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FINITE-DIMENSIONAL REDUCTIONS OF CONSERVATIVE DYNAMICAL SYSTEMS AND NUMERICAL ANALYSIS. I

СКІНЧЕННОВИМІРНІ РЕДУКЦІЇ КОНСЕРВАТИВНИХ ДИНАМІЧНИХ СИСТЕМ І ЧИСЕЛЬНИЙ АНАЛІЗ. I

The paper deals with the infinite-dimensional Liouville – Lax integrable nonlinear dynamical systems for which a problem of finding an appropriate set of initial conditions for corresponding to such their typical solutions as the solitons and travelling waves is studied. An approach for solving the problem is developed which based on the exact reduction of the given nonlinear dynamical system on its finite-dimensional invariant submanifolds and the sequel investigation of the system of ordinary differential equations obtained by means of qualitative analysis. The effectiveness of the proposed approach is demonstrated on examples of Korteweg – de Vries equation, modified nonlinear Schrödinger equation and some hydrodynamical model.

Вивчаються нескінченновимірні інтегровні за Лаксом – Ліувіллем нелінійні динамічні системи, для яких розглядається задача про знаходження множини початкових значень, яким відповідають такі типові їх розв'язки, як солітонні розв'язки та розв'язки вигляду біжучої хвилі. Запропоновано підхід до розв'язання даної задачі, суть якого полягає в редукції вихідної нелінійної динамічної системи на її скінченновимірні інваріантні підмноговиди та в подальшому дослідженні за допомогою методів якісної теорії диференціальних рівнянь одержаних систем. Ефективність запропонованого підходу продемонстровано на прикладі рівняння Кортевега – де Фріза, нелінійного модифікованого рівняння Шредінгера та однієї гідродинамічної моделі.

Introduction. The problem of finding an appropriate set of initial conditions for the infinite-dimensional Liouville – Lax integrable dynamical systems leading to such typical solutions as the travelling waves and solitons has been important problem for numerical analysis of integrable equations [1].

In this paper we make an attempt to develop a regular method of finding various types of initial conditions by employing the method of reductions of the infinite-dimensional integrable systems on finite-dimensional invariant submanifold [2].

The reduced set of equations on a submanifold consists of a pair of Hamiltonian systems integrable in the classical Liouville sense. The first system is associated with the vector field d/dx on the finite-dimensional submanifold and its solutions define a set of initial conditions for the given infinite-dimensional integrable equation. The other finite-dimensional Hamiltonian system corresponds to the vector field d/dt on the submanifold and defines the time evolution of the initial data due to the dynamics of the infinite-dimensional system. The phase portrait of the dynamical system corresponding to the vector field d/dx provides necessary information for identifying the initial conditions for the solitons and travelling waves.

The method can be applied for the numerical analysis of not only the Liouville – Lax integrable dynamical systems but also to the conservative nonlinear dynamical systems possessing several conserved quantities.

The paper is organized as follows. In Section 1 we formulate the basic concepts of the method by Bogoyavlensky and Novikov of finite-dimensional reductions of the Liouville – Lax integrable dynamical systems [2]. In Section 2 these ideas are applied for the numerical study of the Korteweg – de Vries equation. The finite-dimensional reductions of the modified nonlinear Schrödinger equation and the analysis of the corresponding Hamiltonian equations are presented in Section 3. Section 4 demonstrates the applicability of these ideas to one hydrodynamical model possessing four conservative quantities. We conclude the paper with a discussion of our results and perspectives for the future work.

1. Finite-dimensional reductions. Let the dynamical system

$$u_t = K[u] \tag{1}$$

be given on the manifold $M \subset C^{(\infty)}(\mathbf{R}/2\pi; \mathbf{R}^n)$ of smooth 2π-periodic functions.

We denote by $K : M \to T(M)$ the Fréchet smooth tangent vector field on the manifold M representing the dynamical system (1).

Let $\mathcal{D}(M)$ be the space of Fréchet smooth functionals on M. We shall define the operator grad: $\mathcal{D}(M) \to T^*(M)$ by

$$\operatorname{grad} F = \frac{\delta F}{\delta u}$$

for $F \in \mathcal{D}(M)$, where $\delta(\cdot)/\delta u$ is the Euler variational derivative

$$\frac{\delta(\cdot)}{\delta u} = \sum_{k=0}^{\infty} (-1)^k \left(\frac{d}{dx}\right)^k \frac{\partial(\cdot)}{\partial u^{(k)}}.$$

The Poisson bracket of any pair of functionals $F, G \in \mathcal{D}(M)$ is a functional $\{F, G\}_\theta \in \mathcal{D}(M)$ defined by

$$\{F, G\}_\theta = (\operatorname{grad} F, \theta \operatorname{grad} G) \equiv \int_{x_0}^{x_0+2\pi} \langle \operatorname{grad} F, \theta \operatorname{grad} G\rangle \, dx, \tag{2}$$

where θ is a skew-symmetric operator $\theta : T^*(M) \to T(M)$ chosen in such a way, that the Poisson bracket (2) satisfies the Jacobi identity

$$\{\{F, G\}_\theta, H\}_\theta + \{\{G, H\}_\theta, F\}_\theta + \{\{H, F\}_\theta, G\}_\theta = 0$$

for all $F, G, H \in \mathcal{D}(M)$.

Therefore, the operator $\theta : T^*(M) \to T(M)$ determines a symplectic structure on the manifold M.

It is known [3, 4] that the symplectic structure determined by the operator $\theta : T^*(M) \to T(M)$ is invariant with respect to the phase flow of a dynamical system (1) if and only if

$$L_K \theta = 0, \tag{3}$$

where L_K is the Lie derivative along the vector field $K : M \to T(M)$.

An operator possessing this property is called a *Noetherian operator*.

The explicit form of the expression (3) reads

$$\frac{d\theta}{dt} - \theta K'^* - K'\theta = 0, \tag{4}$$

where $K' : T(M) \to T(M)$ is the Fréchet derivative of the vector field $K : M \to T(M)$ and $K'^* : T^*(M) \to T^*(M)$ is the adjoint operator to $K' : T(M) \to T(M)$ with respect to the former bilinear form on $T^*(M) \times T(M)$.

If (4) holds for vector fields like $\vartheta\varphi \in T(M)$ for all $\varphi \in T^*(M)$, then the Jacobi identity for the Poisson bracket (2) is satisfied automatically [3].

Similarly to (3), a functional $\varphi \in T^*(M)$ is related to a conservation law of the dynamical system determined by the vector field $K : M \to T(M)$ if and only if $\varphi' = \varphi'^*$ and

$$L_K \varphi = 0,$$

or, in explicit form,

$$\frac{d\varphi}{dt} + K'^{*}\varphi = 0. \tag{5}$$

The equations (4) and (5) can be solved by using some special asymptotic methods developed in [4, 5].

The typical features of a Liouville – Lax integrable dynamical system are the existence of two nonequivalent solutions to the equation (4), θ and $\tilde{\theta}$, defining two symplectic structures on the phase space and the existence of an infinite hierarchy of conserved quantities

$$H_i = \int_{x_0}^{x_0+2\pi} \mathcal{H}_i[u]\,dx, \tag{6}$$

with $\varphi_i := \operatorname{grad} H_i$, $i \in \mathbf{Z}_+$, satisfying the equation (5).

All conserved quantities are in involution to each other with respect to the Poisson brackets (2) defined by any compatible [3, 4] operators θ and η:

$$\{H_i, H_j\}_\theta = 0 = \{H_i, H_j\}_\eta,$$

for all $i, j \in \mathbf{Z}_+$, and the dynamical system (1) is representable in the bi-Hamiltonian form

$$u_t = K[u] = -\theta \operatorname{grad} H = -\eta \operatorname{grad} \tilde{H},$$

with the Hamiltonian functions H and $\tilde{H}$ being elements of the hierarchy of conservation laws (6) or linear combinations of a finite number of conserved quantities.

The set of fixed points of an invariant functional $\mathcal{L}_N \in \mathcal{D}(M)$, $N \in \mathbf{Z}_+$, is a finite-dimensional submanifold $M_N \subset M$ invariant with respect to the dynamics of (1) and all other vector fields, generated by the hierarchy (6).

The invariant submanifold $M_N \subset M$ can be represented as follows:

$$M_N = \{u \in M : \operatorname{grad} \mathcal{L}_N[u] = 0\},$$

where $\mathcal{L}_N$ is a Lagrangian function chosen for instance like

$$\mathcal{L}_N = H_N + \sum_{j=0}^{N-1} c_j H_j \equiv \int_{x_0}^{x_0+2\pi} \mathcal{L}_N[u]\,dx,$$

and $c_j \in \mathbf{R}$, $j = \overline{0, N-1}$, are arbitrary constants. There exists a natural set of the canonical (Hamiltonian) variables on the manifold $M_N \subset M$.

The system

$$\operatorname{grad} \mathcal{L}_N[u] := \frac{\delta \mathcal{L}_N[u]}{\delta u} = 0 \tag{7}$$

is a Lagrangian dynamical system but it can be represented also in the form of the canonical Hamiltonian equations [6] defining the Liouville integrable dynamical system (vector field) d/dx:

$$\begin{aligned} \frac{dq_i}{dx} &= \frac{\partial h_N^{(x)}}{\partial p_i}, \\ \frac{dp_i}{dx} &= -\frac{\partial h_N^{(x)}}{\partial q_i}, \end{aligned} \tag{8}$$

where

$$q_i = u^{(i-1)}, \quad p_i = \frac{\delta \mathcal{L}_N[u]}{\delta u^{(i)}} = \sum_{k=0}^{\infty} (-1)^k \left(\frac{d}{dx}\right)^k \frac{\delta \mathcal{L}_N[u]}{\partial u^{(k+i)}},$$

$i = \overline{1, \overline{N}(N)}$, $\overline{N}(N) = 1/2 \dim M_N$, and the Hamiltonian function $h_N^{(x)} \in \mathcal{D}(M_N)$ is of the form

$$h_N^{(x)}(p, q) = \sum_{i=1}^{\overline{N}} \langle p_i, u^{(i)} \rangle - \mathcal{L}_N[u],$$

satisfying the equation

$$\frac{dh_N^{(x)}}{dx} = -\langle \operatorname{grad} \mathcal{L}_N[u], u_x \rangle,$$

where $\langle \cdot, \cdot \rangle$ denotes the scalar product in the Euclidian space $\mathbf{E}^{\overline{N}} := (\mathbf{R}^n; \langle \cdot, \cdot \rangle)$.

The expression (7) holds on the submanifold M_N. This implies that the function $h_N^{(x)}$ remains constant as its arguments change according to the system (7).

Any solution to the corresponding Hamiltonian system which belongs to M_N can be used as an initial condition for the given infinite-dimensional dynamical system (1). The time evolution of the integral curves of the vector field d/dx is defined by a finite-dimensional vector field d/dt on M_N that is an exact reduction of the dynamical system (1).

It can be shown (see [5]) that d/dt is also a Liouville integrable Hamiltonian vector field with the canonical Poisson structure and the Hamiltonian function $h_N^{(t)} \in \mathcal{D}(M_N)$ determined by the equation

$$\frac{dh_N^{(t)}}{dt} = -\langle \operatorname{grad} \mathcal{L}_N[u], K[u] \rangle.$$

The Liouville integrability of the vector field d/dt implies [1] the existence of the quasiperiodic dynamics of the solutions starting from some of the integral curves of the vector field d/dx.

We shall demonstrate in the sequel how the phase plane analysis of the dynamical system (8) associated with some integrable or only conservative infinite-dimensional dynamical systems makes it possible to find the initial conditions for such solutions as the solitons and travelling waves that propagate without changing in shape.

2. The Korteweg – de Vries equation. We shall start with the most popular equation of the theory of integrability — the Korteweg – de Vries (KdV) equation:

$$u_t + u_{xxx} + 6uu_x = 0. \tag{9}$$

Let the phase space of (9) be an 2π-periodic manifold. It is known that the phase space associated with the KdV equation possesses a pair of symplectic structures defined by the following implectic Noetherian operators

$$\theta = \frac{d}{dx} := \partial, \quad \eta = \partial^3 + 2u\partial + 2\partial u. \tag{10}$$

The first three terms of the infinite hierarchy of conservation laws for the KdV equation are

$$H_1 = \int_{x_0}^{x_0+2\pi} u\,dx, \quad H_2 = \int_{x_0}^{x_0+2\pi} \frac{u^2}{2}\,dx, \quad H_3 = \int_{x_0}^{x_0+2\pi} \left(-\frac{1}{2}u_x^2 + u^3\right)dx. \tag{11}$$

The bi-Hamiltonian representation of (9) is

$$u_t = -\theta \operatorname{grad} H_3 = -\eta \operatorname{grad} H_2.$$

Let us reduce the dynamics of the KdV equation on the submanifold M_3 of critical points of the following Lagrangian function:

$$\mathcal{L}_3 = c_1 H_1 + c_2 H_2 + H_3 \equiv \int_{x_0}^{x_0+2\pi} \mathcal{L}_3[u]\, dx.$$

The submanifold $M_3 \subset M$ is constituted by all solution of the equation of the form (7) the explicit form of which is

$$c_1 + c_2 u + u_{xx} + 3u^2 = 0. \tag{12}$$

The equation (12) can be written in a form of the canonical Hamiltonian equations by using the following canonical coordinates:

$$q := u, \quad p := \frac{\delta \mathcal{L}_3[u]}{\delta u_x} = -u_x$$

and the Hamiltonian function

$$h_3^{(x)} = c_1 q + \frac{1}{2} p^2 + \frac{c_2}{2} q^2 + q^3,$$

satisfying the equation (8).

Therefore, the Hamiltonian equations defining the vector field d/dx on the submanifold M_3 are

$$\frac{dp}{dx} = c_1 + c_2 q + 3q^2, \qquad \frac{dq}{dx} = -p. \tag{13}$$

There are two fixed points on the phase plane with the coordinates $(p_0, q_0) = (0, (-c_2 \pm \sqrt{c_2^2 - 12c_1})/6)$ and the corresponding eigenvalues of the linearized problem are $\lambda_{1,2} = \pm (c_2^2 - 12c_1)^{1/4}$ and $\lambda_{3,4} = \pm (c_2^2 - 12c_1)^{1/4}$.

For the specific values of the constants $c_1 = 0$, $c_2 = 4$ there is the hyperbolic fixed point at $(p_0, q_0) = (0, -4/3)$ and the elliptic one located at $(0, 0)$.

There are one-dimensional stable and unstable manifolds and the homoclinic separatrix to the hyperbolic fixed point on the phase portrait. The trajectories inside the homoclinic separatrix are periodic in x and can be used as the initial conditions for the periodic KdV equation.

The time evolution of these initial profiles can be found by integrating the canonical Hamiltonian system of equations defined by the Hamiltonian function $h_3^{(t)} \in \mathcal{D}(M_N)$ (10) and associated with the vector field d/dt on the submanifold M_3. The calculation show that $h_3^{(t)}$ and $h_3^{(x)}$ are linearly dependent: $h_3^{(t)} = c_2 h_3^{(x)}$.

One easily demonstrates the travelling wave propagating without change of shape. In fact, the exact analytic solution in terms of the Weierstrass elliptic function for the cnoidal waves associated with the periodic KdV equation [2, 5]

$$x - x_0 = \int_{q_0}^{q} \frac{dq}{\sqrt{2q^3 + c_2 q^2 + 2c_1 q + \beta}}$$

with $u = u(x - c_2 t)$, $c_2 \in \mathbf{R}$, being the wave velocity and $\beta \in \mathbf{R}$ being a parameter, satisfies the set of equation (13).

The most interesting observation is that the homoclinic orbit to the hyperbolic fixed point on the phase portrait of the dynamical system d/dx provides the initial condition for the soliton solution of the KdV equation on the infinite domain with the boundary conditions of Schwarz type.

The propagation in time of the soliton is obtained [7] by integrating numerically the dynamical system d/dt.

3. The modified nonlinear Schrödinger equation. Our next example deals with the finite-dimensional reductions of the modified nonlinear Schrödinger equation

$$\left.\begin{aligned} \psi_t &= i\psi_{xx} - (\psi^2\psi^*)_x \\ \psi_t^* &= i\psi_{xx}^* - (\psi\psi^{*2})_x \end{aligned}\right\} := K[u], \tag{14}$$

where $(\psi, \psi^*) \in M \subset C^{(\infty)}(\mathbf{R}/2\pi; \mathbf{C}^2)$.

The dynamical system (14) is exactly Liouville – Lax integrable [4].

The following implectic Noetherian operators

$$\theta = \begin{pmatrix} 0 & \partial \\ \partial & 0 \end{pmatrix}, \quad \eta = \begin{pmatrix} -\psi\partial^{-1}\psi & i + \psi\partial^{-1}\psi^* \\ -i + \psi^*\partial^{-1}\psi & \psi^*\partial^{-1}\psi^* \end{pmatrix}$$

define two compatible Poisson brackets on the space of functionals $\mathcal{D}(M)$.

The system (14) possesses an infinite hierarchy of conserved quantities the first three terms of which are

$$H_1 = \int\limits_{x_0}^{x_0+2\pi} i\psi\psi^*\, dx, \quad H_2 = \int\limits_{x_0}^{x_0+2\pi} \left(-i\psi^*\psi_x + \frac{\psi^2\psi^{*2}}{2}\right) dx,$$

$$H_3 = \int\limits_{x_0}^{x_0+2\pi} \left(i\psi^*\psi_{xx} - 2\psi\psi^{*2}\psi_x - \frac{\psi^2\psi^*\psi_x^*}{2} - i(\psi^*\psi)^3\right) dx.$$

All conserved quantities are in involution with respect to the pair of the Poisson brackets defined by the operators θ and $\eta : T^*(M) \to T(M)$.

Therefore the corresponding tangent vector fields $K_i := -\theta \operatorname{grad} H_i$, $i \in \mathbf{Z}_+$, are mutually commuting.

The reduction procedure for the nonlinear modified Schrödinger equation (14) on the submanifold of critical points of the first two conserved quantities leads to a two-dimensional Hamiltonian system which doesn't exhibit an interesting dynamics. Therefore we should consider the next submanifold given as a set of critical points of the following high order Lagrangian function:

$$\mathcal{L}_3 = c_1 H_1 + c_2 H_2 + H_3.$$

Applying the same reduction procedure as in the previous section we obtain the four dimensional system corresponding to the vector field d/dx:

$$\begin{aligned} \frac{dp_1}{dx} &= c_1 q_2 - c_2 q_1 q_2^2 + 3q_1^2 q_2^3, & \frac{dq_1}{dx} &= -p_2, \\ \frac{dp_2}{dx} &= c_1 q_1 - c_2 q_1^2 q_2 + 3q_1^3 q_2^2, & \frac{dq_2}{dx} &= -p_1. \end{aligned} \tag{15}$$

The system (15) is a set of the canonical Hamiltonian equations with respect to the Hamiltonian function

$$h_3^{(x)} = c_1 q_1 q_2 + p_1 p_2 - \frac{c_2}{2} q_1^2 q_2^2 + q_1^3 q_2^3,$$

and the canonical coordinates are

$$q_1 = \psi, \quad q_2 = i\psi^*, \quad p_1 = -i\psi_x^*, \quad p_2 = -\psi_x.$$

We shall analyse the phase portrait of the system (15) for some fixed values of the parameters, in particular, we take $c_1 = -3$, $c_2 = 0$. The origin of the phase plane $(q_1, p_1, q_2, p_2) = (0, 0, 0, 0)$ is a hyperbolic fixed point.

The following curves in the four-dimensional phase space

$$p_1 = p_2 = 0, \quad q_1 = \pm\frac{1}{q_2}$$

are the manifolds of the elliptic fixed points.

The typical solutions $q_1(x)$ that can be used as the initial conditions for the infinite-dimensional system (14) can be easily depicted. The time evolution of these initial profiles are obtained by integrating the set of the Hamiltonian equations corresponding to the vector field d/dt on the submanifold M_3.

As in the previous example, the periodic initial profiles lead to the travelling waves propagating without change in the shape. The homoclinic separatrix to the hyperbolic fixed point $(0, 0, 0, 0)$ provides the soliton-type initial condition.

4. A hydrodynamical model. The following hydrodynamical system of equations was used in [7, 8] for the description of the surface evolution of thin fluid jets and fluid sheets:

$$\left.\begin{aligned} u_t &= v_{xxx} - u_x u \\ v_x &= -(uv)_x \end{aligned}\right\} := K[u, v]. \tag{16}$$

Various types of the dynamics of the system (16) were described in [7, 8] by means of numerical methods based on the pseudospectral in space and Runge – Kutta in time technique. The surface instability of fluid sheets were studied as well as a wide range of the quasiperiodic solutions.

We shall demonstrate in this section that a class of the travelling wave solutions can be obtained by using the finite-dimensional reduction approach.

The system (16) is conservative. It possesses the following conserved quantities

$$H_1 = \int_{x_0}^{x_0+2\pi} u\,dx, \quad \tilde{H}_1 = \int_{x_0}^{x_0+2\pi} v\,dx,$$

$$H_2 = \int_{x_0}^{x_0+2\pi} uv\,dx, \tag{17}$$

$$H_3 = \frac{1}{2}\int_{x_0}^{x_0+2\pi} (v_x^2 + u^2 v)\,dx.$$

It was shown in [9] that the dynamical system (16) is Hamiltonian with respect to the canonical Poisson bracket defined by the operator

$$\theta = \begin{pmatrix} 0 & \partial \\ \partial & 0 \end{pmatrix}$$

and the Hamiltonian function $H = H_3$:

$$(u_t, v_t)^T = -\theta \operatorname{grad} H_3[u, v].$$

It was also shown in [9] that the dynamical system (16) possesses the Lax type representation: it is equivalent to the following operational expression

$$L_t = LP - PL$$

where

$$L = v^{-1}\partial + u, \tag{18}$$

$$P = u\partial + \left(vv_{xx} - \frac{v_x^2}{2}\right). \tag{19}$$

The exact integrability of the dynamical system (16) remains still an open question: the first order Lax operator (18) doesn't yield an infinite hierarchy of conserved quantities and no solitary wave solutions can be found in an infinite spatial domain due to the triviality of the Lax spectral problem.

Nevertheless, since the conserved quantities (17) are in involution to each other with respect to the canonical Poisson bracket, the approach used in the previous sections can be applied to the dynamical system (16).

Let us consider the finite-dimensional reductions of the dynamical system (1) on the submanifold M_3 of critical points of the following Lagrangian function

$$\mathcal{L}_3 \equiv \int\limits_{x_0}^{x_0+2\pi} \mathcal{L}_3[u, v]\, dx = c_1(H_1 + \tilde{H}_1) + c_2 H_2 + c_3 H_3.$$

The equation (7) defines the Lagrangian dynamical system and the constraint $2c_3 u + c_1/v + c_2 = 0$.

One choice of the canonical Hamiltonian variables q and p on the submanifold M_3 is

$$q = v, \qquad p = \frac{\delta \mathcal{L}_3[u, v]}{\delta v_x} = c_3 v_x.$$

By using the equations like (7), (8) one can obtain the Hamiltonian functions $h_3^{(x)}$ and $h_3^{(t)}$ determining the Hamiltonian vector fields d/dx and d/dt on M_3:

$$h_3^{(x)} = \frac{1}{4c_3} p^2 + \left(\frac{c_2^2}{4c_3} - c_1\right) q + \frac{c_1^2}{4c_3}\frac{1}{q},$$

$$h_3^{(t)} = \frac{c_2}{8c_3^2} p^2 - \left(\frac{c_1 c_2}{2c_3} - \frac{c_2^3}{8c_3^2}\right) q + \frac{c_1^2 c_2}{8c_3^2}\frac{1}{q}.$$

We integrated the corresponding Hamiltonian equations numerically by using the Runge – Kutta method for the following numerical values of the parameters: $c_1 = 1$, $c_2 = \sqrt{4 + 1/\pi^2}$, $c_3 = 1$.

The phase portrait of the Hamiltonian dynamical system

$$\frac{dq}{dx} = \frac{\partial h_3^{(x)}}{\partial p},$$

$$\frac{dp}{dx} = -\frac{\partial h_3^{(x)}}{\partial q},$$

corresponding to the vector field d/dx can be plotted via a simple integration.

There are two fixed points in the phase space of the system d/dx: the hyperbolic point with coordinates $(-\pi, 0)$ and the elliptic one located at $(\pi, 0)$. The physically realistic solutions corresponding to the subdomain of positive values of q are periodic in x. Some typical solutions defining the initial conditions for the infinite-dimensional dynamical system (16) can be easily extracted.

Notice that the independent and dependent variables can be rescaled to obtain l-periodicity of any curve. We studied the dynamics due to the Hamiltonian dynamical system

$$\frac{dq}{dx} = \frac{\partial h_3^{(l)}}{\partial p},$$

$$\frac{dp}{dx} = -\frac{\partial h_3^{(l)}}{\partial q},$$

corresponding to the vector field d/dx and found the time evolution of the initial data.

The solution has the form of a small amplitude travelling wave. The absence of a homo- or heteroclinic orbit to a hyperbolic fixed point on the phase space nevertheless doesn't indicate that the soliton solutions to the dynamical system (16) don't exist.

Since the system (16) is a natural generalization of the well known Burgers flow possessing both dissipative and soliton like solutions, the corresponding Cauchy data at which solutions are solitonic should be treated via the reduction method based on the well known Moser's mapping approach, devised in [5]. This trend of our studying the system (16) we are going to perform in detail in a work under preparation.

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