ION BEAM DYNAMICS IN SUPERCONDUCTING DRIFT TUBE LINAC

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Ion superconducting linac is based on periodic system from short identical niobium drift tube cavities. This linac supplies typically 1 MV of accelerating potential per cavity. By specific phasing of the RF cavities one can provide a stable particle motion in the whole accelerator. The longitudinal and transverse ion beam dynamics is studied in this linac. The equation of motion in the Hamiltonian form done by means of the smooth approximation. This equation is used for analysis of the nonlinear ion beam dynamics in superconducting linac. It was shown that the connection between the phase acceptance and the transverse emittance can be found by means of the effective potential function. The focusing methods by the solenoid field and RF field are studied. The results of this investigation are compared with the numerical simulation of ion beam dynamics in superconducting linac.

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1. INTRODUCTION

Ion superconducting linac is usually based on the superconducting (SC) independently phased cavities. This linac consists of the niobium cavities which can provide typically 1 MV of accelerating potential per cavity. Such structures can be used for ion acceleration with different charge-to-mass ratio in the low energy region [1] and for proton linac in the high-energy region (SNS, JHF, ESS project). It is desirable to have a constant geometry of the accelerating cavity in order to simplify manufacturing. Such geometry leads to a non-synchronism but a stable longitudinal particle motion can be provided by proper phasing of the RF cavities. The ions are accelerated and slipping relative to the RF wave in dependence of the ratio between the particle velocity β and the phase velocity of the wave β_G in resonator. The geometrical velocity β_G of the RF wave is constant for cavities. The geometric size of a cavity and a wave velocity in the each cavity β_G must be changed step by step from one class to other class. The optimum number of cavity in each class determines the number of classes in SC linac. By controlling the driven phase of the accelerating structure and the distance between the cavities, the beam can be both longitudinally stable and accelerated in the whole system. In this paper two methods of the beam dynamics investigation are compared for low ion velocities and for the charge-to-mass ratio $Z/A = 1/66$. This comparison can be demonstrated with an example the post-accelerator of radioactive ion beams (RIB) linac, where beam velocity increases from $\beta = 0.01$ to $\beta = 0.06$ [1].

Beam focusing can be provided with the help of SC solenoid lenses, following each cavity and with the help of special RF fields. A schematic plot of one period of the accelerator structure is shown in Fig.1. The low-charge-state beams and the low velocity require stronger transverse focusing than one is used in existing SC ion linac. The large radial variation of the axial accelerating field induces a beam energy spread, which will accumulate as the beam passes through successive resonators. Early investigation of beam dynamics shown that for the initial normalized transverse emittance $\varepsilon_T = 0.1\pi\cdot$mm·mrad and the longitudinal emittance $\varepsilon_V = 0.3\pi\cdot$keV/u·nsec the connection between the longitudinal and transverse motion can be neglected if maximum beam envelop $X_m < 3\ldots4$ mm and inner radius of drift tubes $a = 15$ mm.

2. PARTICLE MOTION IN SC LINAC

The general axisymmetric equations of motion for ion moving inside an accelerator can be written as

$$\begin{aligned}\frac{d}{dt}\left(\gamma\frac{dz}{dt}\right) &= \frac{eZ}{Am}E_z(\vec{r},t) - \frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial z}A_\phi^2,\\ \frac{d}{dt}\left(\gamma\frac{dr}{dt}\right) &= \frac{eZ}{Am}E_r(\vec{r},t)(1-\beta\beta_G) - \frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial r}A_\phi^2.\end{aligned} \quad (1)$$

In every cavity the acceleration RF field of periodic H-cavity can be represented as an expansion of spatial harmonics

$$\begin{cases} E_z = E_0\sum I_0(h_n r)\cos(h_n(z-z_i))\cos(\omega t),\\ E_r = E_0\sum I_1(h_n r)\sin(h_n(z-z_i))\cos(\omega t), \end{cases} \quad (2)$$

where E_0 is amplitude of RF field at the axis ($E_0 \neq 0$ if $-L_r/2 < z-z_i < L_r/2$), $h_n = \pi/D + 2\pi n/D$, $n = 0, 1, 2, \ldots$, $D_i = \beta_G\lambda/2$ is the period length of the cavity, L_r is the cavity length, z_i is the coordinate of the i-th cavity center, I_0, I_1 are the modified Bessel functions. Let we call particle which is accelerating on axis and does not have slow phase and transverse oscillations term as reference. In our case the reference particle velocity β_c and the geometrical velocity β_G are closely in each class of the identical cavities. Retaining in (2) only zeroth harmonic we can use the traveling wave system. In this system ωt can be replaced by $h_0(z-z_i) + \varphi_{0i}$, where φ_{0i} is the RF phase when the reference particle traverses the cavity center. In equation (1) the value A_φ is the azimuthal vector-potential of the magnetic field in every solenoid (***B*** = rot***A***).

Superconducting cavities can provide high accelerating gradient in linear accelerators. Together with the higher rate of energy gain in SC linac the defocusing factor is much higher in comparison to the normal conducting linear accelerator. The beam focusing can be provided by SC solenoids which follow each the cavity [1]. The conditions of longitudinal and transverse beam stabilities for the structure consisting from the periodic sequence of the cavities and solenoids were studied early using transfer matrix calculation [2]. In SC linac design, it is very important to know the bucket size since it relates to the longitudinal RF focusing. But the linac

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longitudinal acceptance cannot be calculated by matrix method because of the assumption that the particles have small longitudinal oscillation amplitude. In order to investigate the nonlinear ion beam dynamics in such accelerator structure and to calculate the longitudinal and transverse acceptances it can be used the smooth approximation [3,4]. In this paper, equation of motion for ion beam in the Hamiltonian form is derived in the smooth approximation for superconducting linac.

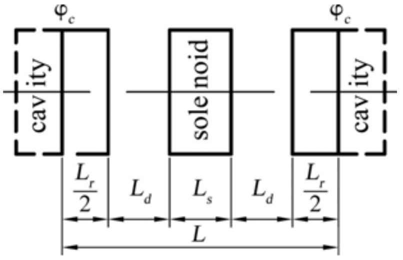

Fig.1. Layout of structure period

In periodical structure, which was shown in Fig.1, RF field can be expanded into a Fourier series as

$$E_z = \frac{U}{L} I_0(k_0 r)\left\{ f_{z,0} + \sum_1^{\infty} f^c_{z,n} \cos k_n z + f^s_{z,n} \sin k_n z \right\},$$
$$E_r = \frac{U}{L} I_1(k_0 r)\left\{ f_{r,0} + \sum_1^{\infty} f^c_{r,n} \cos k_n z + f^s_{r,n} \sin k_n z \right\}. \quad (3)$$

Here $f_{z,0} = S_0 \cos(\varphi_c + \psi)$, $f_{r,0} = -S_0 \sin(\varphi_c + \psi)$, $f^c_{z,n} = (-1)^n T^+_n \cos(\varphi_c + \psi)$, $f^c_{r,n} = (-1)^{n+1} T^+_n \sin(\varphi_c + \psi)$, $f^s_{z,n} = (-1)^n T^-_n \sin(\varphi_c + \psi)$, $f^s_{r,n} = (-1)^{n+1} T^-_n \cos(\varphi_c + \psi)$, $T^{\pm}_n = S^+_n \pm S^-_n$, $S_n^{\pm} = \sin(Y_n^{\pm})/Y_n^{\pm}$, $Y_n^{\pm} = (k_c \pm k_n)L_r/2$. In this expressions: $E = 2U/L_r$, U is the cavity voltage amplitude; $k_n = 2\pi n/L$, $n = 0, 1, 2, \ldots$; k_c is slipping factor, $k_c = (2\pi/\lambda)(1/\beta_c - 1/\beta_G)$. In the coefficients $f_n^{c,s}$ the phase relative to the reference particle ψ defined by $\psi = \omega(t - t_c)$, t_c is the flight time of the reference particle.

In the simple case the vector-potential of the magnetic field $A_\varphi = Br/2$ can be approximated by the step function for every solenoid. If L_s is effective solenoid length and L is the lattice period, the external solenoid magnetic field can be represented as an expansion into spatial harmonics also.

3. BEAM DYNAMICS IN SMOOTH APPROXIMATION

3.1. SOLENOID FOCUSING

Let us consider particle motion in the polyharmonic fields of the cavities (3) and solenoids. The ion dynamics in such periodic structure is complicated. The particles trajectories can be presented as a sum of the slowly term and a fast oscillation term with a period L. The normalized particle velocity deviation with respect to the reference particle velocity, $\Delta\beta$, can be represented as a sum of a slow motion term and a fast oscillation term also.

Following Ref. [5] one can apply an averaging over the fast oscillations and obtain the phase (ψ) and radial ($\rho = h_0 r$) motion equations in smooth approximation:

$$\frac{d^2\psi}{d\xi^2} + 3\left[\frac{d}{d\xi}(\ln \beta\gamma)\right]\frac{d\psi}{d\xi} = -\frac{\partial \bar{U}_{eff}}{\partial \psi},$$
$$\frac{d^2\rho}{d\xi^2} + \left[\frac{d}{d\xi}(\ln \beta\gamma)\right]\frac{d\rho}{d\xi} = -\frac{\partial \bar{U}_{eff}}{\partial \rho}, \quad (4)$$

where $U_{eff} = U_0 + U_1 + U_2$ is effective potential function. We use the following designations:

$$U_0 = 4\alpha[I_0(\rho)\sin(\varphi_c + \psi) - \psi\cos\varphi_c - \sin\varphi_c]S_0 + \frac{1}{2}b\frac{L_c}{L}\rho^2,$$
$$U_1 = \alpha^2 \sum_1^{\infty}\left[\frac{I_0^2(\rho)}{(2\pi n)^2}\left(g^{c\ 2}_{z,n} + g^{s\ 2}_{z,n}\right) + \frac{I_1^2(\rho)}{(2\pi n)^2}\left(g^{c\ 2}_{r,n} + g^{s\ 2}_{r,n}\right)\right], \quad (5)$$
$$U_2 = -4\alpha b\rho I_1(\rho)\frac{L_s}{L}\sum \frac{g^c_{r,n}}{(2\pi n)^2}\frac{\sin X_n}{X_n} + b^2 \sum \frac{1}{(2\pi n)^2}\left(\frac{\sin X_n}{X_n}\right)^2 \rho^2.$$

Here $\alpha = \dfrac{\pi eZUL}{2A\lambda mc^2\beta_g^3\gamma_g^3}$ is interaction parameter, $b = (eZBL/2Amc\beta_c\gamma_c)^2$ is focusing parameter, $X_n = \pi n L_s/L$, $g_{z,0} = S_0(\cos\varphi_c - \cos(\varphi_c + \psi))$,

$$g_{r,0} = -S_0(\sin\varphi_c - \sin(\varphi_c + \psi)),$$
$$g^c_{z,n} = (-1)^n T^+_n(\cos\varphi_c - \cos(\varphi_c + \psi)),$$
$$g^s_{z,n} = (-1)^n T^-_n(\sin\varphi_c - \sin(\varphi_c + \psi)),$$
$$g^c_{r,n} = (-1)^{n+1} T^+_n(\sin\varphi_c - \sin(\varphi_c + \psi)),$$
$$g^s_{r,n} = (-1)^{n+1} T^-_n(\cos\varphi_c - \cos(\varphi_c + \psi)).$$

In this expression for U_{eff} we take into account the coherent oscillations of bunches and the effective potential function describe slowly oscillations in the reference particle frame. Earlier, in [5] the effective potential function was found in the frame where averaged velocity of reference particle, $\bar{\beta}_c = 0$. Now, it is interesting to compare these two cases.

The effective potential U_{eff} provides the full description of the ion dynamics in the smooth one-particle approximation. In our case the analysis of the effective potential function (5) makes it possible to find the longitudinal acceptance. Fig.2 shows the phase acceptance (thin line) and maximum energy width inside the RF bucket (thick line) for $L_r/L = 1/4$, $\varphi_c = -20°$ and different β. In case when the effective potential function was found in the reference particle frame the phase acceptance and maximum energy width shown by gray lines, and when higher harmonics is absent (single wave approximation) by black lines. The influence of the fast oscillations is negligibly for maximum energy width.

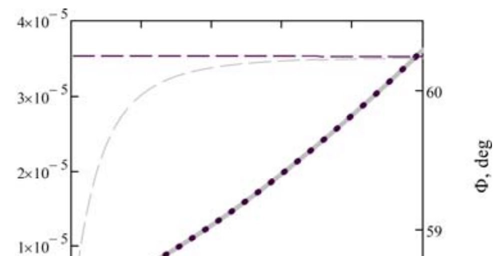

Fig.2. The phase acceptance (Φ) and maximum energy width (Δγ) within bunch for different β

For the charge-to-mass ratio $Z/A = 1/66$, $\varphi_c > -20°$, $\beta = 0.01$ and the transverse emittance $\varepsilon_T = 0.1\pi$·mm·mrad the beam focusing can be realized for the solenoid field above $B \leq 20$ T.

3.2. APF AND SOLENOID FOCUSING

As was shown early the beam focusing can be realized for the solenoid field near $B \sim 20$ T. The value of magnetic field B can be reduced by using of addition alternative phase focusing (APF). The smooth approximation has been applied to the study of APF in RIB linac. By adjusting the drive phase (φ_1 and φ_2) of the two cavities, we can achieve the acceleration and the focusing by less magnitude of magnetic field B [2]. Adding a focusing solenoid into period will also allow separate control of the transverse and longitudinal beam dynamics. It is interesting to investigate the nonlinear ion beam dynamics in such accelerated structure. New effective potential function U_{eff} must be finding for this accelerating structure. The analysis of the effective potential function makes it possible to study the condition at which the phase and radial stability of the beam is achieved and calculate the longitudinal acceptance.

In the simplest case when a slipping factor is $k_c = 0$, $\varphi_1 = -30°$, $\varphi_2 = 20°$ and $\rho = 0$ the longitudinal acceptance is shown on Fig.3. Now the influence of the fast oscillations is considerable. The maximum energy width inside the RF bucket has the minimum in $\beta = 0.017$ and the phase acceptance decreases from 60° to 15°. The value of magnetic field of solenoids B_{min} can be reduced to 9 T in this case.

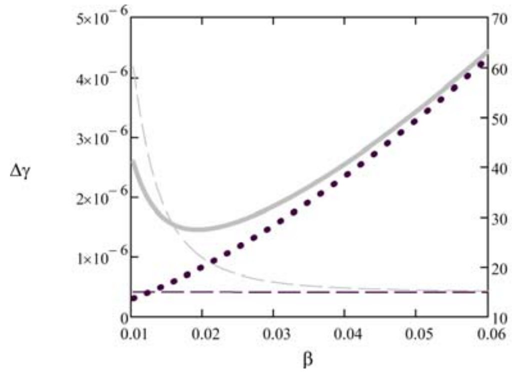

Fig.3. The phase acceptance (Φ) and maximum energy width (Δγ) within bunch for different β

4. NUMERICAL SIMULATION OF ION BEAM DYNAMICS

4.1. SOLENOID FOCUSING

For the analyses of longitudinal and transverse motions the beam dynamic are studied in averaged field. The field components can be easily obtained from effective potential function. Solution can be obtained only by numerical simulation as the field components are nonlinear functions. The results of ion beam numerical simulations with $Z/A = 1/66$ are shown in Fig.4. In this case we use simplified model and propose that the geometrical speed β_G varies in each cavity. On Fig.4,a and 4,b the beam at the front-end of accelerator, and on 4,c and 4,d through 50 periods are represented. At value of magnetic field $B = 20$ T all particles remained within beam envelop around 4 mm. The length of the period is 1 m, the length of the solenoid is 0.2 m, and the cavity is 0.28 m. Accelerating potential per cavity typically is equal 1 MV and an initial particle phase in the cavity length is $\varphi_c = -20°$. Initial beam velocity is $\beta_{in} = 0.01$, final velocity is $\beta_{out} = 0.04$.

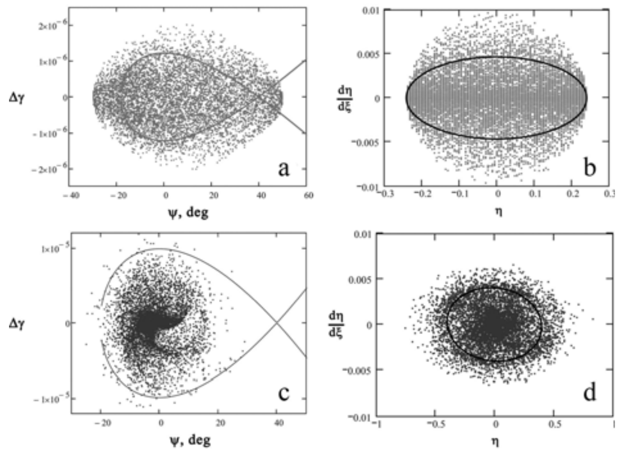

Fig.4. Beam dynamics in the smooth approximation (solenoid focusing)

4.2. APF AND SOLENOID FOCUSING

The initial parameters of numerical simulation for this system are similar to the previous system. The initial particle phase in the cavity are $\varphi_1 = -20°$ and $\varphi_2 = 30°$, and magnetic field of solenoid is $B = 12$ Tl. In this case the beam moved through 20 periods. The results shown in Fig.5 agree with previously calculation.

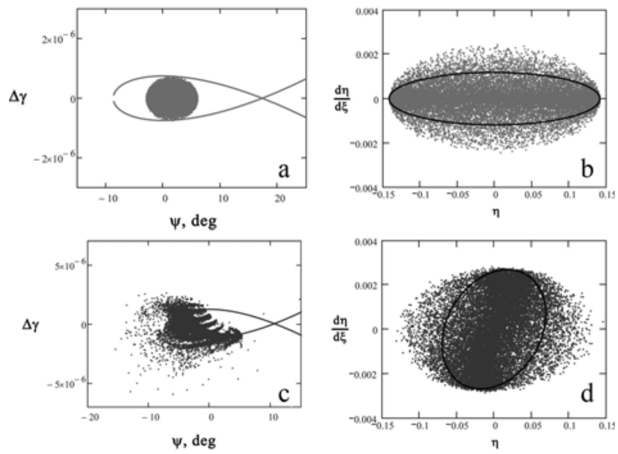

Fig.5. Beam dynamics in the smooth approximation (solenoid and APF focusing)

5. NUMERICAL SIMULATION IN POLYHARMONIC FIELD

The numerical simulation in polyharmonic field was performed to verify of the result obtained bellow. Numerical simulation was spent for the same focusing periods and the same initial parameters as in section 4 accordingly. Geometrical velocity β_G varies in each cavity too. The results of ion beam dynamics numerical simulations in a polyharmonic field are close to results received in smooth approximation. The beam longitudinal volume increase negligible and transverse emittance slowly vying.

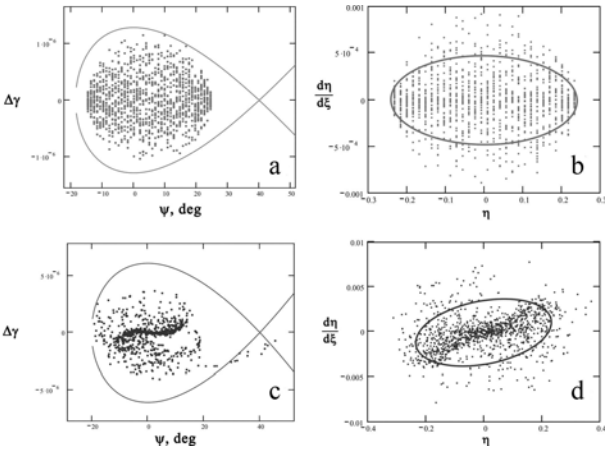

Fig.6. Beam dynamics in the polyharmonic field (solenoid focusing)

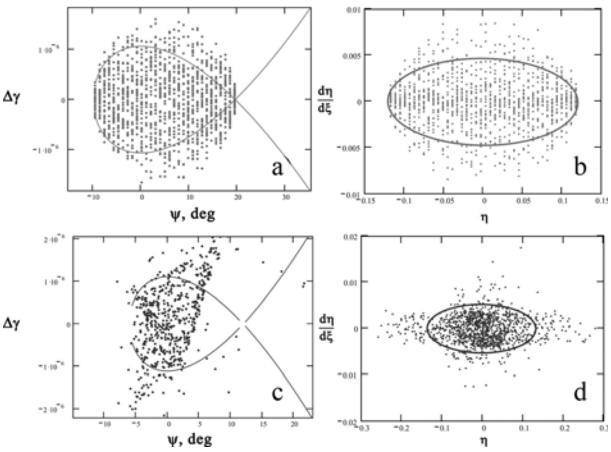

Fig.7. Beam dynamics in the polyharmonic field (solenoid and APF focusing)

CONCLUSIONS

The methods of the beam focusing in SC linac analysis are compared for low ion velocities. By the smooth approximation it was studied more detailed nonlinear ion beam dynamics and founded the beam stability area. It was done the recommendations for choice of the reference particle phases and the value of solenoid magnetic field *B*. It was shown that the smooth approximation gives very good agreement with the simulation in polyharmonic field. By the smooth approximation it is studied nonlinear ion beam dynamics in linac with combined focusing.

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ДИНАМИКА ИОННОГО ПУЧКА В СВЕРХПРОВОДЯЩЕМ УСКОРИТЕЛЕ С ТРУБКАМИ ДРЕЙФА

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Линейный ионный сверхпроводящий ускоритель основан на периодической последовательности коротких идентичных ниобиевых резонаторов с трубками дрейфа. В таком ускорителе каждый резонатор обеспечивает ускоряющее напряжение около 1 МВ. С помощью специальной фазировки поля в каждом резонаторе можно обеспечить устойчивое движение во всем ускорителе. Изучается продольная и поперечная динамика пучка в таком ускорителе. С помощью метода усреднения получено трехмерное уравнение движения в гамильтоновой форме. Это уравнение используется для анализа нелинейной динамики пучка в сверхпроводящем ускорителе. Показано, как найти связь между фазовым аксептансом и поперечным эмиттансом пучка с помощью эффективной потенциальной функции. Изучаются методы фокусировки с помощью магнитного поля соленоидов и ВЧ-поля. Результаты исследований сравниваются с численным моделированием динамики ионного пучка в сверхпроводящем линейном ускорителе.

ДИНАМІКА ІОННОГО ПУЧКА В НАДПРОВІДНОМУ ПРИСКОРЮВАЧІ З ТРУБКАМИ ДРЕЙФУ

Е.С. Масунов, А.С. Пластун, А.В. Самошин

Лінійний іонний надпровідний прискорювач заснований на періодичній послідовності коротких ідентичних ніобієвих резонаторів із трубками дрейфу. У такому прискорювачі кожен резонатор забезпечує прискорювальну напругу близько 1 МВ. За допомогою спеціального фазування поля в кожному резонаторі можна забезпечити стійкий рух у всьому прискорювачі. Вивчається поздовжня і поперечна динаміка пучка в такому прискорювачі. За допомогою методу усереднення отримане тривимірне рівняння руху в гамільтоновій формі. Це рівняння використається для аналізу нелінійної динаміки пучка у надпровідному прискорювачі. Показано, як знайти зв'язок між фазовим аксептансом і поперечним еміттансом пучка за допомогою ефективної потенційної функції. Вивчаються методи фокусування за допомогою магнітного поля соленоїдів і ВЧ-поля. Результати досліджень погоджуються із чисельним моделюванням динаміки іонного пучка в надпровідному лінійному прискорювачі.