

. . . , 15, 49005, . . . ; e-mail: knyshenko@ukr.net

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(Re_d),

(Re_ω)

(Re_s).

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(Re_d),

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(Re_s).

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The aim of this work is to analyze existing analytical and empirical relationships for determining the forces acting on a solid particle in a gas flow which are caused by its rotation (the Magnus force) and the gradient of the carrying gas velocity (the Saffman force). These forces act transversely to the velocity of a particle in a gas flow with restricting walls, and their effect is crucial in terms of the adequacy of calculation of the particle trajectory parameters. The paper analyzes relationships for Magnus and Saffman force determination obtained analytically or by mathematical treatment of experimental data and limited by certain values of criterion parameters. The force determination criteria are the Reynolds numbers based on the particle diameter and flow-past velocity (Re_d), on the square of the particle diameter and the particle rotation angular velocity (Re_ω), and on the particle diameter and the gradient of the carrier gas local velocity (Re_s). Particle rotation and the Magnus force are caused by

[1].

[2] – [5].

[1, 6].

[7].

$$\omega_p^* = \frac{5k_t + 2}{7} \omega_p \pm \frac{10(1 - k_t)}{7d_p} u_p,$$

ω_p^* , ω_p –
 u_p –
 k_t –

; d_p –

) $k_t = 0,3$ (
 $10 /$

10^{-3}

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$$\omega_p^* \approx 10^3 \text{ 1/} .$$

()
 ,

[7]

$$\vec{F} = \frac{\rho d_p^3 \vec{\omega}_p \times \vec{V}_R}{8}, \quad (1)$$

$\vec{\omega}_p$ –

; $\vec{V}_R = \vec{U} - \vec{U}_p$ –

, ρ –

; –

$$Re_d, Re_\omega; Re_d = \frac{\rho |\vec{V}_R| d_p}{\mu}; Re_\omega = \frac{\rho |\vec{\omega}_p| d_p^2}{\mu}; \mu -$$

$$c = F(Re_d, Re_\omega).$$

$$c = F(Re_d, Re_\omega),$$

$$1): \quad Re_d < 1, \quad Re_\omega < 1 \quad c = \pi \quad [8], \quad Re_d \gg 1, \quad Re_\omega \gg 1 -$$

$$c = 8/3 \pi \quad [9].$$

[1], [7], [10], [11], [12].

[2]

(1)

$$F = \frac{\pi}{8} c^* d_p^2 \rho |\vec{V}_R| \frac{|\vec{\omega}_p \times \vec{V}_R|}{|\vec{\omega}_p|}, \quad (2)$$

[3], [4], [5]

$$F = \frac{\pi}{8} c^* d_p^2 \rho \frac{Re_d}{Re_\omega} |\vec{\omega}_p \times \vec{V}_R|. \quad (3)$$

$$c^* \quad Re_d \quad Re_\omega,$$

[11]

(1),

[12]

$$F = \frac{\pi}{6} c^* d_p^3 \rho \vec{\omega}_p \times \vec{V}_R. \quad (4)$$

Таблица 1

Автор, год, источник	Формула для определения силы Магнуса	Выражения для c_w и c_w^*	Диапазон чисел Рейнольдса $Re_d = \frac{\rho \vec{V}_R d_p}{\mu}$	Диапазон чисел Рейнольдса $Re_o = \frac{\rho \vec{\omega}_p d_p^2}{\mu}$	Метод получения зависимостей: – расчетный (расчет); – по данным экспериментов (эксперимент)
Рубинов, 1961, [8]		$c_w = \pi$	$Re_d < 1$	$Re_o < 1$	Расчет
Соу, 1971, [17-18]; Нигматулин, 1978 [9]		$c_w = 8\pi/3$	$Re_d \rightarrow \infty$	$Re_o \rightarrow \infty$	Расчет
Шрайбер, 1980, [1]		$c_w = 0.72$	не определен	не определен	Эксперимент
Наумов, 2006, [7]		$c_w = \pi$	не определен	$Re_o < 100$	$f(Re_o) = 6,05 \pi Re_o^{-0,39}$ (Эксперимент)
Вараксин, 2003, [10]	$F_w = \frac{c_w}{8} \rho d_p^3 \vec{\omega}_p \times \vec{V}_R$	$c_w = f(Re_o)$	не определен	$100 < Re_o < 36000$	$f(Re_o, Re_d) = 0,534 Re_d^{0,715} Re_o^{-0,64}$ (Эксперимент)
Яценко, 2001, [11]		$c_w = f(Re_o, Re_d)$	$360 < Re_d < 13500$	$590 < Re_o < 45000$	$f(Re_o, Re_d) = 0,667 \pi Re_d^{0,447} Re_o^{-0,529}$ (Эксперимент)
Sommerfeld, 2003, [2]	$F_w = \frac{\pi}{8} d_p^2 c_w^* \vec{V}_R \frac{ \vec{\omega}_p \times \vec{V}_R }{ \vec{\omega}_p }$	$c_w^* = Re_o / Re_d$ $c_w^* = \frac{\pi Re_d}{Re_o}$	$Re_d < 1$	не определен	$f(Re_o, Re_d) = 0,45 + (Re_o / Re_d - 0,45)^*$ $\exp(-0,05684 Re_o^{0,4} Re_d^{0,3})$ (Эксперимент)
Kharlamov A., 2008 [12]	$F_w = \frac{\pi}{6} c_w d_p^3 \vec{\omega}_p \times \vec{V}_R$	$c_w^* = \frac{1 + c Re_d}{a + b Re_d}$ $c_w^* = \frac{4\pi}{3} c_w$ $a = a_1 + a_2 \Gamma + a_3 \Gamma^2$ $b = (b_1 + \Gamma) / (b_2 + b_3 \Gamma)$ $c = 1 / (c_1 + c_2 \Gamma)$	$0,5 < Re_d < 40000$	$0,2 Re_d < Re_o < 14 Re_d$	$\Gamma = 0,5 d_p \vec{\omega}_p / \vec{V}_R $ $a_1 = 1,333; a_2 = -0,061;$ $a_3 = 0,029; b_1 = 5,9;$ $b_2 = 38; b_3 = 4,6;$ $c_1 = 25; c_2 = 21$ (Эксперимент)

Таблица 1 (продолжение)

Автор, источник	Формула для определения силы Магнуса	Выражения для c_u и c_u^*	Диапазон чисел Рейнольдса $Re_d = \frac{\rho \bar{V}_R d_p}{\mu}$	Диапазон чисел Рейнольдса $Re_o = \frac{\rho \bar{\omega}_p d_p^2}{\mu}$	Метод получения зависимостей: – расчетный (расчет); – по данным экспериментов (эксперимент)
Lain, 2013 [4]. Souza, 2013, [5]	$F_M = \frac{\pi}{8} c_u^* d_p^3 \frac{Re_d \bar{\omega}_p}{Re_o} \times \vec{V}_R$	$c_u^* = Re_o / Re_d$	$Re_d < 1$	не определен	$f(Re_d) = (0,178 + 0,822 Re_d^{-0,522})$ (Эксперимент)
		$c_u = c_u^* \frac{Re_o}{Re_d} f(Re_d)$	$1 < Re_d < 1000$		
		$c_u^* = \frac{Re_o}{Re_d} f(Re_o, Re_d)$	$Re_d < 2000$		

(1) (. . . 1).

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Re_d Re_ω ,

1,) [2] 1 - 5

$Re_\omega = 1000, 500, 200, 100, 50;$ 1,) [10] -

$Re_\omega = 4800, 3600, 2400, 1200, 590;$. 1,) [7] - $Re_\omega = 20000, 5000, 2000,$

1000, 100; . 1, [5] - $Re_\omega = 1000, 500, 200, 100, 50;$. 1,) [11] -

$Re_\omega = 6000, 4000, 2000, 1000, 590;$. 1,) [12] - $Re_\omega = 40000, 30000,$

20000, 4000, 2000.

Re_d Re_ω ,

Re_ω , . . .

Re_d Re_ω

. 1, $Re_d = Re_\omega = 1000$, 3,0 (. 1,) 0,1 (. 1,)).

[2], [4], [5], [12]

[1], [10], [7].

500

Re_d Re_ω ,

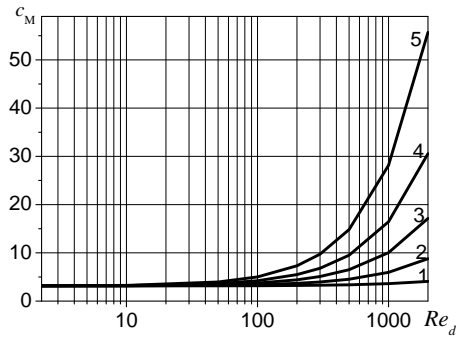
[13]

$$F_S = c_S d_p^2 \rho v^{0.5} \bar{V}_R \left(\frac{dU}{dy} \right)^{0.5}, \quad (5)$$

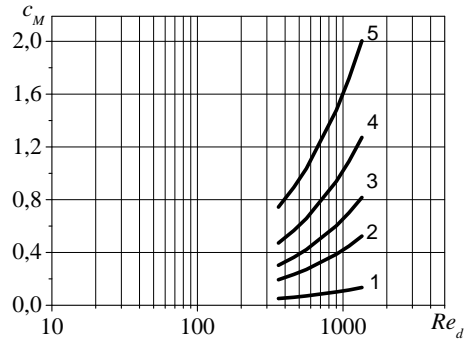
s -

$$Re_d \quad Re_s; \quad Re_s = \frac{\rho d_p^2}{\nu} \left| \frac{dU}{dy} \right|; \quad \frac{dU}{dy} -$$

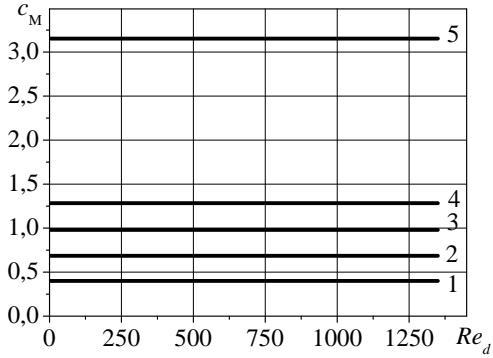
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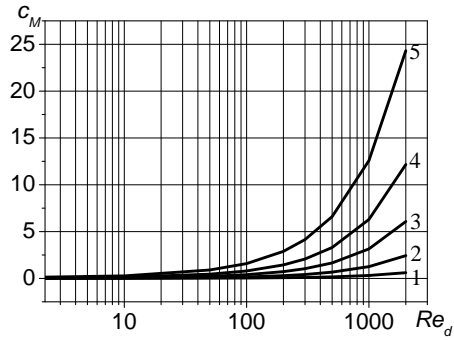
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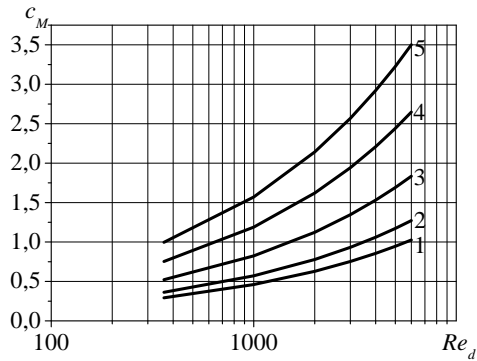
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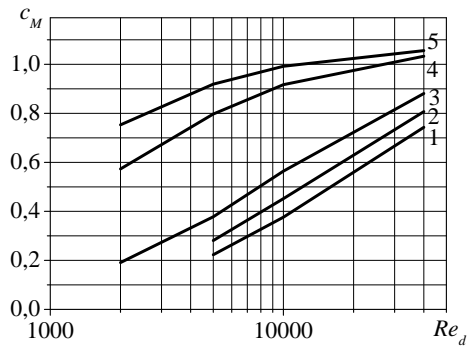
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$$Re_d \ll 1, Re_s \ll 1$$

$$A = Re_d / Re_s^{0.5} \ll 1,$$

$$s = \frac{0}{s} = 1,615$$

-

$$Re_d \quad Re_s.$$

2

s

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$$s = c_s^0 f(Re_d, Re_s),$$

Re_d

Re_s

Таблица 2

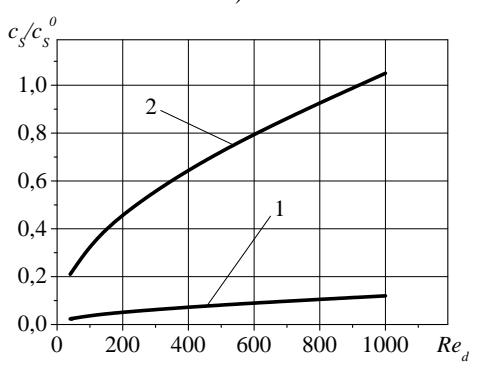
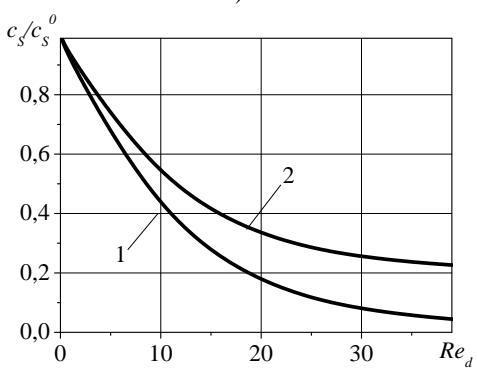
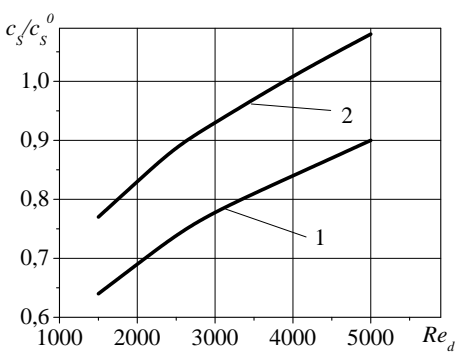
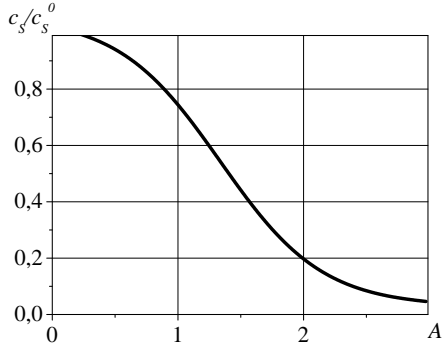
Автор, год, источник	Формула для определения силы Сэфмена	Выражение для c_S	Диапазон чисел Рейнольдса $Re_d = \frac{\rho \bar{V}_R d_p }{\mu}$	Диапазон чисел Рейнольдса $Re_s = \frac{\rho d_p^2 dU/dy }{\mu}$	Метод получения зависимостей: – расчетный (расчет); – по данным экспериментов различных авторов (эксперимент)
Saffman, 1961, [13]		$c_S^0 = 1,615$	$Re_d \ll 1$	$Re_s \ll 1$	$A = \frac{Re_d}{Re_s^{0,5}} \ll 1$ (Расчет)
Асмолов, 2015, [14]		$\frac{c_S}{c_S^0} = f(Re_d, Re_s)$	$Re_d \ll 1$	$Re_s \ll 1$	$f(Re_d, Re_s) = (1 + 0,581A^2 - 0,439A^3 + 0,203A^4)^{-1}$ $0 \leq A \leq 3$ (Расчет)
Dandy, 1990, [16], McLaughlin, 1991, [17], Наумов, 2006, [7], Sommerfeld, 2012, [2]	$F_S = c_S d_p^2 \rho v^{0,5} \bar{V}_R \left(\frac{dU}{dy}\right)^{0,5}$	$\frac{c_S}{c_S^0} = f(Re_d, Re_s)$	$Re_d < 40$		$f(Re_d, Re_s) = (1 - 0,3314\beta^{0,5}) \times \exp(-0,1Re_d) + 0,3314\beta^{0,5}$ (Эксперимент)
			$Re_d > 40$	$0,01Re_d < Re_s < 0,8Re_d$	$f(Re_d, Re_s) = 0,0524(\beta Re_d)^{0,5}$ $\beta = 0,5 Re_s / Re_d ; 0,005 < \beta < 0,4$ (Эксперимент)
Яценко, 2002, [15]		$\frac{c_S}{c_S^0} = f(Re_d, Re_s)$ $\frac{c_S}{c_S^0} = f(Re_d, Re_s)$	$1500 < Re_d < 5000$	$1000 < Re_s < 2600$	$f(Re_d, Re_s) = 0,034 \cdot Re_d^{0,286} \cdot Re_s^{0,191}$ (Эксперимент)

.2,)
 [14] $A \quad 0 \leq A \leq 0,3,$

c_s , $Re_s^{0,5}$, Re_d , s/s^0 , .2,)
 [15] Re_s (1)
 $Re_s=1000,$ $2 - 2600$).

Re_s , s/s^0 , .2,) 2,)
 [7,] [2], [16], [17] $Re_d < 40$ $Re_d > 40$, 1
 $\beta = 0,005, (\beta = 0,5 Re_s / Re_d),$ $2 - \beta = 0,4.$

β , c_s/c_s^0 ,
 $Re_d,$ Re_d 0 40 s/s^0 ,
 $Re_d > 40$ s/s^0 $Re_d.$



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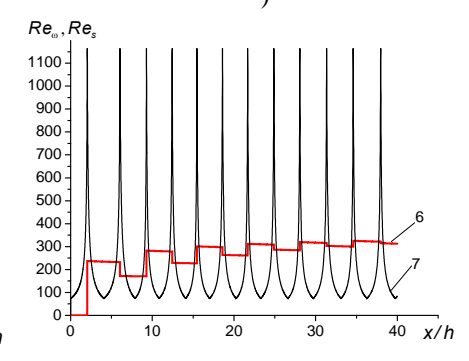
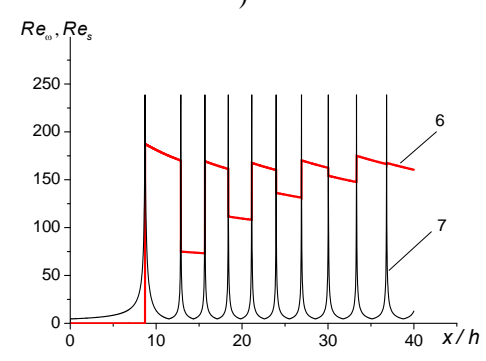
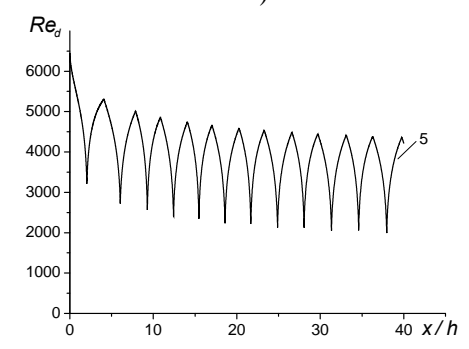
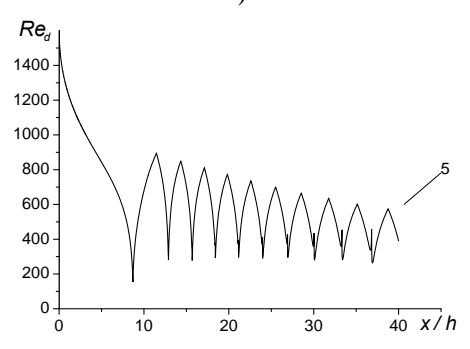
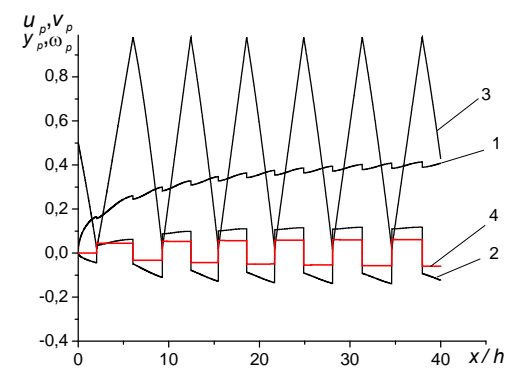
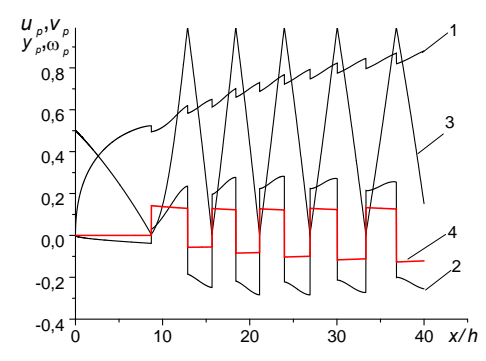
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[19].



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$$y_p \quad \left(\begin{array}{l} 3 \\ \omega_p \quad (4) \end{array} \right) \quad u_p, \quad v_p \quad u, \quad h, \quad (1 \quad 2) -$$

(5, 6, 7), Re_d, Re_ω, Re_s

$$Re_d \quad [7] \quad [2]. \quad 1$$

Re_s

Re_ω

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