F.A.Aliev[1,2], N.S.Hajieva[1], A.A.Namazov[1], N.A.Safarova[1]

THE IDENTIFICATION PROBLEM FOR DEFINING THE PARAMETERS OF DISCRETE DYNAMIC SYSTEM

[1]*Institute of Applied Mathematics,*
Baku State University, Z. Khalilov str.23, AZ1148, Baku, Azerbaijan;
e-mail: f_aliev@yahoo.com, nazile.m@mail.ru, atif.namazov@gmail.com, narchis2003@yahoo.com.
[2]*Institute of Information Technology of NAS Azerbaijan,*
B. Vahabzade str.9, AZ1141, Baku, Azerbaijan.

Abstract. An identification problem is considered. It allows to determine the parameters of dynamic system in the discrete case. First, the nonlinear discrete equation is linearized by the method of quasi-linearization. Then, the quadratic functional and its gradient are derived using the statistical data. A calculation algorithm is proposed to the solution of problem in hand. It is shown on an example that the statistical value of the coefficient of hydraulic resistance differs from the obtained value on the order 10^{-4}. It shows an adequacy of the mathematical model.

Key words: dynamic system, nonlinear discrete equation, method of quasi-linearization, gradient of functional, identification, statistical data, coefficient of hydraulic resistance.

1. Introduction.

As it is known, the identification problem plays an important role in solutions of the applied problems from physics, hydrodynamic, oil production [1, 6, 7, 10, 13, 14, 16] and etc. There are different methods for solution of such problems. One of these methods is the optimization method. Choosing the corresponding optimized functional plays an important role in this process. Firstly, the motion of the object is described by the nonlinear system of equations in such applied problems, and further choosing such functional and solving the corresponding problem is problematical. One of the ways to overcome these difficulties is using the iterative method of quasilinearization [4, 9, 17]. Note that if the motion of the object is written as the system of fractional-derivative differential equations then the analogical method may be considered [2].

In the paper the identification problem in the discrete case is considered to determine the parameters, of which in the right part of the system of nonlinear equations with the initial and final conditions [12]. The solution of this problem is reduced to the solution of optimization problem. The given system is reduced to the linear system with respect to the phase coordinates and vector parameters with the method of quasilinearization. Using the least quadratic method a quadratic functional is formed and the expression of the functional gradient is derived. Calculating fundamental matrices in the continuous case [5] is a difficult process than in the discrete case. The computational algorithm is proposed for finding the optimal solution that allows one to define the finding parameters. The results are illustrated in a specific practical example.

2. Problem statement.

Let the motion of the object be described by the system of nonlinear discrete equations:

$$y(i+1)=f\left(y(i),\alpha\right),\ i=\overline{0,N-1}, \tag{1}$$

where y is a n-dimensional phase vector, f is a n-dimensional continuously differentiable function in the interval $(0,T)$, α is a finding m-dimensional constant vector, N is a given natural number.

Let the following initial conditions be given

$$y_j(0)=y_{0j},\quad j=\overline{1,M}, \tag{2}$$

where M is a given natural number, y_{0j} is a given n-dimensional vector.

The problem consists of the finding of the vector α such that the solution of the Cauchy problem (1) – (2) satisfies the given condition

$$y_j(N)=y_{Nj},\ j=1,N. \tag{3}$$

In these cases it is required to find the vector α such that the solution of the problem by initial data (2) be maximally close to the measured data at the end points.

The solution of the problem (1) – (3) can be solved with the different numerical methods, for example the method of quasilinearization [9]. So in the first step we linearize the equation (1) for the solution of the problem (1) – (3). Then selecting some nominal trajectory $y^0(i)$ and the parameter α^0, we assume that (*k*-1)-*th* iteration has been already fulfilled. If we linearize the equation (1) relatively these data in the order $o\left(y-y^0,\ \alpha-\alpha^0\right)$

$$\begin{aligned} y^k(i+1)=f\left(y^{k-1}(i),\alpha^{k-1}\right)+\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1}\right)}{\partial y(i)}\left(y^k(i)-y^{k-1}(i)\right)+\\ +\frac{\partial f\left(y^{k-1}(i),\alpha^{i-1}\right)}{\partial\alpha}\left(\alpha^k-\alpha^{k-1}\right). \end{aligned} \tag{4}$$

After some transformations the equation (4) can be reduced to the following form

$$y^k(i+1)=A^{k-1}(i)y^k(i)+B^{k-1}(i)\alpha^k+C^{k-1}(i), \tag{5}$$

where

$$A^{k-1}(i)=\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1}\right)}{\partial y(i)};\quad B^{k-1}(i)=\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1}\right)}{\partial\alpha};$$

$$C^{k-1}(i)=f\left(y^{k-1}(i),\alpha^{k-1}\right)-\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1}\right)}{\partial y(i)}y^{k-1}(i)-\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1}\right)}{\partial\alpha}\alpha^{k-1}.$$

Then $y(N)$ from the equation (5) has the form

$$y^k(N)=\Phi^{k-1}(i)y^k(0)+\Phi_1^{k-1}(i)\alpha^k+\Phi_2^{k-1}(i) \tag{6}$$

and

$$\Phi^{k-1}(i)=\prod_{i=N-1}^{0}A^{k-1}(i);\ \Phi_1^{k-1}(i)=\sum_{p=1}^{N-1}\left(\prod_{i=N-1}^{p}A^{k-1}(i)\right)B^{k-1}(p-1);$$

$$\Phi_2^{k-1}(i)=\sum_{p=1}^{N-2}\left(\prod_{i=N-1}^{p+1}A^{k-1}(i)\right)C^{k-1}(p)+B(N-1)+C(N-1),i=\overline{0,N-1}.$$

To provide that the solution $y_j(N)$ of the linearized differential equation (5) with initial data (2) coincides with the values of the measurements y_{Nj}, we construct the following quadratic functional in the *k- th* iteration

$$I^k = \sum_{s=1}^{n}\left(y_s^k(N) - y_{NS}^k\right)^T A\left(y_S^k(N) - y_{NS}^k\right), \tag{7}$$

where the symbol *T* means the operation of transpore, *A* is a $n \times n$ dimensional constant symmetric weight matrix, that is chosen in each iteration, considering the spesifics of the concrete problem, y_{NS}^k is a $n \times 1$ dimensional vector of observation, $y_s^k(N)$ is a $n \times 1$ dimensional vector defined by (6). Then the solution of the stated problem is reduced to the problem: Find a constant vector α, by which the solution of the equation (1) with initial data (2) minimizes the functional (7).

Then substituting $y^k(N)$ from (6) into (7) we get

$$\begin{aligned} J^k = \sum_{s=1}^{n}\Big(\Phi_s^{k-1}(i)\, y_s^k(0) + \Phi_{1s}^{k-1}(i)\alpha^k + \Phi_{2s}^{k-1}(i) - y_{NS}^k\Big)^T A \times \\ \times\Big(\Phi_s^{k-1}(i)\, y_s^k(0) + \Phi_{1s}^{k-1}(i)\alpha^k + \Phi_{2s}^{k-1}(i) - y_{NS}^k\Big). \end{aligned} \tag{8}$$

The gradient of the functional J^k with respect to the parameter α^k has the form

$$\begin{aligned} \frac{\partial J^k}{\partial \alpha^k} = \sum_{s=1}^{n}\Big(y_s^{kT}(0)\Phi_s^{k-1T}(i) A\Phi_{1s}^{k-1}(i) + \Phi_{1s}^{k-1T}(i) A\Phi_s^{k-1} y_s^k(0) + \\ +\Phi_{1s}^{k-1T}(i) A\Phi_{2s}^{k-1}(i) - \Phi_{1s}^{k-1T}(i)\, y_{NS}^{k\ T} + \Phi_{2s}^{k-1T}(i)\ \Phi_{1s}^{k-1}(i) - \\ - y_{NS}^{k\ T}\Phi_{1s}^{k-1}(i) + 2\Phi_{1s}^{k-1}(i) A\, \Phi_{1s}^{k-1}(i)\alpha^k\Big). \end{aligned} \tag{9}$$

Equating the expression (9) to zero we get

$$\begin{aligned} \sum_{s=1}^{n}\Big(2\Phi_{1s}^{k-1}(i) A\ \Phi_{1s}^{k-1}(i)\alpha^k\Big) = -\sum_{s=1}^{n}\Big(y_s^{kT}(0)\Phi_s^{k-1T}(i) A\, \Phi_{1s}^{k-1}(i) + \\ +\Phi_{1s}^{k-1T}(i) A\, \Phi_s^{k-1} y_s^k(0) + \Phi_{1s}^{k-1T}(i) A\Phi_{2s}^{k-1}(i) - \Phi_{1s}^{k-1T}(i)\, y_{NS}^{k\ T}\Big). \end{aligned} \tag{10}$$

Solving the equation (10) with respect to the parameter α^k we get the expression for the parameter α^k

$$\begin{aligned} \alpha^k = -\frac{1}{2}\sum_{s=1}^{n}\Big[\Big(\Phi_{1s}^{k-1}(i) A\Phi_{1s}^{k-1}(i)\alpha^k\Big)^{-1} \times y_s^{kT}(0)\Phi_s^{k-1T}(i) A\Phi_{1s}^{k-1}(i) + \\ +\Phi_{1s}^{k-1T}(i) A\Phi_{2s}^{k-1}(i) - \Phi_{1s}^{k-1T}(i)\, y_{NS}^{k\ T} + \Phi_{2s}^{k-1T}(i)\Phi_{1s}^{k-1}(i) - y_{NS}^{k\ T}\Phi_{1s}^{k-1}(i)\Big], \end{aligned}$$

where is assumed that $\left(\Phi_{1s}^{k-1}(i) A\Phi_{1s}^{k-1}(i)\ \alpha^k\right)^{-1}$ exists.

Thus we offer the following computational algorithm for the solution of the identification problem (1) – (3):

Algorithm.

1. The initial data and parameters from (5) are introduced;
2. The nominal trajectory $y^0(i)$ and the parameter α^0 are selected;
3. $A^{k-1}(i),\ B^{k-1}(i),\ C^{k-1}(i)$ from (5) are calculated;
4. The fundamental matrix $\Phi^{k-1}(i)$ and the matrices $\Phi_1^{k-1}(i),\ \Phi_2^{k-1}(i)$ from (6) are calculated;
5. The functional J^{κ} from 8) is formed and the solution of the equation (10) is found;
6. For the sufficiently small number ε the condition

$$\left|\frac{\partial J^{\kappa}}{\partial a^k}\right| < \varepsilon \tag{11}$$

is checked. If the condition (11) is satisfied, the calculation process stops, otherwise go to the step 2.

Example. For illustration of the offered algorithm we consider the example of the gas-lift process [6, 15]. It is known that the mathematical model describing the gas-lift process is in the form of the system of partial differential equations

$$-\frac{\partial P}{\partial x} = \frac{\partial(\rho\omega_c)}{\partial t} + 2a\rho\omega_c; \tag{12}$$

$$-\frac{\partial P}{\partial t} = c^2\frac{\partial(\rho\omega_c)}{\partial x}, \tag{13}$$

where $P = P(x, t)$ is a pressure of gas and or gas-liquid mixture (GLM; t, x – the time and space variables, respectively c is a speed of sound in the gas or GLM; $2a = \frac{g}{\omega_c} + \frac{\lambda\omega_c}{2D}$, g – acceleration of free fall; λ – coefficient of hydraulic resistance; ω_c – averaged over the cross section velocity of the mixture; D – effective internal diameter of the lift and the annular space; $\rho\omega_c = Q/F$, $Q = \rho\omega_c F$ – mass flow rate of injected gas in the annular space and the GLM in the lift; F – constant cross section area of the pump-compressor pipe.

Using time-averaging method the partial differential equations of flow of gas and GLM may be replaced by the ordinary differential equations

$$\dot{Q} = \frac{2\alpha(\lambda_c)\rho F Q^2}{c^2\rho^2F^2 - Q^2};\ Q(0) = u; \tag{14}$$

$$\dot{P} = -\frac{2\alpha c^2\rho^2 F Q^2}{c^2\rho^2F^2 - Q};\ P(0) = P_0. \tag{15}$$

The equation (14) doesn't depend on the equation (15), so this equation can be solved by the method of the variables separated.

The discrete nonlinear equations corresponding to the system of the nonlinear differential equations (14), (15) have the following form:

$$Q(i+1) = Q(i) + h\frac{2\alpha(\lambda_c)\rho F Q^2(i)}{c^2\rho^2F^2 - Q^2(i)};\quad Q(0) = u_0; \tag{16}$$

$$P(i+1) = P(i) - h\frac{2\alpha c^2\rho^2 F Q(i)}{c^2\rho^2F^2 - Q^2(i)};\quad P(0) = P_0. \tag{17}$$

The equation (16) includes the coefficient of hydraulic resistance λ_c [3, 8, 11, 16], which affects to the correctness of calculation of Q in the annular space and lift.

The equation (16) in the intervals $0 \le i \le N-1$ and $N+1 \le i \le 2N$ is substituted with the following equations:

$$Q(i+1) = Q(i) + h\frac{2\alpha_1\rho_1 F_1 Q^2(i)}{c_1^{\,2}\rho_1^{\,2}F_1^{\,2} - Q^2(i)}; \quad 0 \le i \le N-1; \tag{18}$$

$$Q(i+1) = Q(i) + h\frac{2\alpha_2\rho_2 F_2 Q^2(i)}{c_2^{\,2}\rho_2^{\,2}F_2^{\,2} - Q^2(i)}; \quad N+1 \le i \le 2N; \tag{19}$$

$$Q(0) = u. \tag{20}$$

At the point N the equations (18), (19) are connected with each other as follows:

$$Q(N+1) = \gamma Q(N-1) + \left(-\delta_3\left(Q(N-1)-\delta_2\right)^2 + \delta_1\right)\bar{Q}, \tag{21}$$

where γ and δ_1, δ_2, δ_3 are constant real numbers. For simplicity, assume that the parameters γ, δ_1, δ_2, δ_3 are known, $\bar{Q}$ is the volume of fluids in the mixture zone.

Some nominal trajectory $Q^0(i)$ and the parameter α^0 are selected and assume that (k-1)-*th* iteration has been already fulfilled. We linearize the system (18), (19) relatively these data and get

$$Q^k(i+1) = A_1\left(Q^{k-1}(i),\alpha^{k-1}\right)Q^k(i) + B_1\left(Q^{k-1}(i),\alpha^{k-1}\right)\alpha^k +$$
$$+C_1\left(Q^{k-1}(i),\alpha^{k-1}\right), \quad 0 \le i \le N-1; \tag{22}$$

$$Q^k(i+1) = A_2\left(Q^{k-1}(i),\alpha^{k-1}\right)Q^k(i) + B_2\left(Q^{k-1}(i),\alpha^{k-1}\right)\alpha^k +$$
$$+C_2\left(Q^{k-1}(i),\alpha^{k-1}\right), \quad N+1 \le i \le 2N, \tag{23}$$

where

$$A_j\left(Q^{k-1}(i),\alpha^{k-1}\right) = E + h\frac{4\alpha_j^{k-1}c_j^{\,2}\rho_j^{\,3}F_j^{\,3}Q^{k-1}(i)}{\left(c_j^{\,2}\rho_j^{\,2}F_j^{\,2} - Q^{2k-1}(i)\right)^2};$$

$$B_j\left(Q^{k-1}(i),\alpha^{k-1}\right) = h\frac{2\alpha_j^{k-1}c_j\rho_j F_j Q^{2k-1}(i)}{c_j^{\,2}\rho_j^{\,2}F_j^{\,2} - Q^{2k-1}(i)};$$

$$C_j\left(Q^{k-1}(i),\alpha^{k-1}\right) = Q^{k-1}(i) + h\frac{2\alpha_j^{\,k-1}\rho_j F_j Q^{2k-1}(i)}{c_j^{\,2}\rho_j^{\,2}F_j^{\,2} - Q^{2k-41}(i)} -$$
$$-\left(E + h\frac{4\alpha_j^{k-1}c_j^{\,2}\rho_j^{\,3}F_j^{\,3}Q^{k-1}(i)}{\left(c_j^{\,2}\rho_j^{\,2}F_j^{\,2} - Q^{2k-1}(i)\right)^2}\right)Q^{k-1}(i) - h\frac{2\rho_j F_j Q^{2k-1}(i)}{c_j^{\,2}\rho_j^{\,2}F_j^{\,2} - Q^{2k-1}(i)}\alpha^{k-1} \quad (j = 1,\ 2)$$

and h sufficiently small number.

The equations (22) – (23) may be written at the end of the intervals as

$$Q^k(N-1) = \Phi_1^{k-1}(i)Q^k(0) + \Phi_{11}^{k-1}\alpha^k + \Phi_{21}^{k-1}(i),\ 0 \le i \le N-1; \tag{24}$$

$$Q^k(2N) = \Phi_2^{k-1}(i)Q^k(N+1) + \Phi_{12}^{k-1}(i)\alpha^k + \Phi_{22}^{k-1}(i),\ N+1 \le i \le 2N, \tag{25}$$

where

$$\Phi_1^{k-1}(i) = \prod_{i=N-2}^{0} A_1\left(Q^{k-1}(i),\alpha^{k-1}\right);\quad \Phi_2^{k-1}(i) = \prod_{i=2N-1}^{N+1} A_2\left(Q^{k-1}(i),\alpha^{k-1}\right);$$

$$\Phi_{11}^{k-1}(i) = \sum_{j=1}^{N-2}\left(\prod_{i=N-2}^{j} A_1\left(Q^{k-1}(i),\alpha^{k-1}\right)\right)B_1\left(Q^{k-1}(j-1),\alpha^{k-1}\right) + B_1\left(Q^{k-1}(N-2),\alpha^{k-1}\right);$$

$$\Phi_{12}^{k-1}(i) = \sum_{j=1}^{2N-1}\left(\prod_{i=2N-1}^{j} A_2\left(Q^{k-1}(i),\alpha^{k-1}\right)\right)B_2\left(Q^{k-1}(j-1),\alpha^{k-1}\right) + B_2\left(Q^{k-1}(2N-1),\alpha^{k-1}\right);$$

$$\Phi_{21}^{k-1}(i) = \sum_{j=0}^{N-3}\left(\prod_{i=N-3}^{j+1} A_1\left(Q^{k-1}(i),\alpha^{k-1}\right)\right)C_1\left(Q^{k-1}(j),\alpha^{k-1}\right) + C_1\left(Q^{k-1}(N-2),\alpha^{k-1}\right);$$

$$\Phi_{22}^{k-1}(i) = \sum_{j=0}^{2N-2}\left(\prod_{i=2N-2}^{j+1} A_2\left(Q^{k-1}(i),\alpha^{k-1}\right)\right)C_2\left(Q^{k-1}(j),\alpha^{k-1}\right) + C_2\left(Q^{k-1}(2N-1),\alpha^{k-1}\right).$$

Let's have some statistical data that at the given initial volumes of gas $\tilde{Q}_s(0)$ the debit $\tilde{Q}_s(2N)$ is measured at the output, i.e. $\tilde{Q}_s(0)$ and $\tilde{Q}_s(2N)$ are known ($s=\overline{1,5}$), where s is the number of measurements.

As in for determining the coefficient of hydraulic resistance we introduce the next quadratic functional:

$$J^k = \sum_{s=1}^{5}\left|Q_s^k(2N) - \tilde{Q}_s^k(2N)\right|^2. \tag{26}$$

Putting (25) into (26) we get:

$$J^k = \sum_{s=1}^{5}\left|\Phi_{2s}^{k-1}(i)Q_s^k(N+1) + \Phi_{12s}^{k-1}(i)\alpha^k + \Phi_{22s}^{k-1}(i) - \tilde{Q}_s^k(2N)\right|^2. \tag{27}$$

For the solution of the initial optimization problem оптимизации (20), (24) – (26) we obtain the gradient of the functional J^k:

$$\frac{\partial J^k}{\partial \alpha^k} = 2\sum_{s=1}^{5}\left|\Phi_{2s}^{k-1}(i)Q_s^k(N+1) + \Phi_{12s}^{k-1}(i)\alpha^k + \Phi_{22s}^{k-1}(i) - \tilde{Q}_s^k(2N)\right|\Phi_{12s}^{k-1}(i) =$$

$$= 2\sum_{s=1}^{5}\left|\Phi_{2s}^{k-1}(i)\Phi_{12s}^{k-1}(i)Q_s^k(N+1) + \Phi_{22s}^{k-1}(i)\Phi_{12s}^{k-1}(i) - \Phi_{12s}^{k-1}(i)\tilde{Q}_s^k(2N) + \left(\Phi_{12s}^{k-1}(i)\right)^2\alpha^k\right|. \tag{28}$$

Equating (29) to zero we get:

$$\sum_{s=1}^{5}\left|\left(\Phi_{12s}^{k-1}(i)\right)^2\right|\alpha^k =$$

$$=-\sum_{s=1}^{5}\left|\Phi_{2s}^{k-1}(i)\Phi_{12s}^{k-1}(i)Q_s^k(N+1)+\Phi_{22s}^{k-1}(i)\Phi_{12s}^{k-1}(i)-\Phi_{12s}^{k-1}(i)\tilde{Q}_s^k(2N)\right|. \qquad (29)$$

Solving the equation (29) with respect to α^k we have [18]

$$\alpha^k=-\sum_{s=1}^{5}\left|\left(\Phi_{12s}^{k-1}(i)\right)^2\right|^{-1}\sum_{s=1}^{5}\left|\Phi_{2s}^{k-1}(i)\Phi_{12s}^{k-1}(i)Q_s^k(N+1)+\Phi_{22s}^{k-1}(i)\Phi_{12s}^{k-1}(i)-\Phi_{12s}^{k-1}(i)\tilde{Q}_s^k(2N)\right|,$$

where is assumed that $\left|\left(\Phi_{12s}^{k-1}(i)\right)^2\right|^{-1}$ exists.

Let the parameters from equations (16) – (17) be given by $l=1485$ m, $c=331$m/san, $\rho=0,717$кg / m^3, $d=\sqrt{114^2-73^2}\cdot 10^{-3}$m, λ = 0,01 for $0\le i\le N-1$; $c=850$m/san, $\rho=0,700$кg / m^3, $d=0,073$m, $\lambda=0,23$ for $N+1\le i\le 2N$.

By using algorithm described above, we see that to achieve the accuracy 10^{-3} is needed 44 iterations and finally the following result is obtained $\lambda_c=0,298342$. Here $\tilde{\lambda}_c=0,23$, where $\tilde{\lambda}_c$ is the hydraulic resistance value from practice. Note that λ_c differs from $\tilde{\lambda}_c$ to the order 10^{-4}, and it shows the efficiency of the proposed algorithm.

3. Conclusions.

In the considered problem, the quadratic functional is formed for the solution of the identification problem and it allows to find the coefficient of hydraulic resistance. The offered calculation algorithm confirm the adequacy of the constructed mathematical model with practice.

Р Е З Ю М Е. Розглянуто задачу ідентифікації, яка дозволяє визначити параметри динамічної системи у дискретному випадку. Спочатку нелінійне дискретне рівняння лінеаризується методом квазілінеаризації. Далі за допомогою статистичних даних отримується квадратичний функціонал і його градієнт. Тоді пропонується алгоритм обчислення для задачі, що розглядається. Показано на прикладі, що статистичне значення коефіцієнта гідравлічного опору відрізняється від отриманого чисельно на порядок 10^{-4}. Це свідчить про адекватність використаної математичної моделі.

1. *Akbarov S.D.* Forced Vibration of the Hydro-Viscoelastic and-Elastic Systems Consisting of the Viscoelastic or Elastic Plate, Compressible Viscous Fluid and Rigid Wall: A Review // Appl. Comput. Math. – 2018. – **17**, N 3. – P. 221 – 245.
2. *Aliev F.A., Aliev N.A., Safarova N.A., Gasimova K.G., Velieva N.I.* Solution of Linear Fractional-Derivative Ordinary Differential Equations with Constant Matrix Coefficients // Appl. Comput. Math. – 2018. – **17**, N 3. – P. 317 – 322.
3. *Aliev F.A., Ismailov N.A., Haciyev H., Guliev M.F.* A method of determine the coefficient of hy draulic resistance in different area of pump-compressor pipes // TWMS J. Pure Appl. Math. – 2016. – **7**, N 2. – P. 211 – 217.
4. *Aliev F.A., Ismailov N.A., Mamedova E.V., Mukhtarova N.S.* Computational algorithm for solving problem of optimal boundary-control with nonseparated boundary conditions // J. Comput. Syst. Sci. Int. – 2016. – **55**, N 5. – P. 700 – 711.
5. *Aliev F.A., Ismayilov N.A., Gasimov Y.S, Namazov A.A.* On an identification problem on the definition of the parameters of the dynamic system // Proceedings of IAM. – 2014. – **3**, N 2. – P. 139 – 151 (in Russian).
6. *Aliev F.A., Ilyasov M.Kh., Nuriev N.B.* Problems of Modeling and Optimal Stabilization of the Gas-Lift Process // Int. Appl. Mech. – 2010. – **46**, N 6. – P. 709 – 717.
7. *Aliev T.A., Musaeva N.F., Rzaeva N.E., Sattarova U.E.* Algorithms for Forming Correlation Matrices Equivalent to Matrices of Useful Signals of Multidimensional Stochastic Objects // Appl. Comput. Math. – 2018. – **17**, N 2. – P. 205 – 216.
8. *Altshul D.M.* Hydraulic resistance. – M: Nedra, 1970. – 216 p. (in Russian).

9. *Bellman P.E., Kalaba P.E,* Quasilinearization and nonlinear boundary problems. – M: Mir, 1968. – 153 p. (in Russian).
10. *Brayson A., Yu-Shi X.* Applied theory of optimal control. – M: Mir, 1972. – 554p.
11. *Hajieva N.S., Safarova N.A, Ismailov N.A.* Algorithm defining the hydraulic resistance coefficient by lines method in gas-lift process // Miskolc Mathematical Notes. – 2017. – **18**, N 2. – P. 771 – 777.
12. *Ismailov N.A., Mukhtarova N.S.* Method for solution of the problem of discrete optimization with boundary control // Proc. of IAM. – 2013. – **2**, N 1. – P. 20 – 27 (in Russian).
13. *Jamalbayov M.A., Veliyev N.A.* The technique of early determination of reservoir drive of gas condensate and velotail oil deposits on the basis of new diagnosis indicators // TWMS J. Pure Appl. Math. – 2017. – **8**, N 2. – P. 237 – 250.
14. *Ljung L.* System Identification. – Wiley Encyclopedia of Electrical and Electronics Engineering, 1999. – P. 263-282.
15. *Mirzadjanzadeh A.Kh., Akhmetov I.M., Khasaev A.M, Gusev V.I.* Technology and Technique of Oil production. – M: Nedra, 1986. – 382 p. (in Russian).
16. *Mukhtarova N.S.* Algorithm to solution the identification problem for finding the coefficient of hydraulicnresistance in gas-lift proceses // Proc. of IAM. – 2015. – **4**, N 2. – P. 206 – 213 (in Russian).
17. *Mukhtarova N.S., Ismailov N.A.* Algorithm to solution of the optimization problem with periodic condition and boundary control // TWMS J. Pure Appl. Math. – 2014. – **5**, N 1. – P. 130 – 137.
18. *Triki H., Ak T., Biswas A.*, New types of solution-like solutions for a second order wave equation of Korteweg-de Vries type // Appl. Comput. Math. – 2017. – **14**, N 2. – P. 168 – 176.

From the Editorial Board: The article corresponds completely to submitted manuscript.

Поступила 04.07.2017 Утверждена в печать 22.11.2018