

A Method of Virtual Design of the Fatigue Life of a Dynamic Structure

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Метод виртуального анализа усталостной долговечности динамической конструкции

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В современных инженерных расчетах важное место занимают расчеты усталостной долговечности и оценка надежности динамических систем при их случайном нагружении. При проведении этих расчетов необходимо знать плотность спектра напряжений для исследуемого компонента динамической системы. Современный уровень развития компьютерных технологий и теорий численного моделирования позволяет моделировать плотность спектра напряжений динамических систем. На основании этого предложен метод численного моделирования для прогнозирования усталостной долговечности динамической конструкции, который базируется на модельных уравнениях для плотности спектра напряжений и усталостной долговечности. В качестве примера выполнен виртуальный анализ усталостной долговечности и надежности большегрузного грузового автомобиля SW-200k. Показаны применимость метода для оценки существующих динамических конструкций и его перспективность для оценки усталостной долговечности разрабатываемых новых конструкций.

Ключевые слова: система со случайными колебаниями, плотность спектра напряжений, прогнозирование усталостной долговечности, виртуальный анализ.

Introduction. The fatigue life prediction and reliability analysis of dynamic systems under random excitement are important topics in modern engineering design [1–8]. The vibration characteristics of fatigue damage of beam-type structural components has been researched [9, 10] and a computational and experimental assessment of the sensitivity of structural materials to stress concentration under high-cycle asymmetrical loading has been performed [11]. A numerical calculation of the frequencies of natural vibrations of beams with a cross section varying linearly in height under different conditions of fixing their ends is presented [12]. However, the stress power spectral density of a component in a dynamic system must be known when one predicts its fatigue life and analyzes its reliability. A traditional way is to measure the stress power spectral density, which has played an important role in practice [4]. However, the stress power spectral density of a dynamic system will change with the alteration of the structure and dynamic parameters of a system. So the engineering application in this way is limited. Moreover, one is not able to obtain the stress power spectral density of a new dynamic system before it has been designed. With the rapid development of computer technology and numerical computation theories, it has been possible to simulate stress power spectral density of dynamic systems. Based on this,

the method of simulating stress power spectral density is put forward in this paper, and as an example the fatigue life prediction and reliability analysis of CW-200k [4] vehicle truck are made.

Computational Method of Stress Power Spectral Density.

Random Response Spectra of Dynamic System Computation [1, 2]. In general, the vibration equation of a linear system can be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{z}}(t)+\mathbf{C}\dot{\mathbf{z}}(t)+\mathbf{K}\mathbf{z}(t)=\mathbf{F}(t), \tag{1}$$

where \mathbf{M} , \mathbf{C} , and \mathbf{K} are $N \times N$ symmetric matrices representing the masses, damping, and stiffness of the system, $\ddot{\mathbf{z}}(t)$, $\dot{\mathbf{z}}(t)$, and $\mathbf{z}(t)$ are column vectors representing acceleration, velocity, and displacement, respectively, and $\mathbf{F}(t)$ is matrix of excitation function.

From Eq. (1), the frequency response matrix is obtained by taking the Fourier transform as follows [3]:

$$\mathbf{H}(\omega)=\mathbf{F}(\omega)[\mathbf{K}-\omega^2\mathbf{M}+j\omega\mathbf{C}]^{-1}, \tag{2}$$

where $\mathbf{H}(\omega)=F(\mathbf{z}(t))/F(\mathbf{F}(t))$.

Therefore, the power spectral density matrix of system can be deduced as

$$\mathbf{G}_z(\omega)=\mathbf{H}^{*T}(\omega)\mathbf{G}_x(\omega)\mathbf{H}(\omega), \tag{3}$$

where $\mathbf{H}^{*T}(\omega)$ is conjugate transpose matrix of $\mathbf{H}(\omega)$ and $\mathbf{G}_x(\omega)$ is input power spectral density matrix of a system.

If the time difference τ exists in the excitation process, according to the time difference property of the Fourier transform, $\mathbf{G}_x(\omega)$ may be expressed as

$$\mathbf{G}_{x_i x_j}(\omega)=\mathbf{G}_x(\omega) \begin{bmatrix} 1 & e^{-j\omega\tau_{12}} & e^{-j\omega\tau_{13}} & \dots & e^{-j\omega\tau_{1n}} \\ e^{j\omega\tau_{21}} & 1 & e^{-j\omega\tau_{23}} & \dots & e^{-j\omega\tau_{2n}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{j\omega\tau_{n1}} & e^{j\omega\tau_{n2}} & e^{-j\omega\tau_{n3}} & \dots & 1 \end{bmatrix}. \tag{4}$$

Similarly, the acceleration amplitude of the system response is given by

$$\ddot{\mathbf{z}}=3 \left[\int_{\omega_{\min}}^{\omega_{\max}} \omega^4 \mathbf{G}_z(\omega) d\omega \right]^{1/2}. \tag{5}$$

The Unit Load Stress Matrix Computation. In a linear elastic system, the stress state of the structure may be expressed as

$$\sigma(x, y, z, t)=\mathbf{S}(x, y, z) \mathbf{b}^T(t), \tag{6}$$

where $\mathbf{S}(x, y, z)$ is defined as unit load stress matrix. Vector $\mathbf{b}(t)$ is the load state vector. The load state vectors may be expressed as

$$\mathbf{b}(t)=(\mathbf{F}(t), \mathbf{M}(t))=(\mathbf{W}\mathbf{z}(t), \mathbf{V}\mathbf{z}(t))=\mathbf{J}\mathbf{z}(t), \tag{7}$$

where $\mathbf{F}(t)=(\mathbf{F}_1(t), \mathbf{F}_2(t), \dots, \mathbf{F}_n(t))$ is called the forces in the translation sense, $\mathbf{M}(t)=(\mathbf{M}_1(t), \mathbf{M}_2(t), \dots, \mathbf{M}_n(t))$ is called the moments in rotation, $\mathbf{J}=(\mathbf{W}, \mathbf{V})^T$ is

called linear operator or coefficients' matrix, and $z(t)$ is vector representing the response of each degree of freedom.

According to the theory of correlation function [3], the correlation function matrix of stress state of stationary random process can be written as

$$\begin{aligned} \mathbf{R}_s(\tau) &= E[\sigma(t)\sigma^T(t+\tau)] = \mathbf{S}(x, y, z)E[b(t)b^T(t+\tau)]\mathbf{S}^T(x, y, z) = \\ &= \mathbf{S}(x, y, z)\mathbf{J}E[z(t)z^T(t+\tau)]\mathbf{J}^T\mathbf{S}^T(x, y, z) = \mathbf{S}(x, y, z)\mathbf{J}\mathbf{R}_z(\tau)\mathbf{J}^T\mathbf{S}^T(x, y, z), \end{aligned} \quad (8)$$

where E represents mathematical expectation.

Therefore, the stress power spectral density matrix can be obtained by taking Fourier transform as

$$\begin{aligned} \mathbf{G}_s(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{R}_s(\tau)e^{-j\omega\tau} d\tau = \mathbf{S}(x, y, z)\mathbf{J} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{R}_z(\tau)e^{-j\omega\tau} d\tau \right] \mathbf{J}^T\mathbf{S}^T(x, y, z) = \\ &= \mathbf{S}(x, y, z)\mathbf{J}\mathbf{G}_z(\omega)\mathbf{J}^T\mathbf{S}^T(x, y, z). \end{aligned} \quad (9)$$

Then the stress square deviation is

$$\mathbf{D}_s = \mathbf{R}_s(0) = \int_{-\infty}^{\infty} \mathbf{G}_s(\omega)d\omega. \quad (10)$$

If $z(t) = [z_1(t), z_2(t), \dots, z_n(t)]$ represents response vector matrix of a system, then the response spectra will be of the form

$$\mathbf{G}_z(\omega) = \begin{bmatrix} \mathbf{G}_{z_1z_1}(\omega) & \mathbf{G}_{z_1z_2}(\omega) & \cdots & \mathbf{G}_{z_1z_n}(\omega) \\ \mathbf{G}_{z_2z_1}(\omega) & \mathbf{G}_{z_2z_2}(\omega) & \cdots & \mathbf{G}_{z_2z_n}(\omega) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{G}_{z_nz_1}(\omega) & \mathbf{G}_{z_nz_2}(\omega) & \cdots & \mathbf{G}_{z_nz_n}(\omega) \end{bmatrix}. \quad (11)$$

Now, the relationship between the response spectra of the system and stress power spectral density of a structure is established. Based on this formulation, one is able to estimate fatigue life and analyze the reliability of the structure.

The Fatigue Life Prediction of the CW-200k type Truck.

The Stress Power Spectral Density Computation of the CW-200k Type Truck of a Passenger Vehicle. The stress power spectral density of the frame of a truck can be researched in the vertical vibration of six-degree-of-freedom system because in high speed railways track is either straight or has a big radius. The dynamic model of vehicle is as shown in Fig. 1.

The equations of motion was obtained as follows:

$$\begin{cases} m_c \ddot{z}_c + 2C_2 \dot{z}_c - C_2 \dot{z}_{b1} - C_2 \dot{z}_{b2} + 2K_2 z_c - K_2 z_{b1} - K_2 z_{b2} = 0, \\ J_c \ddot{\varphi}_c + 2C_2 l^2 \dot{\varphi}_c - C_2 l \dot{z}_{b1} + C_2 l \dot{z}_{b2} + 2K_2 l^2 \varphi_c - K_2 l z_{b1} + K_2 l z_{b2} = 0, \\ m_b \ddot{z}_{b1} - C_2 \dot{z}_c - C_2 l \dot{\varphi}_c + (2C_1 + C_2) \dot{z}_{b1} - K_2 z_c - K_2 l \varphi_c + (2K_1 + K_2) z_{b1} = \\ = C_1 (\dot{\eta}_1 + \dot{\eta}_2) + K_1 (\eta_1 + \eta_2), \\ m_b \ddot{z}_{b2} - C_2 \dot{z}_c + C_2 l \dot{\varphi}_c + (2C_1 + C_2) \dot{z}_{b2} - K_2 z_c + K_2 l \varphi_c + (2K_1 + K_2) z_{b2} = \\ = C_1 (\dot{\eta}_3 + \dot{\eta}_4) + K_1 (\eta_3 + \eta_4), \end{cases} \quad (12a)$$

$$\begin{cases} J_b \ddot{\varphi}_{b1} + 2C_1 l_1^2 \dot{\varphi}_{b1} + 2K_1 l_1^2 \varphi_{b1} = C_1 l_1 (\dot{\eta}_1 - \dot{\eta}_2) + K_1 l_1 (\eta_1 - \eta_2), \\ J_b \ddot{\varphi}_{b2} + 2C_1 l_1^2 \dot{\varphi}_{b2} + 2K_1 l_1^2 \varphi_{b2} = C_1 l_1 (\dot{\eta}_3 - \dot{\eta}_4) + K_1 l_1 (\eta_3 - \eta_4), \end{cases} \quad (12b)$$

and Eqs. (12) are written in matrix form as

$$[M]\{\ddot{z}\} + [C]\{\dot{z}\} + [K]\{z\} = [C_\eta]\{\dot{\eta}\} + [K_\eta]\{\eta\}, \quad (13)$$

where $\{z\}$ is displacement vector, $\{z\} = [z_c \ \varphi_c \ z_{b1} \ \varphi_{b1} \ z_{b2} \ \varphi_{b2}]^T$, $\{\eta\}$ is the input vector, $\{\eta\} = [\eta_1 \ \eta_2 \ \eta_3 \ \eta_4]^T$, $[M]$, $[C]$, and $[K]$ are 6×6 symmetric matrices representing the mass, damping, and stiffness of a vehicle system, respectively, and $[C_\eta]$ and $[K_\eta]$ are matrices of coefficients. They correspond to the CW-200k type truck and 25 type passenger rail vehicle.

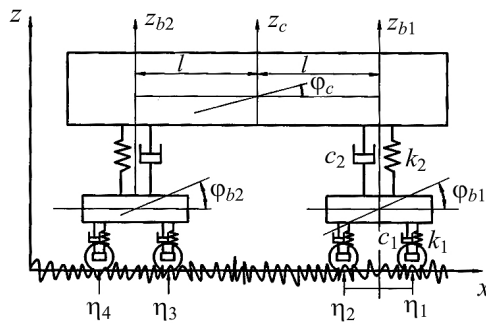


Fig. 1. The vertical vibration model of vehicle.

The expression for the track spectrum is given by [4]

$$G_\eta(\omega) = \frac{A_v \Omega_c^2}{(\omega^2 + \Omega_r^2) + (\omega^2 + \Omega_c^2)}. \quad (14)$$

The track spectrum come from the Office for Research and Experiments of the International Union of Railways (ORE B176), the high level ORE parameters are as follows: $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, and $A_v = 1.08 \cdot 10^{-6}$ m · rad.

As a result of symmetry of the structure and the force, the whole model of the side frame of truck and the loading model are shown in Fig. 2a and b, respectively.

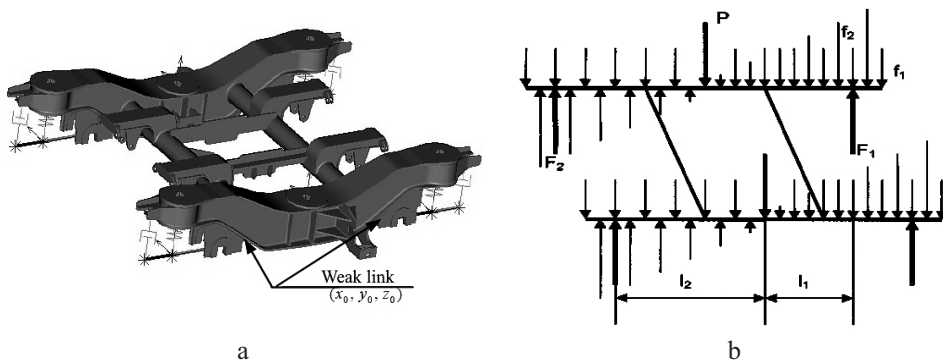


Fig. 2. The model of the CW-200k type truck (a); loading model of the CW-200k truck (b).

$$G_z(\omega) = \begin{bmatrix} G_{\ddot{z}_{b1}\ddot{z}_{b1}} & G_{\ddot{z}_{b1}\dot{z}_{b1}} & G_{\ddot{z}_{b1}z_{b1}} & G_{\ddot{z}_{b1}\ddot{\varphi}_{b1}} & G_{\ddot{z}_{b1}\dot{\varphi}_{b1}} & G_{\ddot{z}_{b1}\varphi_{b1}} & G_{\ddot{z}_{b1}\dot{\eta}_1} & G_{\ddot{z}_{b1}\eta_1} \\ G_{\dot{z}_{b1}\ddot{z}_{b1}} & G_{\dot{z}_{b1}\dot{z}_{b1}} & G_{\dot{z}_{b1}z_{b1}} & G_{\dot{z}_{b1}\ddot{\varphi}_{b1}} & G_{\dot{z}_{b1}\dot{\varphi}_{b1}} & G_{\dot{z}_{b1}\varphi_{b1}} & G_{\dot{z}_{b1}\dot{\eta}_1} & G_{\dot{z}_{b1}\eta_1} \\ G_{z_{b1}\ddot{z}_{b1}} & G_{z_{b1}\dot{z}_{b1}} & G_{z_{b1}z_{b1}} & G_{z_{b1}\ddot{\varphi}_{b1}} & G_{z_{b1}\dot{\varphi}_{b1}} & G_{z_{b1}\varphi_{b1}} & G_{z_{b1}\dot{\eta}_1} & G_{z_{b1}\eta_1} \\ G_{\dot{\varphi}_{b1}\ddot{z}_{b1}} & G_{\dot{\varphi}_{b1}z_{b1}} & G_{\dot{\varphi}_{b1}\dot{z}_{b1}} & G_{\dot{\varphi}_{b1}\ddot{\varphi}_{b1}} & G_{\dot{\varphi}_{b1}\dot{\varphi}_{b1}} & G_{\dot{\varphi}_{b1}\varphi_{b1}} & G_{\dot{\varphi}_{b1}\dot{\eta}_1} & G_{\dot{\varphi}_{b1}\eta_1} \\ G_{\varphi_{b1}\ddot{z}_{b1}} & G_{\varphi_{b1}\dot{z}_{b1}} & G_{\varphi_{b1}z_{b1}} & G_{\varphi_{b1}\ddot{\varphi}_{b1}} & G_{\varphi_{b1}\dot{\varphi}_{b1}} & G_{\varphi_{b1}\varphi_{b1}} & G_{\varphi_{b1}\dot{\eta}_1} & G_{\varphi_{b1}\eta_1} \\ G_{\dot{\eta}_1\ddot{z}_{b1}} & G_{\dot{\eta}_1\dot{z}_{b1}} & G_{\dot{\eta}_1z_{b1}} & G_{\dot{\eta}_1\ddot{\varphi}_{b1}} & G_{\dot{\eta}_1\dot{\varphi}_{b1}} & G_{\dot{\eta}_1\varphi_{b1}} & G_{\dot{\eta}_1\dot{\eta}_1} & G_{\dot{\eta}_1\eta_1} \\ G_{\eta_1\ddot{z}_{b1}} & G_{\eta_1\dot{z}_{b1}} & G_{\eta_1z_{b1}} & G_{\eta_1\ddot{\varphi}_{b1}} & G_{\eta_1\dot{\varphi}_{b1}} & G_{\eta_1\varphi_{b1}} & G_{\eta_1\dot{\eta}_1} & G_{\eta_1\eta_1} \end{bmatrix}, \quad (19)$$

substituting Eqs. (16), (18), and (19) into Eq. (9) and substitution of the parameters of the truck then numbering computation, the SPSD $G_s(\omega)$ of the weak link can be obtained. As an example of stress power spectral density that the train runs at a speed of 180 km per hour is given in Fig. 4.

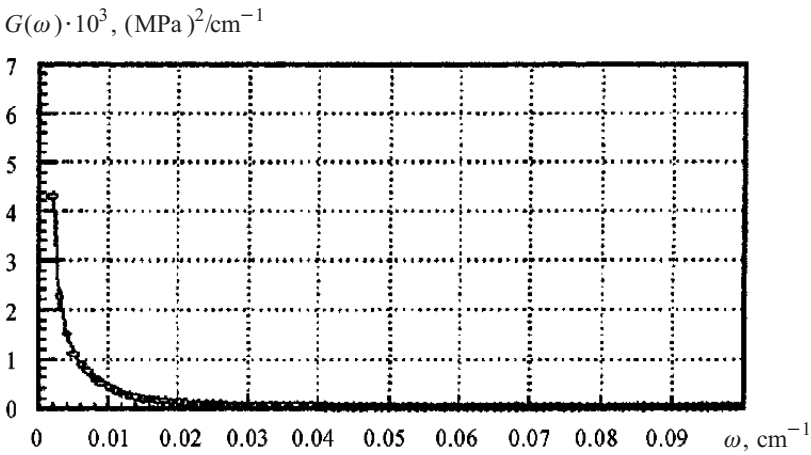


Fig. 4. Stress power spectral density (SPSD).

The Fatigue Life Prediction of the Side Frame of The CW-200k Type Truck. After the stress power spectral density are obtained, based on it, the distribution density of stress peak can be deduced as [5, 6]

$$f(s) = \frac{\sqrt{-R''S}}{2\pi R\sqrt{R}} \exp\left(\frac{-S^2}{2R}\right), \quad (20)$$

where

$$R = \int_0^\infty G_s(\omega)d\omega, \quad R'' = \int_0^\infty -\omega^2 G_s(\omega)d\omega,$$

S is random stress when average value is zero. Generally, the fatigue curve can be described by

$$S^m N = C, \quad (21)$$

where S is reversed stress amplitude, N is number of cycles to failure, and m and C are material constants of the structure.

According to the Miner cumulative-damage law [7, 8], the fatigue life computational equation can be obtained as follows:

$$T_f = \left[\int_0^\infty \frac{f(S)S^m}{C} dS \right]^{-1}. \tag{22}$$

From statistics, the fatigue life of structure accords with log normal distribution. In the light of this, the fatigue life equation in different reliabilities can be deduced as

$$N(R) = \exp \left\{ \sqrt{\ln[1 + \eta^2(N)]} \Phi^{-1}[1 - R(N)] + \ln \bar{N} - \frac{1}{2} \ln[1 + \eta^2(N)] \right\}, \tag{23}$$

where

$$R(N) = 1 - \Phi \left[\frac{\ln N - \ln(\bar{N} / \sqrt{1 + \eta^2(N)})}{\sqrt{\ln(1 + \eta^2(N))}} \right], \tag{24}$$

$\eta(N) = \sigma_N / \mu_N$ is called change coefficient, μ_N and σ_N are average value and standard deviation, and \bar{N} is the life when the reliability is 50%. Its value is equal to T_f .

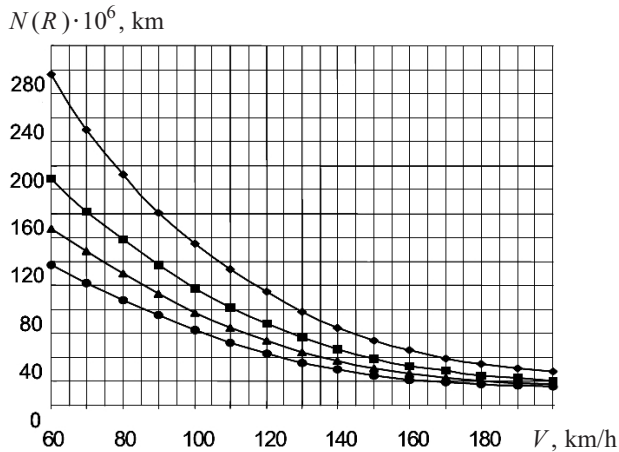


Fig. 5. The fatigue life curve of the side frame of the CW-200k type truck for different reliabilities [(♦) $P = 50\%$; (■) $P = 90\%$; (▲) $P = 99\%$; (●) $P = 99.9\%$].

In this paper, we computed the fatigue life of a side frame for four reliability, when the train runs at 60 to 200 km/h, as shown in Fig. 5.

Conclusions. A method for calculating the stress power spectral density of a general linear dynamic system on the basis of the unit load stress matrix is put forward. The relationship between the stress power spectral density of dynamic components and the response spectra of dynamic system is established, and the corresponding formulas are derived. It is very important for the numerical emulation in dynamic fatigue design and reliability fatigue life prediction by computer. The dynamic response computation and fatigue life prediction of the CW-200k type vehicle truck are developed using the method and the high level ORE track spectrum.

The proposed method is suitable not only for the running dynamic system but also for new design dynamic systems.

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Резюме

У сучасних інженерних розрахунках важливе місце займають розрахунки втомної довговічності й оцінка надійності динамічних систем при їх випадковому навантаженні. Для проведення цих розрахунків необхідно знати густину спектра напруги для досліджуваного компонента динамічної системи. Сучасний рівень розвитку комп'ютерних технологій і теорій числового моделювання дозволяє моделювати густину спектра напруг динамічних систем. На основі цього запропоновано метод числового моделювання для прогнозування втомної довговічності динамічної конструкції, що базується на модельних рівняннях для густини спектра напруг і втомної довговічності. Як приклад виконано віртуальний аналіз втомної довговічності і надійності великовантажного грузового автомобіля CW-200k. Показано застосування методу для оцінки існуючих динамічних конструкцій та його перспективи для оцінки втомної довговічності розроблюваних нових конструкцій.

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