z)}{c k_4'(z)\sqrt{K_2(z)}} \exp[-I_2].$$

2) *$\Omega_c^2<1$:*

For the amplitudes A_1^+ only pole $\Omega_p^2=1$ makes contribution to the integral, so

$$A_1^+ = -\frac{\pi e v_0 \kappa_z \kappa_\perp}{c} \frac{d^2}{d(\Omega_p^2)^2} \left\{ \frac{\alpha(\Omega_p^2)}{\sqrt{K_1(\Omega_p^2)}} \times \right. \quad (23)$$
$$\left. \times \exp\left[-i\left(\kappa_z z + K_1(\Omega_p^2)\right)\right] \right\}\Big|_{\Omega_p^2=1}.$$

For A_1^- amplitude contribution from Cherenkov resonant point is the same as in previous case.

7. DISCUSSION

1. For charged particle moving along the plasma density gradient in the magnetic field, parallel to the density gradient, transition radiation is emitted both forward and backward relatively to the particle motion direction, and current wave is transformed into ordinary and extraordinary waves.

2. Transformation of the current wave into extraordinary wave occurs in the vicinities of local plasma resonance point and Cherenkov resonance point.

3. In the case of current wave transformation into ordinary wave the local plasma resonance point is a branch point, and calculation of transition radiation needs more detailed analysis.

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ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ ЗАРЯЖЕННОЙ ЧАСТИЦЫ В НЕОДНОРОДНОЙ ПЛАЗМЕ С ПРОДОЛЬНЫМ МАГНИТНЫМ ПОЛЕМ

И.А. Анисимов, Ю.Н. Борох

Найдены области наиболее эффективной трансформации волны тока в электромагнитные волны для заряда, движущегося вдоль градиента концентрации слабонеоднородной холодной плазмы в магнитном поле, также направленном вдоль градиента концентрации плазмы. Найдена амплитуда переходного излучения необыкновенной волны.

ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ ЗАРЯДЖЕНОЇ ЧАСТИНКИ В НЕОДНОРІДНІЙ ПЛАЗМІ З ПОВЗДОВЖНІМ МАГНІТНИМ ПОЛЕМ

І.О. Анісімов, Ю.Н. Борох

Знайдені області найбільш ефективної трансформації хвилі струму в електромагнітні хвилі для заряду, що рухається вздовж градієнту концентрації слабконеоднорідної холодної плазми в магнітному полі, що також спрямоване вздовж градієнту концентрації плазми. Знайдена амплітуда перехідного випромінювання незвичайної хвилі.