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ON CLOSED FORM SOLUTIONS OF TRIPLE SERIES EQUATIONS INVOLVING LAGUERRE POLYNOMIALS

ПРО РОЗВ'ЯЗКИ ЗАМКНЕНОЇ ФОРМИ ДЛЯ РІВНЯНЬ ПОТРІЙНИХ РЯДІВ, ЩО МІСТЯТЬ ПОЛІНОМИ ЛАГЕРРА

We consider some triple series equations involving generalized Laguerre polynomials. The equations are reduced to triple integral equations of Bessel functions. Closed form solutions for the triple integral equations of Bessel functions are obtained and finally closed form solutions of triple series equations of Laguerre polynomials are obtained.

Розглянуто деякі рівняння потрійних рядів, що містять узагальнені поліноми Лагерра. Рівняння зведено до потрійних інтегральних рівнянь функцій Бесселя. Отримано розв'язки замкненої форми для потрійних інтегральних рівнянь функцій Бесселя, а також розв'язки замкненої форми для рівнянь потрійних рядів з поліномами Лагерра.

1. Introduction. Srivastava [1] was the first mathematician to solve dual series equations of Laguerre polynomials. Later on Lowndes [2], Srivastava [3 – 6] and Srivastava and Panda [7] generalized the dual series equations discussed by Srivastava [1]. In recent years Singh, Rokne and Dhaliwal [8, 9] have also discussed dual series equations of Laguerre polynomials and have obtained solutions in closed form.

Lowndes [10] and Dwivedi and Trivedi [11] solved triple series equations involving Laguerre polynomials and obtained solutions through Fredholm integral equations of the second kind which can be solved numerically. Further Lowndes and Srivastava [12] solved triple series equations of Laguerre polynomials and obtained a closed form solution by using the Erdélyi – Kober operator of fractional integration. In this paper we consider the triple series equations of Laguerre polynomials given by

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{\alpha}(x^2/2)}{\Gamma(\alpha+n+1)} = f_1(x), \quad 0 < x < a, \tag{1}$$

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{\beta}(x^2/2)}{\Gamma(\beta+n+1)} = g_2(x), \quad a < x < b, \tag{2}$$

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{\gamma}(x^2/2)}{\Gamma(\gamma+n+1)} = f_3(x), \quad b < x < \infty, \tag{3}$$

with three given functions $f_1(x)$, $g_2(x)$ and $f_3(x)$ of sufficient smoothness and three given parameters $\alpha, \beta, \gamma > -1$. Here $L_n^p(x)$, $p > -1$, $x \in \mathbf{R}$, $n \geq 0$, are Laguerre polynomials.

The series equations (l) – (3) have a more general form than the series equations discussed by Lowndes and Srivasava [12]. Particular cases of the series equations of the paper are discussed in Section 6. The aim of this paper is to find a closed form for the series equations (l) – (3).

2. Useful results. We need the following results for studying of the triple series equations (1), (2) and (3).

From the book of Bateman and Erdély ([13], Ch. 10, 10.12, (18)) we find the following result:

$$\sum_{n=0}^{\infty} \frac{t^{2n}}{2^n \Gamma(p+n+1)} L_n^p(x^2/2) = e^{t^2/2} \left(\frac{xt}{2} \right)^{-p} J_p(xt), \quad p > -1, \tag{4}$$

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where $J_p(\cdot)$ is the Bessel function of the first kind.

From Bateman and Erdély ([13], Ch. 10, 10.12, (21)) we find the integral

$$\int_0^\infty t^{p_1+2n+1}e^{-t^2/2}J_{p_1}(xt)\,dt = 2^n n!\,x^{p_1}e^{-x^2/2}L_n^{p_1}\left(\frac{x^2}{2}\right), \quad p_1 > -1. \tag{5}$$

If $\lambda_1 > \mu_1 > -1$ then the integral

$$\int_0^\infty J_{\lambda_1}(a\xi)J_{\mu_1}(b\xi)\xi^{1+\mu_1-\lambda_1}\,d\xi = \begin{cases} 0, & 0 < a < b, \\ \dfrac{b^{\mu_1}(a^2-b^2)^{\lambda_1-\mu_1-1}}{2^{\lambda_1-\mu_1-\lambda_1}\Gamma(\lambda_1-\mu_1)}, & 0 < b < a, \end{cases} \tag{6}$$

is a well-known result in the theory of Bessel function of Watson ([14, p. 401], equations (1) and (3)).

3. Reduction of triple series equations to triple integral equations. In this section we shall reduce the triple series equations (1), (2) and (3) to triple integral equations of Bessel functions.

Following Lowndes and Srivastava [12] the coefficients A_n in equations (1), (2) and (3) can be written in terms of an unknown function $A(t)$:

$$A_n = 2^{-n}\int_0^\infty t^{2n}e^{-t^2/2}A(t)\,dt, \quad n = 0, 1, 2, 3, \ldots . \tag{7}$$

Substituting equation (7) into equations (1), (2) and (3) and interchanging the order of summation and integrations and using equations (4) we find the following form of the triple integral equations for $\alpha, \beta, \gamma > -1$:

$$\int_0^\infty t^{-\alpha}A(t)J_\alpha(xt)\,dt = \left(\frac{x}{2}\right)^\alpha f_1(x), \quad 0 < x < a, \tag{8}$$

$$\int_0^\infty t^{-\beta}A(t)J_\beta(xt)\,dt = \left(\frac{x}{2}\right)^\beta g_2(x), \quad a < x < b, \tag{9}$$

$$\int_0^\infty t^{-\gamma}A(t)J_\gamma(xt)\,dt = \left(\frac{x}{2}\right)^\gamma f_3(x), \quad b < x < \infty. \tag{10}$$

In Sections A and B we find two different explict integral formulae for $A(x)$ for the two following cases:

$$0 < \alpha-\gamma < 1, \quad 0 < \beta-\gamma < 1, \quad \alpha, \beta, \gamma > -1, \tag{11}$$

$$0 < \beta-\gamma < 1, \quad 0 < \gamma-\alpha < 1, \quad \alpha, \beta, \gamma > -1. \tag{12}$$

4. Section A. In this section we shall find the solution of the triple integral equations (8), (9) and (10) subject to conditions (11).

We assume that

$$\int_0^\infty t^{-\gamma}A(x)J_\gamma(xt)\,dt = \psi(x), \quad 0 < x < b. \tag{13}$$

Using Hankel transforms and equations (10) and (13) we obtain

$$A(t) = t^{1+\gamma}\int_0^b u\psi(u)J_\gamma(ut)\,du + \frac{t^{1+\gamma}}{2^\gamma}\int_b^\infty u^{1+\gamma}f_3(u)J_\gamma(ut)\,du, \qquad 0 < t < \infty. \tag{14}$$

Substituting the value of $A(t)$ from equation (14) into equation (8) and interchanging the order of integrations, using the integral defined by equation (6) we get

$$\int_0^x \frac{u^{1+\gamma}\psi(u)\,du}{(x^2-u^2)^{1+\gamma-\alpha}} = \frac{x^{2\alpha}\Gamma(\alpha-\gamma)f_1(x)}{2^{\gamma+1}}, \qquad 0 < x < a, \qquad \alpha > \gamma > -1. \tag{15}$$

The above equation is an Abel's type integral equation if $\gamma > \alpha - 1$. Hence the solution of equation (15) can be written in the form

$$\psi(u) = \frac{u^{-1-\gamma}\sin[(\alpha-\gamma)\pi]\Gamma(\alpha-\gamma)}{2^\gamma\pi}\frac{d}{du}\int_0^u \frac{x^{1+2\alpha}f_1(x)\,dx}{(u^2-x^2)^{\alpha-\gamma}} = \frac{u^\gamma F_1(u)}{2^\gamma}, \quad 0 < x < a. \tag{16}$$

For obtaining equation (16) we have the condition

$$0 < 1-(\alpha-\gamma) < 1. \tag{17}$$

Substituting equation (14) into (9) and interchanging the order of integrations and using the result defined by equation (6) and assuming $\beta > \gamma$ we find that

$$\int_0^x \frac{u^{1+\gamma}\psi(u)\,du}{(x^2-u^2)^{1+\gamma-\beta}} = \frac{x^{2\beta}\Gamma(\beta-\gamma)g_2(x)}{2^{\gamma+1}}, \qquad a < x < b, \tag{18}$$

where

$$0 < 1+\gamma-\beta < 1. \tag{19}$$

With equation (16) we can write equation (18) in the form

$$\int_a^x \frac{u^{1+\gamma}\psi(u)\,du}{(x^2-u^2)^{1+\gamma-\beta}} = \frac{x^{2\beta}\Gamma(\beta-\gamma)g_2(x)}{2^{\gamma+1}} - \frac{1}{2^\gamma}\int_0^a \frac{p^{1+2\gamma}F_1(p)\,dp}{(x^2-p^2)^{1+\gamma-\beta}}, \qquad a < x < b. \tag{20}$$

Equation (20) is an Abel's type integral equation if $\gamma > \beta - 1$ and the right-hand side of equation (20) is known function of x. Hence the solution of equation (20) can be written as

$$\psi(u) = \frac{u^{-\gamma-1}\Gamma(\beta-\gamma)\sin[(\beta-\gamma)\pi]}{2^\gamma\pi}\frac{d}{du}\int_a^u \frac{x^{1+2\beta}g_2(x)\,dx}{(u^2-x^2)^{\beta-\gamma}} -$$

$$- \frac{u^{-\gamma-1}}{2^{\gamma-1}\pi}\sin[(\beta-\gamma)\pi]\frac{d}{du}\int_a^u \frac{x\,dx}{(u^2-x^2)^{\beta-\gamma}}\int_0^a \frac{p^{2\gamma+1}F_1(p)\,dp}{(x^2-p^2)^{1+\gamma-\beta}}, \qquad a < u < b. \tag{21}$$

From the integral

$$\frac{d}{du}\int_a^u \frac{x\,dx}{(u^2-x^2)^{\beta-\gamma}(x^2-p^2)^{1-(\beta-\gamma)}} = \frac{u(a^2-p^2)^{\beta-\gamma}}{(u^2-p^2)(u^2-a^2)^{\beta-\gamma}}, \quad p < a < u, \tag{22}$$

and interchanging the order of integrations in integrals of equation (21) we find that

$$\psi(u) = \frac{u^{-\gamma-1}\Gamma(\beta-\gamma)\sin[(\beta-\gamma)\pi]}{2^{\gamma}\pi}\frac{d}{du}\int_a^u \frac{x^{1+2\beta}g_2(x)\,dx}{(u^2-x^2)^{\beta-\gamma}} -$$

$$-\frac{u^{-\gamma}}{2^{\gamma-1}\pi}\frac{\sin[(\beta-\gamma)\pi]}{(u^2-a^2)^{\beta-\gamma}}\int_0^a \frac{p^{2\gamma+1}F_1(p)(a^2-p^2)^{\beta-\gamma}\,dp}{u^2-p^2} = \tag{23}$$

$$= \frac{u^{\gamma}G_2(u)}{2^{\gamma}}, \quad a < u < b. \tag{24}$$

Equation (14) together with equations (16) and (24) lead to

$$A(t) = 2^{-\gamma}t^{1+\gamma}\left[\int_0^a p^{\gamma+1}F_1(p)J_{\gamma}(pt)\,dp + \right.$$

$$\left. + \int_a^b p^{\gamma+1}G_2(p)J_{\gamma}(pt)\,dp + \int_b^{\infty} f_3(p)p^{1+\gamma}J_{\gamma}(pt)\,dp\right]. \tag{25}$$

From equation (25) and the integral (5) we can write equation (7) in the form

$$A_n = n!\,2^{-\gamma}\left[\int_0^a F_1(p) + \int_a^b G_2(p) + \int_b^{\infty} f_3(p)\right]p^{1+2\gamma}e^{-p^2/2}L_n^{\gamma}\left(\frac{p^2}{2}\right)dp, \quad n = 0, 1, 2, \ldots . \tag{26}$$

Conditions (17) and (19) can be written as:

$$0 < \alpha - \gamma < 1, \quad 0 < \beta - \gamma < 1. \tag{27}$$

Equation (26) represents the solution of equations (1), (2) and (3) subject to the conditions (27).

5. Section B. In this section we find the solution of the triple series equations (1), (2) and (3) when $\beta > \alpha$, $\gamma > \alpha$.

For solving the triple series equations we will solve the triple integral equations (8), (9) and (10) when $\beta > \alpha$, $\gamma > \alpha$. Making use of equation (8), we assume that

$$\int_0^{\infty} t^{-\alpha}A(t)J_{\alpha}(xt)\,dt = \begin{cases} f_1(x)\left(\dfrac{x}{2}\right)^{\alpha}, & 0 < x < a, \\ \varphi(x), & a < x < \infty, \end{cases} \tag{28}$$

where $\varphi(x)$ is an unknown function to be determined.

Using Hankel transforms we get from equation (28) that

$$A(t) = \frac{t^{1+\alpha}}{2^{\alpha}}\int_0^a f_1(u)u^{\alpha+1}J_{\alpha}(ut)\,du + t^{1+\alpha}\int_a^{\infty} u\varphi(u)J_{\alpha}(ut)\,du, \quad 0 < t < \infty. \tag{29}$$

Substituting equation (29) into equation (9), interchanging the order of integrations and using the integral (6) we find that

$$\int_a^x \frac{u^{\alpha+1}\varphi(u)\,du}{(x^2-u^2)^{1-(\beta-\alpha)}} = \frac{\Gamma(\beta-\alpha)x^{2\beta}}{2^{\alpha+1}}g_2(x) - \frac{1}{2^{\alpha}}\int_0^a \frac{u^{2\alpha+1}f_1(u)\,du}{(x^2-u^2)^{1-(\beta-\alpha)}}, \quad a < x < b. \tag{30}$$

The left-hand side of equation (30) is an Abel's type integral equation with corresponding condition $\beta-\alpha<1$ and the right-hand side is a known function of x hence we get the solution of the above equation as

$$\varphi(u)=\frac{u^{-\alpha-1}\sin[(\beta-\alpha)\pi]\Gamma(\beta-\alpha)}{2^{\alpha}\pi}\frac{d}{du}\int_a^u\frac{x^{1+2\beta}g_2(x)\,dx}{(u^2-x^2)^{\beta-\alpha}}-$$

$$-\frac{\sin[(\beta-\alpha)\pi]u^{-\alpha-1}}{\pi 2^{\alpha-1}}\frac{d}{du}\int_a^u\frac{x\,dx}{(u^2-x^2)^{\beta-\alpha}}\int_0^a\frac{p^{2\alpha+1}f_1(p)\,dp}{(x^2-p^2)^{1-(\beta-\alpha)}},\quad a<u<b, \tag{31}$$

where

$$0<1-(\beta-\alpha)<1. \tag{32}$$

Interchanging the order of integrations in the second integral of equation (31) and using the result (22) we can write equation (31) in the following form:

$$\varphi(u)=\frac{\Gamma(\beta-\alpha)u^{-\alpha-1}\sin[(\beta-\alpha)\pi]}{2^{\alpha}\pi}\frac{d}{du}\int_a^u\frac{x^{1+2\beta}g_2(x)\,dx}{(u^2-x^2)^{\beta-\alpha}}-$$

$$-\frac{\sin[(\beta-\alpha)\pi]u^{-\alpha}}{\pi 2^{\alpha-1}(u^2-a^2)^{\beta-\alpha}}\int_0^a\frac{p^{2\alpha+1}(a^2-p^2)^{\beta-\alpha}f_1(p)\,dp}{u^2-p^2}=$$

$$=\frac{u^{\alpha}l_2(u)}{2^{\alpha}},\quad a<u<b. \tag{33}$$

Substituting equation (29) into equation (10) and interchanging the order of integrations we can find that as in equation (33) that

$$\varphi(u)=\frac{\Gamma(\gamma-\alpha)u^{-\alpha-1}\sin[(\gamma-\alpha)\pi]}{2^{\alpha}\pi}\frac{d}{du}\int_a^u\frac{x^{1+2\gamma}f_3(x)\,dx}{(u^2-x^2)^{\gamma-\alpha}}-$$

$$-\frac{\sin[(\gamma-\alpha)\pi]u^{-\alpha}}{\pi 2^{\alpha-1}(u^2-a^2)^{\gamma-\alpha}}\int_0^a\frac{p^{2\alpha+1}(a^2-p^2)^{\gamma-\alpha}f_1(p)\,dp}{u^2-p^2}= \tag{34}$$

$$=\frac{u^{\alpha}l_3(u)}{2^{\alpha}},\quad b<u<\infty, \tag{35}$$

where

$$0<1-(\gamma-\alpha)<1. \tag{36}$$

From equations (33) and (35), equation (29) can be written as

$$A(t)=\frac{t^{1+\alpha}}{2^{\alpha}}\left\{\int_0^a f_1(u)+\int_a^b l_2(u)+\int_b^\infty l_3(u)\right\}u^{\alpha+1}J_{\alpha}(ut)\,du. \tag{37}$$

With equations (37) equation (7) can be written as

$$A_n=n!\,2^{-\alpha}\left\{\int_0^a f_1(u)+\int_a^b l_2(u)+\int_b^\infty l_3(u)\right\}u^{1+2\alpha}e^{-u^2/2}L_n^{\alpha}\left(\frac{u^2}{2}\right)du,\qquad n=0,1,2,\ldots. \tag{38}$$

Conditions (32) and (36) can be writtend in the following form:

$$0 < \beta - \alpha < 1, \quad 0 < \gamma - \alpha < 1. \tag{39}$$

Equation (38) is a solution of the triple series equations (1), (2) and (3) subject to conditions (39).

6. Particular cases $(\alpha = \gamma)$. In this section we shall find the results for particular cases of Section A and Section B when $\alpha = \gamma$. Hence we shall obtain the results of Lowndes and Srivastava [12].

Making use of equation

$$\sin m_1\pi = \frac{\pi}{\Gamma(m_1)\Gamma(1-m_1)}, \tag{40}$$

we find from equation (16) that

$$F_1(u) = f_1(u), \quad 0 < u < a. \tag{41}$$

If we assume

$$\beta - \gamma = \beta - \alpha = \lambda, \quad 0 < \lambda < 1, \tag{42}$$

then we find from equations (23), (24), (41) that

$$G_2(p) = \frac{p^{-1-2\alpha}}{\Gamma(1-\lambda)} \frac{d}{dp} \int_a^p \frac{x^{1+2\beta} g_2(x)\, dx}{(p^2 - x^2)^\lambda} -$$

$$- \frac{2p^{-2\alpha} \sin(\pi\lambda)}{\pi(p^2 - a^2)^\lambda} \int_0^a \frac{x^{1+2\alpha} f_1(x)(a^2 - x^2)^\lambda\, dx}{p^2 - x^2}, \quad a < p < b. \tag{43}$$

Making use of equation (41) we can write equation (26) in the following manner:

$$A_n = n!\, 2^{-\gamma} \left[\int_0^a f_1(p) + \int_a^b G_2(p) + \int_b^\infty f_3(p) \right] p^{1+2\gamma} e^{-p^2/2} L_n^\gamma \left(\frac{p^2}{2} \right) dp. \tag{44}$$

Equation (44) is the solution the triple series equations (1), (2) and (3) when $\alpha = \gamma$, $\beta \geq \alpha > -1$.

If we compare this solution defined by equation (44) to ([12, p. 186], 4.11) of Lowndes and Srivastava solution.

Both are the same if

$$G_2(p) = f_2(p), \tag{45}$$

where $f_2(p)$ is defined by equation (4.9) in reference [12, p. 186]. Making $m = 0$ in equation (2.8) in [12, p. 183], then the relation (45) is correct. Without using the derivative formula in [12, p. 183] ((2.5)) both solutions are same. Otherwise are different when m is not zero.

In Section B, assming $\gamma = \alpha$ in equations (33), (34) and (35) we find by making use of equation (40) that

$$l_2(u) = \frac{u^{-1-2\alpha}}{\Gamma(1-\lambda)} \frac{d}{du} \int_a^u \frac{x^{1+2\beta} g_2(u)\, du}{(u^2 - x^2)^\lambda} -$$

$$- \frac{2u^{-2\alpha} \sin(\pi\lambda)}{\pi(u^2 - a^2)^\lambda} \int_0^a \frac{p^{1+2\alpha} f_1(p)(a^2 - p^2)^\lambda\, dp}{u^2 - p^2}, \quad a < u < b, \tag{46}$$

$$l_3(u) = f_3(u), \quad b < u < \infty. \tag{47}$$

Making use of equation (47), equations (38) can be written in the following form:

$$A_n = n!\,2^{-\alpha}\left[\int_0^a f_1(u) + \int_a^b l_2(u) + \int_b^\infty l_3(u)\right] u^{1+2\alpha} e^{-u^2/2} L_n^\alpha\left(\frac{u^2}{2}\right) du, \quad n = 0, 1, 2, \ldots . \tag{48}$$

We also find from equations (43), (45) and (46) that

$$l_2(u) = f_2(u). \tag{49}$$

Making use of equation (49) the equation (48) converge to equation (44).

Finally solutions of the triple series equations (1), (2) and (3) of Sections A and B converge to Lowndes and Srivastava [12] with $m = 0$ when $\alpha = \gamma$, $\beta \geq \alpha > -1$ and m is defined in the paper [12, p. 183] ((2.4)).

Making use of the derivative formula discussed in [12, p. 183] ((2.5)) to the series equation (2) we can find the solution of the triple series equations (1), (2) and (3) in the same form as discussed in [12] for general values of m when $\alpha = \gamma$, $\beta \geq \alpha > -1$.

7. Conclusion. In Sections A and B we have developed a method for finding a closed form of solutions of triple series equations subject to the conditions (11) and (12). Results for particular cases $\alpha = \gamma$ and $\beta \geq \alpha > -1$ of Sections A and B are given in Section 6 and they include results of Lowndes and Srivastava [12]. If we put $m = 0$ in equation (2.8) of the paper [12, p. 183] then the results of this paper and those of [12] are the same. We have therefore obtained a simplified solution. When m is not zero the solution discussed in [13] has a more complex form and can be obtained in this paper by using the derivative formula ([12, p. 183], (2.5)) to the series equations (2).

1. *Srivastava K. N.* On dual series relations involving Laguerre polynomials // Pacif. J. Math. – 1966. – **19**. – P. 529 – 533.
2. *Lowndes J. S.* Some dual series equations involving Laguerre polynomials // Ibid. – 1968. – **25**. – P. 123 – 127.
3. *Srivastava H. M.* A note on certain dual series equations involving Laguerre polynomials // Ibid. – 1969. – **30**. – P. 525 – 527.
4. *Srivastava H. M.* Dual series relations involving generalized Laguerre polynomials // J. Math. Anal. and Appl. – 1970. – **31**. – P. 587 – 594.
5. *Srivastava H. M.* A further note on certain dual equations involving Fourier – Laguerre series // Ned. Akad. Wetensch. Indag. Math. – 1973. – **35**. – P. 137 – 141.
6. *Srivastava H. M.* Certain dual series equations involving Jacobi polynomials I and II // Atti Accad. naz. Lincei. Rend. CI. sci., fis., mat. e natur. – 1979. – **67**, № 8. – P. 395 – 401; – 1980. – **68**. – P. 34 – 41.
7. *Srivastava H. M., Panda R.* A certain class of dual equations involving series of Jacobi and Laguerre polynomials // Ned. Akad. Wetensch. Indag. Math. – 1978. – **40**. – P. 502 – 514.
8. *Singh B. M., Rokne J., Dhaliwal R. S.* Dual series equations involving generalized Laguerre polynomials // Int. J. Math. and Math. Sci. – 2005. – P. 1135 – 1139.
9. *Singh B. M., Rokne J., Dhaliwal R. S.* The study of dual series equations involving Laguerre polynomials // Submitted Ital. J. Pure and Appl. Math.
10. *Lowndes J. S.* Triple series equations involving Laguerre polynomials // Pacif. J. Math. – 1969. – **29**. – P. 167 – 173.
11. *Dwivedi A. P., Trivedi T. M.* Some triple series equations involving generalized Laguerre polynomials // Indian J. Pure and Appl. Math. – 1974. – **5**. – P. 674 – 681.
12. *Lowndes J. S., Srivastava H. M.* Some triple series and triple integral equations // J. Math. Anal. and Appl. – 1990. – **150**. – P. 181 – 187.
13. *Bateman H., Erdély A.* Tables of integral transforms. – McGraw Hill, 1954. – Vol. 1.
14. *Watson G. N.* A treatise on the theory of Bessel functions. – Cambridge: Cambridge Univ. Press, 1944.

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