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ANALYTICAL METHOD OF PROFILING AXIAL-RADIAL COMPRESSOR IMPELLERS

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A new analytical method for constructing axial-radial compressor impellers with compound lean leading and trailing edges is proposed allowing us to describe a wide class of flow paths based on a limited (small) number of parameterized quantities. With the aid of this method there has been designed a new flow path with a typical axial radial impeller for turbo-expander aggregate compressors with flow coefficients in the range from 0.03 to 0.06. To test the method, a numerical study of spatial viscous flows was carried out in the existing and new modifications of the flow path for the typical axial-radial compressor of a low-temperature turbo-expander aggregate. To do that, the IPMFlow software package was used, which is the development of the FlowER and FlowER-U programs. The computational grid consisted of over 600 thousand cells. The developed impeller has a substantially spatial shape, with the leading edges having compound circumferential lean. The new design is shown to have a more favorable flow structure in which there are almost no flow separations. This is ensured due to the spatial shape of the new impeller, including compound radial lean of the leading edges. This form contributes to "pressing" the flow to the peripheral contour in the region of the flow path turn from the axial direction to the radial one, consequently preventing the occurrence of separable vortices. Due to the absence of separable formations in the nominal mode, and thanks to relatively insignificant separations in the off-design mode, there is provided a high level of aerodynamic perfection (high efficiency) of a new typical compressor in the whole range of turbo-detander aggregate operating modes. Thus, in the nominal mode, the compressor efficiency is 6% higher as compared with that of the prototype. The compressor impeller has been introduced in the turbo-detander aggregates for complex gas processing facilities at the extractive enterprises of gas-condensate deposits of Uzbekistan.

Keywords: *axial-radial compressor, analytical method of profiling, spatial flow, numerical simulation.*

Introduction

One of the main requirements to turbo-machines is their high profitability. Today, there are significant reserves for increasing turbomachinery efficiency due to the gas-dynamic improvement of their flow paths, using spatial profiling technology of [1–4].

Turbomachines include turbo-expander aggregates, which are used in the systems for drying gas during its preparation for transportation [5–7]. Drying is an energy-intensive process that occurs by cooling the gas to low temperatures sufficient to condense unwanted fractions with their subsequent separation and removal. The main elements of a turbo-expander are a turbine and compressor. In the turbine the gas expands and gets cooler. The power received on the turbine shaft is not lost but is used for the compressor drive. In the turbo-expander compressor, the gas is pre-compressed, which reduces the energy consumption of the compressor drive where the gas is pressed to the required value. The turbine and compressor influence on the overall turbo-expander aggregate efficiency is determining. The authors of this paper have extensive experience in creating methods for calculating and designing flow paths for radial-axial turbines [8].

Thus, the high gas-dynamic efficiency of the flow paths for radial-axial turbines is achieved due to the essentially spatial shape of impellers with compound lean leading and trailing edges. For the first time for this type of turbines, there have been offered blades with cross-sections close to classical aerodynamic profiles, which ensure high efficiency not only in the nominal modes, but also in the ones with off-design flow angles.

On the basis of the experience gained during the research of radial-axial turbines, the authors have developed a new analytical method for constructing complex-shaped axial-radial compressor impellers, whose description, as well as that of its approbation results are the main purpose of this paper. This method allows us to describe a broad class of axial-radial compressor impellers based on a limited (small) number of

parameterized quantities. The use of such approaches is most acceptable when solving optimization problems [9–12]. The method was tested during the development of a new typical axial-radial impeller for turbo-expander aggregate compressors with flow coefficients in the range from 0.03 to 0.06. The proposed impeller has a substantially spatial shape, with its leading edges having compound circumferential lean, and ensures flow efficiency increase over the entire range of operating modes.

Method of Analytical Profiling of Axial-Radial Impellers

The method for the analytical profiling of axial-radial impellers is based on the approach used by the authors of this paper for developing radial-axial turbine rotors [8]. The process of creating the three-dimensional geometry of a turbine rotor can be divided into four main stages:

– calculation of the coordinates of meridional contours;

– calculation of the coordinates of the profile midline in the root section;

– calculation of the coordinates of the symmetric profile relative to the midline;

– blade completion in height with leans taken into account.

The method for calculating the coordinates of the meridional contours of axial-radial impellers completely coincides with the method used for radial-axial turbine rotors [8], with the difference that the geometry (Fig. 1) is mirrored with respect to the plane perpendicular to the axis of rotation (Fig. 1, a), and the impeller inlet and outlet change places. The leading edge is set perpendicular to the axis of rotation, and the trailing one is set parallel to it (in case of lean absence).

The root and peripheral contours are described by curves each consisting of an arc of a circle and a line tangent to the arc. The initial data for constructing meridional contours is the following: $r_{\max}$, $r_{\min}$ – impeller maximum and minimum radii, $x_{\max}$ – impeller width, l_{in}, l_{out} – heights of impeller inlet and outlet channels, $\alpha_{\text{in}}^{\text{rt}}$, $\alpha_{\text{out}}^{\text{rt}}$, $\alpha_{\text{in}}^{\text{p}}$, $\alpha_{\text{out}}^{\text{p}}$ – angles of root and peripheral inlet and outlet contours, respectively (Fig. 1, a).

The blade is described on the basis of the basic section, which lies on a surface of revolution and coincides with the root contour. This section is determined in the coordinates associated with surfaces of revolution: $r\varphi$ – scalar product of the radial and angular coordinates; s – distance from a blade leading edge along the corresponding section (along the root contour) in a projection onto the meridional plane (Fig. 1, c, d). The coordinates of the profile on a surface of revolution are obtained by summing the coordinates of the profile midline $r\varphi_{\text{сл}}$ (Fig. 1, c) and the coordinate of the profile relative to the midline $\Delta r\varphi$ (Fig. 1, d) $r\varphi(s) = r\varphi_{\text{ml}}(s) + \Delta r\varphi(s)$).

The profile midline can be described by two types of curves. The first profile is a third-degree polynomial that looks like

$$r\varphi_{\text{ml}} = \sum_{i=0}^{3} a_i s^i , \qquad (1)$$

where a_i – constants.

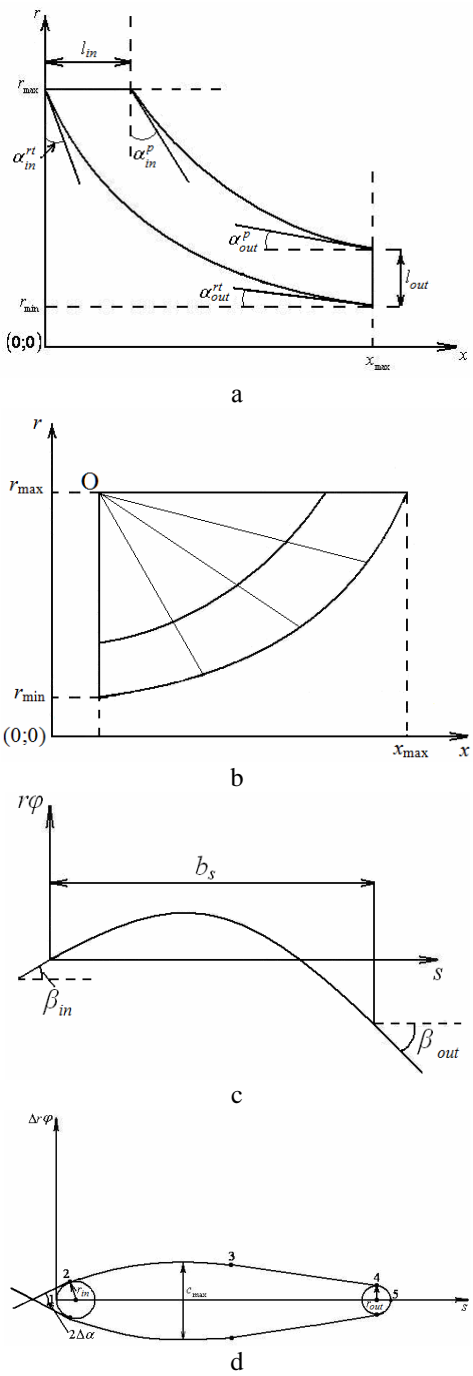

Fig. 1. Impeller design diagram:
a – meridional contours;
b – intersections of similar points;
c – profile midline;
d – profile view relative to midline

To determine the constants in equation (1), the initial data is the following: b_s – profile width; β_{in}, β_{out} – profile midline inlet and outlet angles; d – distance to the point behind the trailing edge (in fractions of the profile width), where the second derivative of the midline is 0 (Fig. 1, c). This type of curve has proven itself well during the construction of radial-axial rotors, but it turned out that in many cases it does not ensure satisfactory results for axial-radial structures. Therefore, a second method was proposed for calculating a midline – a curve with linear changes in the angles of tangent lines between nodal (given) points

$$\beta = \beta_i \frac{s_{i+1} - s}{s_{i+1} - s_i} + \beta_{i+1} \frac{s - s_i}{s_{i+1} - s_i}, \quad s_i \le s \le s_{i+1},$$

where β_i – the angles of tangent lines to the profile midline curve at the nodal points.

In this case, the profile curve midline shape is determined by integrating the equation:

$$d(r\varphi_{\text{ml}}) = \text{tg}(\beta) dS .$$

The profile is described by two symmetric curves relative to the midline. For radial-axial rotors, only one type of symmetric profile was used, in which each side consists of four connected sections: 1–2 – leading edge; 2–3 – arc of a circle; 3–4 – straight line; 4–5 – trailing edge (Fig. 1, d). The initial data for building profiles is the following: b_s – profile width; r_{in}, r_{out} – radii of the leading and trailing edges; $c_{\max}$ – maximum profile thickness; $\Delta\alpha$ – sharpening inlet angle. For axial-radial impellers, which are typical for compressors, blades with the smallest possible width proved to be better. They are given in the form of constant- or variable-thickness plates with rounded edges.

In case if the blade is without lean, then the position of the profile midline on the surface of revolution, located on the peripheral contour, is calculated for its completion. For this purpose, a point is first found on the peripheral contour, similar to that on the root one. There are used several methods of determining similar points. In one (as in [8]), similar are the points in which on the corresponding contours there are the same dimensionless distances from the leading edges

$$\bar{s} = s / s_b ,$$

where s_b is the distance between the leading edges on the corresponding contour.

In another one, similar are the points that lie on the beams crossing the root and peripheral contours. The beam originates from the point O, which corresponds to the leading edge axial coordinate and the trailing edge radial coordinate on the root contour (Fig. 1, b). At a similar point on the peripheral contour, the profile midline angular coordinate is defined equal to the corresponding point on the root contour. The coordinates of the peripheral profile $\Delta r\varphi$ relative to the midline are determined in the same way. The blade midline is defined by straight segments connecting similar points where the profile coordinates $\Delta r\varphi$ relative to the midline are constant.

If a blade is to be designed with leans, then the main difference from the approach described above consists in defining the midline angular coordinate change law at similar points as a function of the distance from the root contour

$$\Delta\varphi_{\text{ml}} = \Delta\varphi_{\text{ml}}(l / l_r, \bar{s}), \tag{2}$$

where l – distance from the root contour on the line of similar points; l_r – distance between similar points on the root and peripheral contours.

The angular coordinate change function (2) is given so that the line of similar points in the profile midline is either a straight line or an arc. The dependence of function (2) $\bar{s}$ is usually chosen as linear.

3D Design Development of an Axial-Radial Compressor Flow Path

In turbo-expander aggregates, there are widely used compressors with flow coefficient in the range from 0.03 to 0.06. For such characteristics there exists a typical radial compressor (existing compressor, Fig. 2), whose flow path view is shown in Fig. 2, a, c. The number of blades in the impeller is 17. The impeller has no cover plate and is at a distance of 0.5 mm (radial clearance) from the peripheral meridional contour.

For the same operating conditions as for the existing radial compressor, there has been developed a new typical axial-radial compressor (according to the method described above), whose flow path view is shown in Fig. 2, b, d. The new compressor impeller has no cover plate either and its radial clearance is 0.5 mm.

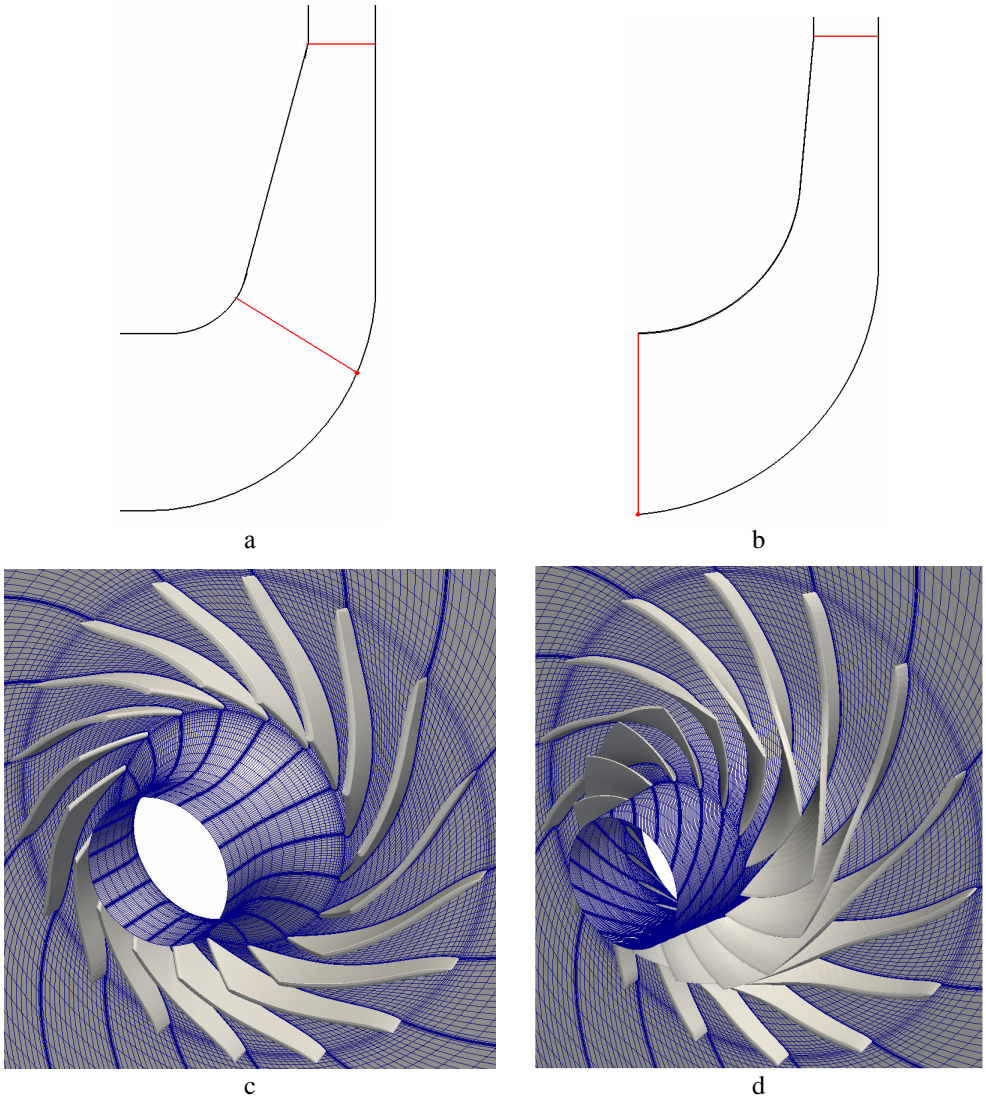

Fig. 2. Flow path view:
a, b – meridional section; c, d – isometry; a, c – existing compressor; b, d – new compressor

The main differences of this variant from the previous one are:

– fewer blades (15 instead of 17);

– compound lean leading edges (spatial profiling technology);

–lower blade outlet height (18.4 mm instead of 20.3 mm without taking into account the 0.5 mm radial clearance);

–peripheral outlet contour inclination relative to the radial direction is -5º instead of 15º.

The calculations of the two compressor variants were carried out under conditions that correspond to the parameters of one of the real oil and gas deposits.

The rotor rotational speeds of 9000, 10,000 and 11000 rpm are considered. The mass flow rate varied from 70 to 110%. The values of total inlet temperature and pressure did not change and were equal to 308 K and 4.6 MPa, respectively. The compressor (bladeless diffuser) outlet pressure was determined by calculation in accordance with a given mass flow rate. The numerical simulation of spatial viscous flows was performed using the IPMFlow software package, which is the development of the FlowER and FlowER-U programs [13–16]. The computational grid consisted of over 600 thousand cells.

Figs. 3 and 4 are the visualizations of the flow in the existing and new compressors both in the nominal mode and in that with 70% flow coefficient, respectively.

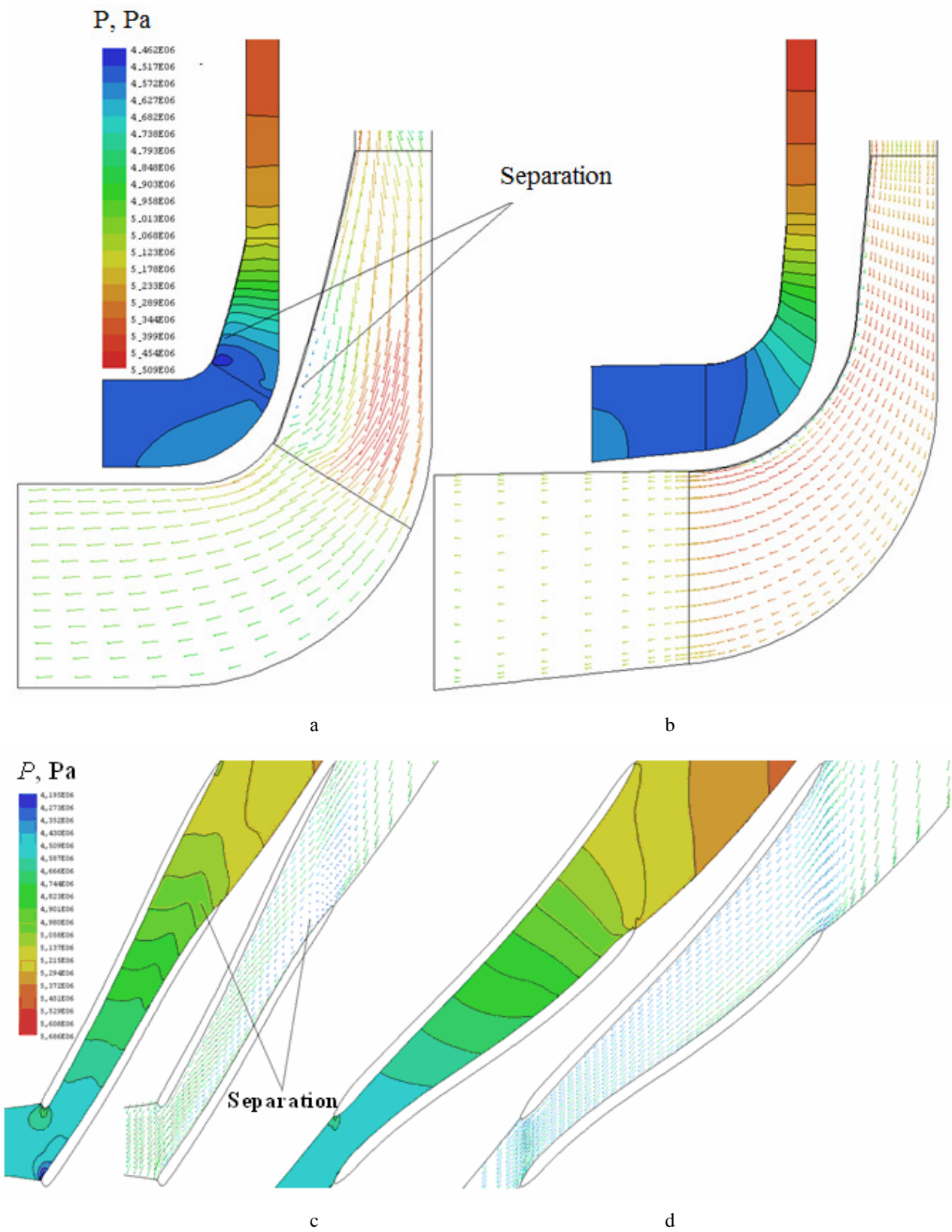

Fig. 3. Visualization of static pressure and velocity vectors. Average section. Nominal mode:
a, b – meridional surface; c, d – tangential surface; a, c – existing compressor; b, d – new compressor

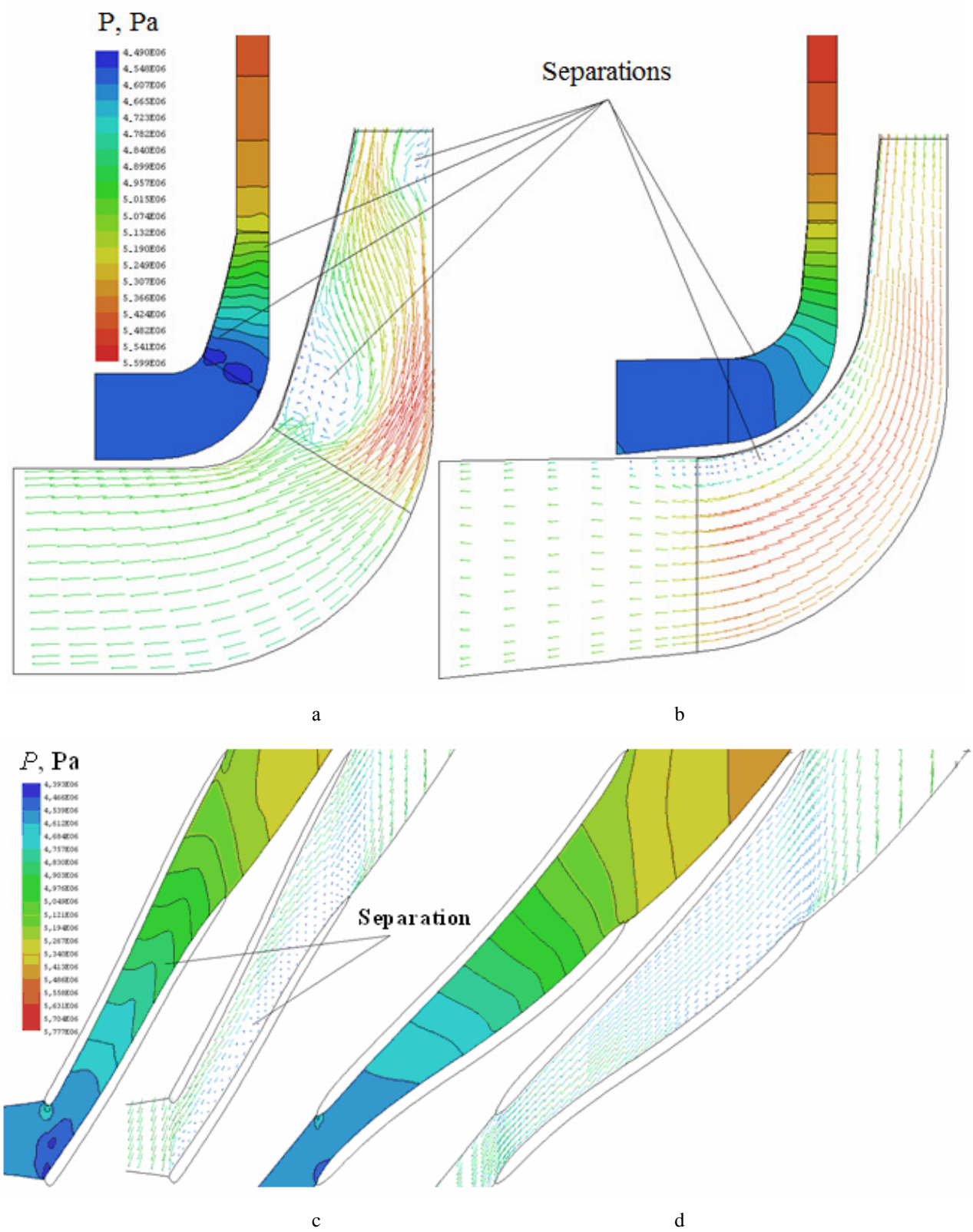

Fig. 4. Visualization of static pressure and velocity vectors. Average section. 70% mass flow rate mode:
a, b – meridional surface; c, d – tangential surface; a, c – existing compressor; b, d – new compressor

The above results clearly show that at the nominal mode (Fig. 3) in the existing compressor there is a separation of the flow in the middle sections both in the meridional (Fig. 3, a) and tangential (Fig. 3, c) planes, while in the new compressor (Fig. 3, b, d) there are no separations. It is worth noting that in the existing compressor, there is clearly pronounced roughness on the static pressure isolines in the areas of vortex flows. In the new compressor, the static pressure isolines are quite smooth, which indicates that the flow is free from vortices. The absence of separations (favorable flow structure) is ensured by the new compressor spatial shape, including the compound lean of the leading edges. This shape makes it possible to "press" the flow to the peripheral contour in the region of the flow path turn from the axial direction to the radial one, consequently preventing flow separation.

In the existing compressor, in the 70% mass flow rate mode, flow separation increases significantly. Thus, in the middle meridional section, there are clearly two large separations (Fig. 4, a), whereas in the average tangential section, flow separation can be observed almost in one half of the inter-blade channel along the entire dilution side (Fig. 4, c). In the new compressor, in the middle meridional section, there is one separation (Fig. 4, b), but its dimensions are much smaller (compared to those in the existing compressor), and its location area is localized, that is, it does not penetrate deeply into the channel. In the tangential plane near the dilution side, the blade leading edge has a flow region which is prone to separation (Fig. 4, d), but despite this, there is no separation of the flow there.

The absence of flow separations in the nominal mode and relatively small separations in off-design modes, that is, a favorable flow pattern, in the new compressor is the main factor that ensured a high level of gas-dynamic perfection (high efficiency) over the entire range of the operating modes of the developed compressor.

Fig. 5 shows the integral characteristics of the existing and new compressors. For the existing compressor, given are both the theoretical characteristic and the one obtained by calculation. Their comparison shows a good reconciliation of design and experimental data.

The above results clearly show that the new flow path developed for a new standard compressor has a high level of gas-dynamic perfection and ensures a satisfactory flow pattern throughout the range of considered (design and off-design) operating modes. At the nominal point, the new compressor flow path efficiency exceeds that of the existing design by 6%. The compressor impeller has been introduced in the turbo-detander units for complex gas processing facilities at the extractive enterprises of gas-condensate deposits of Uzbekistan.

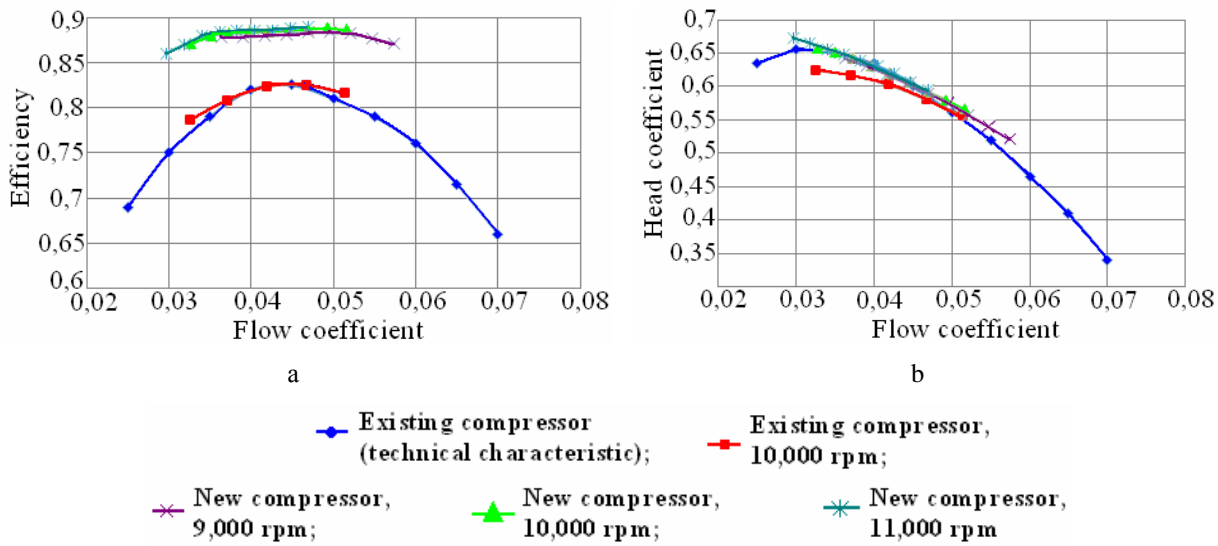

Fig. 5. Integral characteristics of the existing and new compressors:
a – dependence of efficiency on flow coefficient; b – dependence of theoretical head coefficient on flow coefficient

Conclusions

A new analytical method for constructing radial-axial compressor impellers with compound lean leading and trailing edges has been developed. With the aid of this method there was designed a new flow path of a typical axial-radial impeller for turbo-expander aggregate compressors with flow coefficients in the range from 0.03 to 0.06. The impeller has a substantially spatial shape, with the leading edges having com-

pound circumferential lean. In contrast to the existing typical impeller, in the new design there is a more favorable flow structure where there are almost no flow separations. The flow path with a new impeller has a high level of gas-dynamic perfection and its flow efficiency exceeds that of the existing one throughout the entire range of its operating modes, including the nominal one where it is 6% higher.

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Аналітичний метод профілювання робочих коліс осерадіальних компресорів

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Запропоновано новий аналітичний метод побудови осерадіальних робочих коліс компресорів зі складними навалами вхідних і вихідних кромок, який дозволяє описувати широкий клас проточних частин на основі обмеженої (невеликої) кількості параметризованих величин. За допомогою цього методу створено нову проточну частину типового осерадіального робочого колеса для компресорів турбодетандерних агрегатів із коефіцієнтами витрати в діапазоні від 0,03 до 0,06. Для апробації методу виконано чисельне дослідження просторових в'язких течій у існуючій та новій модифікації проточної частини типового осерадіального компресора низькотемпературного турбодетандерного агрегату з використанням програмного комплексу IPMFlow, що є розвитком програм FlowER і FlowER-U. Розрахункова сітка складалася з понад 600 тисяч комірок. Розроблене робоче колесо має істотно просторову форму зі складним коловим навалом вхідних кромок. Показано, що у новій конструкції спостерігається більш сприятлива структура течії, в якій майже відсутні відриви потоку. Це забезпечується за рахунок просторової форми нового робочого колеса, у тому числі складним коловим навалом вхідних кромок. Така форма сприяє «притисненню» потоку до периферійного обводу в області розвороту каналу від осьового до радіального напрямку, і, як наслідок, запобігає виникненню відривних вихорів. Завдяки відсутності відривних утворень на номінальному режимі і відносно незначним відривам на нерозрахованих режимах забезпечено високий рівень аеродинамічної досконалості (високий коефіцієнт корисної дії – ККД) нового типового компресора в усьому діапазоні режимів експлуатації турбодетандерного агрегату. Так, у розрахунковій точці ККД запропонованого компресора на 6% вище порівняно з прототипом. Розробку впроваджено у турбодетандерних агрегатах установок комплексної підготовки газу на видобувних підприємствах газоконденсатних родовищ Узбекистану.

***Ключові слова**: осерадіальний компресор, аналітичний метод профілювання, просторова течія, чисельне моделювання.*

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