

**АНАЛИЗ ЧИСЛЕННОГО
МОДЕЛИРОВАНИЯ
НЕИЗОТЕРМИЧЕСКИХ ПРОЦЕССОВ
В ГРУНТОВОМ МАССИВЕ**

1.

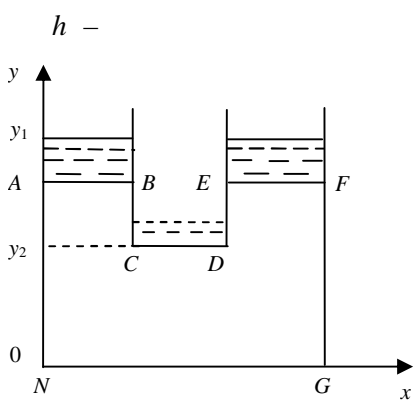
I [1] $\Omega_{\tilde{T}} = \Omega \times (0, \tilde{T}]$ (. 1) -

$$\begin{aligned} \tilde{\mu} \frac{\partial h}{\partial t} - \frac{\partial}{\partial x} \left(K(T, \theta) \frac{\partial h}{\partial x} \right) - \frac{\partial}{\partial y} \left(K(T, \theta) \frac{\partial h}{\partial y} \right) = 0, \\ c_T \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left(\lambda_T \frac{\partial T}{\partial x} - c v_x T \right) - \\ - \frac{\partial}{\partial y} \left(\lambda_T \frac{\partial T}{\partial y} - c v_y T \right) = 0, \end{aligned}$$

.....

$$\frac{\partial^2 u}{\partial t^2} - \mu \Delta u - (\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \frac{\partial P}{\partial x} = 0,$$

$$\frac{\partial^2 v}{\partial t^2} - \mu \Delta v - (\lambda + \mu) \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \frac{\partial P}{\partial y} = -g, \quad (x, y, t) \in \Omega_{\tilde{T}}, \quad (1)$$



; $T =$; $w = (u, v)^T$ -
; $P = \rho g(h - y)$ -
; $K(T, \theta) =$ -
; $\theta = \partial u / \partial x + \partial v / \partial y$ -
; $(v_x, v_y)^T =$ -
; $\lambda, \mu =$; $\tilde{\mu} =$ -
; $c_T, c =$ -
; $\lambda_T =$ -
; $\rho =$ -

. 1.

:

$$h(x, y, t) = y_1, \quad (x, y, t) \in \Gamma^1 \times (0, \tilde{T}], \quad h(x, y, t) = y_2 + \tilde{h}(t), \quad (x, y, t) \in CD \times (0, \tilde{T}], \quad (2)$$

$$\frac{\partial h}{\partial n} = 0, \quad (x, y, t) \in (\Gamma^2 \cup GN) \times (0, \tilde{T}], \quad \frac{\partial T}{\partial n} = 0, \quad (x, y, t) \in \Gamma^2 \times (0, \tilde{T}], \quad (3)$$

$$T(x, y, t) = T_1, \quad (x, y, t) \in (\Gamma^1 \cup CD) \times (0, \tilde{T}], \quad T(x, y, t) = T_2, \quad (x, y, t) \in GN \times (0, \tilde{T}], \quad (4)$$

$$u = 0, \quad (x, y, t) \in \Gamma^2 \times (0, \tilde{T}], \quad u = v = 0, \quad (x, y, t) \in GN \times (0, \tilde{T}], \quad (5)$$

$$\tau_s = 0, \quad (x, y, t) \in (\partial\Omega \setminus GN) \times (0, \tilde{T}], \quad \sigma_n = 0, \quad (x, y, t) \in (\Gamma^1 \cup CD) \times (0, \tilde{T}], \quad (6)$$

$\Gamma^1 = AB \cup EF$, $\Gamma^2 = N \cup BC \cup DE \cup FG$, $n =$,
 $\sigma_n, \tau_s =$ -

$$: K(T, \theta) = \bar{K}(T) e^{f(\theta)},$$

$$\bar{K}(T) = \bar{K}(10)(0.7 + 0.03T), \quad \bar{K}(10) = \begin{cases} \bar{K}, & P \geq 0, \\ \bar{K} e^{2.48P}, & P < 0, \end{cases} \quad \bar{K} = 0.3 / ,$$

$$\tilde{h}(t) = \frac{1}{|CD|} \int_{CD} \int_0^t v_y(x, y, \tau) d\tau d\Gamma, \quad v_y(x, y, t) = -K(T, \theta) \frac{\partial h}{\partial y}, \quad \rho = 1.94 / ^3,$$

$$\tilde{\mu} = 10^{-3}, \quad E = 5000 / ^3, \quad \nu = 0.3.$$

:

$$\begin{aligned}
 h(x, y, 0) &= H_0(x, y), \quad T(x, y, 0) = T_0(x, y), \quad u(x, y, 0) = U_0(x, y), \\
 v(x, y, 0) &= V_0(x, y), \quad \frac{\partial u}{\partial t}(x, y, 0) = \frac{\partial v}{\partial t}(x, y, 0) = 0, \quad (x, y) \in \bar{\Omega}. \quad (7) \\
 \frac{\partial h}{\partial t}(x, y, 0) &= \tilde{H}_0(x, y),
 \end{aligned}$$

$$\frac{\partial T}{\partial t}(x, y, 0) = \tilde{T}_0(x, y), \quad (x, y) \in \bar{\Omega}. \quad Z$$

$$\begin{aligned}
 - \quad w(x, y, t) &= (h(x, y, t), T(x, y, t), u(x, y, t), v(x, y, t)) = \\
 &= (w_1, w_2, w_3, w_4), \quad ,
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial w}{\partial t}(x, y, t), \frac{\partial^2 w}{\partial t^2}(x, y, t) \quad \forall t \in (0, \tilde{T}], \quad \frac{\partial w}{\partial t}(x, y, 0), \quad w(x, y, 0) \\
 W_2^1(\Omega) \quad (2), (4), (5), \quad -
 \end{aligned}$$

[2]:

$$m\left(\frac{\partial^2 w}{\partial t^2}, z\right) + \bar{m}\left(\frac{\partial w}{\partial t}, z\right) + a(w, z) = (F, z), \quad w \in Z, \quad \forall z \in Z_0 \quad \forall t \in (0, \tilde{T}], \quad (8)$$

$$(w(\cdot, \cdot, 0), z) = (w_0, z), \quad \left(\frac{\partial w}{\partial t}(\cdot, \cdot, 0), z\right) = (\tilde{w}_0, z) \quad \forall z \in Z_0, \quad (9)$$

$$m\left(\frac{\partial^2 w}{\partial t^2}, z\right) = \rho \iint_{\Omega} \left(\frac{\partial^2 w_3}{\partial t^2} z_3 + \frac{\partial^2 w_4}{\partial t^2} z_4 \right) d\Omega,$$

$$\bar{m}\left(\frac{\partial w}{\partial t}, z\right) = \iint_{\Omega} \left(\tilde{\mu} \frac{\partial w_1}{\partial t} z_1 + c_T \frac{\partial w_2}{\partial t} z_2 \right) d\Omega,$$

$$a(w, z) = W_1(w, z) + W_2(w, z) + W_3(w, z),$$

$$W_1(w, z) = \iint_{\Omega} \left(K(T, \theta) \left(\frac{\partial w_1}{\partial x} \frac{\partial z_1}{\partial x} + \frac{\partial w_1}{\partial y} \frac{\partial z_1}{\partial y} \right) + \lambda_T \left(\frac{\partial w_2}{\partial x} \frac{\partial z_2}{\partial x} + \frac{\partial w_2}{\partial y} \frac{\partial z_2}{\partial y} \right) \right) d\Omega,$$

$$\begin{aligned}
 W_2(w, z) &= \iint_{\Omega} \left((\lambda + 2\mu) \left(\frac{\partial w_3}{\partial x} \frac{\partial z_3}{\partial x} + \frac{\partial w_4}{\partial y} \frac{\partial z_4}{\partial y} \right) + \lambda \left(\frac{\partial w_3}{\partial y} \frac{\partial z_4}{\partial x} + \frac{\partial w_4}{\partial x} \frac{\partial z_3}{\partial y} \right) \right. \\
 &\quad \left. + \mu \left(\frac{\partial w_3}{\partial y} + \frac{\partial w_4}{\partial x} \right) \left(\frac{\partial z_3}{\partial y} + \frac{\partial z_4}{\partial x} \right) \right) d\Omega,
 \end{aligned}$$

$$W_3(w, z) = \iint_{\Omega} \left(c K(T, \theta) w_2 \left(\frac{\partial w_1}{\partial x} \frac{\partial z_2}{\partial x} + \frac{\partial w_1}{\partial y} \frac{\partial z_2}{\partial y} \right) + \rho g (w_1 - y) \left(\frac{\partial z_3}{\partial x} + \frac{\partial z_4}{\partial y} \right) \right) d\Omega,$$

$$(F, z) = - \iint_{\Omega} \rho g z_4 d\Omega, \quad w_0 = (H_0, T_0, U_0, V_0)^T, \quad \tilde{w}_0 = (\tilde{H}_0, \tilde{T}_0, 0, 0)^T.$$

$$Z_0 \quad - \quad z(x, y) = (z_1, z_2, z_3, z_4)^T \quad -$$

$$W_2^1(\Omega), \quad \cdot$$

$$(8), (9) \quad w^N(x, y, t) \in Z^N$$

$$Z^N \subset Z.$$

II

[1, 3]

$$(1-m)\rho \frac{\partial^2 u}{\partial t^2} + \frac{\rho g m^2}{\bar{K}(T)} \left(\frac{\partial u}{\partial t} - \frac{\partial u}{\partial t} \right) - \left(\mu \Delta u + (\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) -$$

$$-(1-m) \frac{\partial P}{\partial x} = 0,$$

$$(1-m)\rho \frac{\partial^2 v}{\partial t^2} + \frac{\rho g m^2}{\bar{K}(T)} \left(\frac{\partial v}{\partial t} - \frac{\partial v}{\partial t} \right) - \left(\mu \Delta v + (\lambda + \mu) \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) -$$

$$-(1-m) \frac{\partial P}{\partial y} = -(1-m)\rho g,$$

$$m\rho \frac{\partial^2 u}{\partial t^2} + \frac{\rho g m^2}{\bar{K}(T)} \left(-\frac{\partial u}{\partial t} + \frac{\partial u}{\partial t} \right) - m \frac{\partial P}{\partial x} = 0,$$

$$m\rho \frac{\partial^2 v}{\partial t^2} + \frac{\rho g m^2}{\bar{K}(T)} \left(-\frac{\partial v}{\partial t} + \frac{\partial v}{\partial t} \right) - m \frac{\partial P}{\partial y} = -m\rho g,$$

$$\frac{\partial P}{\partial t} - \frac{M}{m} \left((1-m) \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + m \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) = 0,$$

$$c_T \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left(\lambda_T \frac{\partial T}{\partial x} - c \frac{\partial u}{\partial t} T \right) - \frac{\partial}{\partial y} \left(\lambda_T \frac{\partial T}{\partial y} - c \frac{\partial v}{\partial t} T \right) = 0, \quad (x, y, t) \in \Omega, \quad (10)$$

$$w = (u, v)^T \quad ; \quad \bar{K}(T) \quad -$$

$$, \quad ; \quad \rho \quad -$$

$$m \quad - \quad (\rho = \rho(1-m) + \rho m); \quad m = 0.39; \quad M \quad -$$

$$; \quad \rho = 2.54 / 3; \quad M = 203 \cdot 10^3 / 2.$$

$$\begin{aligned}
 & : \\
 & u = 0, \quad u = 0, \quad \tau_{xy} = 0, \quad \frac{\partial T}{\partial n} = 0, \quad (x, y, t) \in \Gamma^2 \times (0, \tilde{T}], \\
 & u = v = 0, \quad v = 0, \quad T(x, y, t) = T_2, \quad (x, y, t) \in GN \times (0, \tilde{T}], \\
 & \sigma_n = 0, \quad \tau_s = 0, \quad T(x, y, t) = T_1, \quad (x, y, t) \in (\Gamma^1 \cup CD) \times (0, \tilde{T}], \\
 & P = -\rho g y_1, \quad (x, y, t) \in \Gamma^1 \times (0, \tilde{T}], \quad P = -\rho g w_n, \quad (x, y, t) \in CD \times (0, \tilde{T}], \quad (11) \\
 & w_n = \dots \\
 & w = w - w \dots
 \end{aligned}$$

$$\begin{aligned}
 & : \\
 & u(x, y, 0) = u(x, y, 0) = U_0(x, y), \quad v(x, y, 0) = v(x, y, 0) = V_0(x, y), \\
 & P(x, y, 0) = \rho g(H_0(x, y) - y), \quad T(x, y, 0) = T_0(x, y), \\
 & \frac{\partial u}{\partial t}(x, y, 0) = \frac{\partial v}{\partial t}(x, y, 0) = \frac{\partial u}{\partial t}(x, y, 0) = \frac{\partial v}{\partial t}(x, y, 0) = 0, \quad (x, y) \in \bar{\Omega}. \quad (12) \\
 & - \quad (10) - (12), \quad - \\
 & \frac{\partial T}{\partial t}(x, y, 0) = \tilde{T}_0(x, y), \quad (x, y) \in \bar{\Omega},
 \end{aligned}$$

$$\begin{aligned}
 & - \quad w(x, y, t) = (u(x, y, t), v(x, y, t), u(x, y, t), v(x, y, t), T(x, y, t)) = \\
 & = (w_1, w_2, w_3, w_4, w_5) \in Z, \quad (8), (9),
 \end{aligned}$$

$$\begin{aligned}
 & m\left(\frac{\partial^2 w}{\partial t^2}, z\right) = \iint_{\Omega} \left(\rho(1-m) \left(\frac{\partial^2 w_1}{\partial t^2} z_1 + \frac{\partial^2 w_2}{\partial t^2} z_2 \right) + \rho m \left(\frac{\partial^2 w_3}{\partial t^2} z_3 + \frac{\partial^2 w_4}{\partial t^2} z_4 \right) \right) d\Omega, \\
 & \bar{m}\left(\frac{\partial w}{\partial t}, z\right) = \iint_{\Omega} \left(\frac{\rho g m^2}{\bar{K}(T)} \left(\frac{\partial(w_1 - w_3)}{\partial t} (z_1 - z_3) + \frac{\partial(w_2 - w_4)}{\partial t} (z_2 - z_4) \right) + T \frac{\partial w_5}{\partial t} z_5 \right) d\Omega,
 \end{aligned}$$

$$a(w, z) = W_1(w, z) + W_2(w, z) + W_3(w, z),$$

$$\begin{aligned}
 W_1(w, z) = & \iint_{\Omega} \left((\lambda + 2\mu) \left(\frac{\partial w_1}{\partial x} \frac{\partial z_1}{\partial x} + \frac{\partial w_2}{\partial y} \frac{\partial z_2}{\partial y} \right) + \lambda \left(\frac{\partial w_1}{\partial y} \frac{\partial z_2}{\partial x} + \frac{\partial w_2}{\partial x} \frac{\partial z_1}{\partial y} \right) + \right. \\
 & \left. + \mu \left(\frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial x} \right) \left(\frac{\partial z_1}{\partial y} + \frac{\partial z_2}{\partial x} \right) \right) d\Omega,
 \end{aligned}$$

$$\begin{aligned}
 W_2(w, z) = & \frac{M}{m} \iint_{\Omega} \left((1-m) \left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y} \right) + m \left(\frac{\partial w_3}{\partial x} + \frac{\partial w_4}{\partial y} \right) \right) \left((1-m) \left(\frac{\partial z_1}{\partial x} + \frac{\partial z_2}{\partial y} \right) + \right. \\
 & \left. + m \left(\frac{\partial z_3}{\partial x} + \frac{\partial z_4}{\partial y} \right) \right) d\Omega,
 \end{aligned}$$

$$W_3(w, z) = \iint_{\Omega} \left(\lambda_T \left(\frac{\partial w_5}{\partial x} \frac{\partial z_5}{\partial x} + \frac{\partial w_5}{\partial y} \frac{\partial z_5}{\partial y} \right) - c \frac{\partial w_3}{\partial t} w_5 \frac{\partial z_5}{\partial x} - c \frac{\partial w_4}{\partial t} w_5 \frac{\partial z_5}{\partial y} \right) d\Omega +$$

$$+ \int_{CD} \rho g (w_4 - w_2) ((1-m)z_2 + mz_4) d\Gamma,$$

$$(F, z) = \iint_{\Omega} \left(-(1-m)\rho g z_2 - m\rho g z_4 + \left(\frac{M}{m} \left(\frac{\partial U_0}{\partial x} + \frac{\partial V_0}{\partial y} \right) - \rho g (H_0 - y) \right) \times \right.$$

$$\left. \times \left((1-m) \left(\frac{\partial z_1}{\partial x} + \frac{\partial z_2}{\partial y} \right) + m \left(\frac{\partial z_3}{\partial x} + \frac{\partial z_4}{\partial y} \right) \right) \right) d\Omega - \int_{\Gamma^1} \rho g y_1 ((1-m)z_2 + mz_4) d\Gamma,$$

$$w_0 = (U_0, V_0, U_0, V_0, T_0)^T, \tilde{w}_0 = (0, 0, 0, 0, \tilde{T}_0)^T,$$

Z -

$$W_2^1(\Omega), \forall t \in (0, \tilde{T}]. \quad Z_0$$

$$z(x, y) = (z_1(x, y), z_2(x, y), z_3(x, y), z_4(x, y), z_5(x, y)) ,$$

$$z_i \in W_2^1(\Omega), \quad i = \overline{1, 5},$$

$$\{W^j(x, y)\}_{j=0}^J \subset Z_t^N$$

[2, 3].

$$H_0(x, y), T_0(x, y), U_0(x, y), V_0(x, y)$$

I

$$\tilde{H}_0(x, y) \equiv 0, \tilde{T}_0(x, y) \equiv 0.$$

2.

T_1

$$y_1 = |NA| + H \quad (H$$

l

I (. 2).

$$d\tilde{h} = (\tilde{h}(25) - \tilde{h}(15)) / \tilde{h}(15)$$

$$(\tilde{h}(T_1))(t)$$

$$T_1 = 25^\circ C \quad T_1 = 15^\circ C \quad (T_2 = 5^\circ C)$$

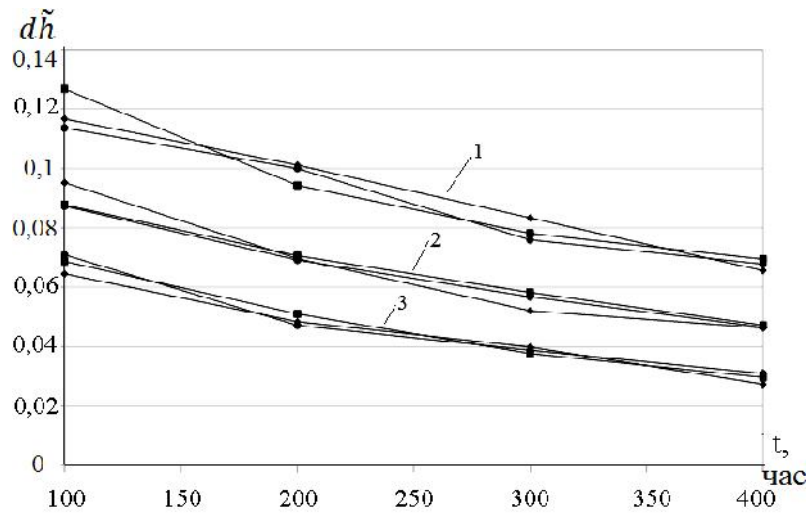
$$t = 400 \quad \therefore \quad l = 2 \quad d\tilde{h}$$

$$H \quad 0.07, \quad l = 5 \quad - 0.05, \quad l = 7 \quad - 0.03, \quad \dots,$$

II $d\tilde{h}$

$H \quad l$

$$(\quad t = 400 \quad \dots \quad 0.03).$$

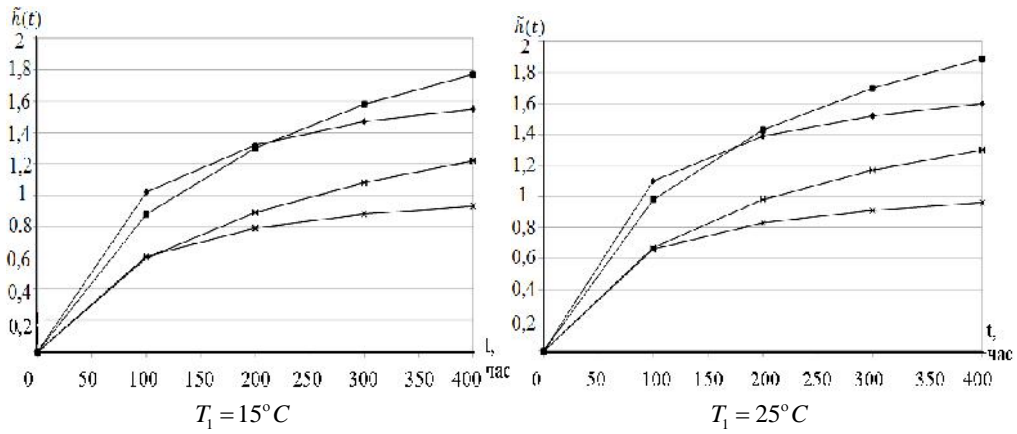


2. $d\tilde{h}$ I: 1 $l=2$,
 $2-l=5$, $3-l=7$; \blacklozenge $H=0.2$; \blacksquare $H=0.6$;
 \bullet $H=1.2$

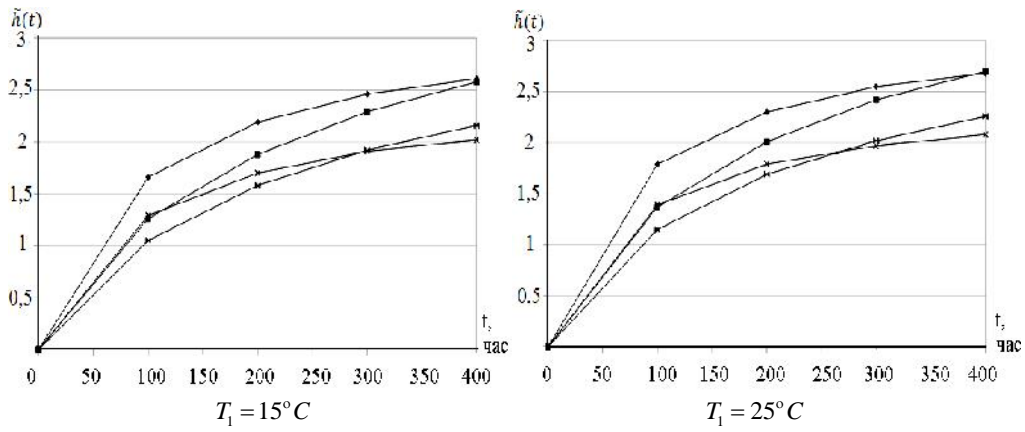
3, 4 $\tilde{h}(T_1)$ -
 : \blacklozenge I $H=0.2$, \blacksquare $H=1.2$;
 \times II $H=0.2$, \bullet $H=1.2$.
 $(u, v)^T$ -

II
 T_1 , u , v ,
 $T_1 = 15^\circ C$ $T_1 = 25^\circ C$,
 $du = \frac{u(25) - u(15)}{u(15)} (-0.03) H,$
 $dv = \frac{v(25) - v(15)}{v(15)} - (-0.35) H = 0.2 (-0.38) H = 1.2 .$
 u T_1 I (du -
 $(-0.05) - (-0.17) H,$
 dv $0.006 H = 0.2$ $0.025 H = 1.2$).

u v , I,
 II (
 60% u 40% v).



3. $l = 2$



4. $l = 5$

II

400 . T_1 15°C 25°C w -

$H = 1.2$ $l = 2$

($u - 7\%$, $v - 4.7\%$).

... , ... , ...

V.O. Bohaienko, O.O. Marchenko, T.A. Samoilenko

**AN ANALYSIS OF NUMERICAL NON-ISOTHERMAL PROCESSES
IN A SOIL MASSIF MODELING**

The problem of water inflow into the pit and comparative analysis of numerical realization of two different mathematical models describing non-isothermal processes of unsteady pressure filtration in the soil massif subject to deformation are considered.

1. 2014. 4. . 34 – 41.
2. 2012. 5. . 142 – 154.
3. 2009. 4. . 69 – 80.

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