Algebra and Discrete Mathematics
Number 4. **(2008).** pp. 90 – 99

SURVEY ARTICLE

Miguel Ferrero

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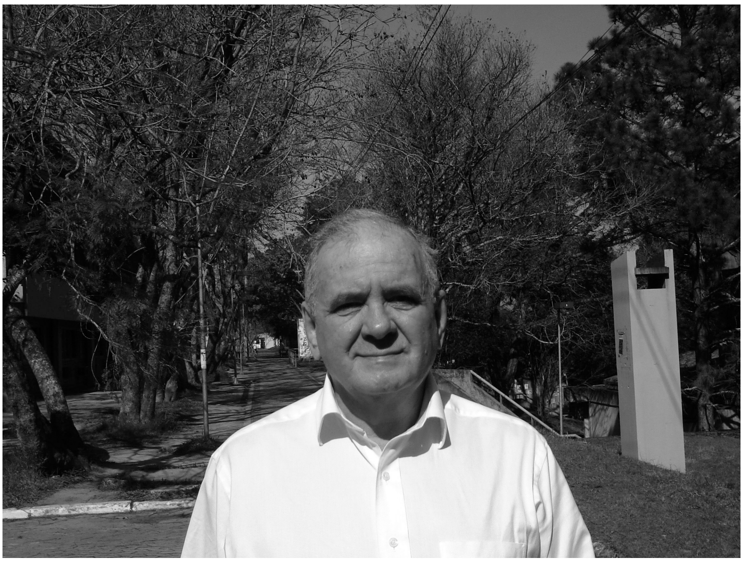

Abstract. This is a short survey of Miguel Ferrero's academic activity written on the occasion of his 70^{th} birthday.

Miguel Angel Alberto Ferrero, internationally known shortly as Miguel Ferrero, was born on September, 14, 1938, in Cañada Rosquín of the Province of Santa Fé in Argentina. In 1963 he graduated from Universidad Nacional de Rosario and in 1970 he obtained his PhD Degree at Universidad de Buenos Aires, defending the PhD Thesis "Teoria de Galois para Anillos Graduados" ("Galois theory for Graded Rings") under the supervision of Professor Orlando Villamayor. He was working at Universidad Nacional de Rosario from 1959 to 1976, occupying various positions from Auxiliary Professor to a Titular Professor.

Miguel Ferrero came to Brazil in 1976 starting his teaching and research activities at the Universidade Estadual de Campinas (Unicamp). Since 1977 he is a Professor at Universidade Federal do Rio Grande do Sul (UFRGS), where he was one of the founders of the Graduate Course in Mathematics.

Miguel Ferrero research interests include such topics as Galois Theory, the theory of radicals, ring extensions, the structure of prime ideals, distributivity of rings and modules, polynomial and skew polynomial rings, (skew) Laurent polynomial rings, partial actions on rings, as well as some other related subjects.

The intensive algebraic research activity of Miguel Ferrero started with Galois Theory (see [70] - [77]), one of the most beautiful areas in Mathematics which keeps him occupied till nowadays. Influenced partly by the results of his supervisor O. E. Villamayor obtained with D. Zelinsky in the sixties, and by the classical paper of S. Chase, D. K. Harrison and A. Rosenberg on Galois Theory of commutative rings (1965), Miguel obtained in a number of articles (some of them in collaboration with other authors) generalizations and refinements of fundamental results from both of these references, including his first algebraic paper [77] published in 1970, the improvement of Galois Theory of commutative rings given with A. Paques in [27] (1997), as well one of the most recent steps, the development of Galois Theory based on partial actions in collaboration with M. Dokuchaev and A. Paques (2007, [4]). Other significant contributions to Galois Theory include results on outer Galois Theory (1976, [70]), connectedness of Galois extensions (with K. Kishimoto, 1983, [63]), $\mathbb{Z}_p$-extensions (with A. Paques and A. Solecki, 1991, [50]) and dihedral Galois extensions (with A. Paques, 1999, [22]).

Starting from late seventies Miguel Ferrero's interest broadened including a number of topics of classical ring theory and resulting in remarkable contributions. Some of the main streams of ring theory, such as the study of polynomial, skew, Laurent polynomial rings, more general ring extensions, their prime ideals, radicals and automorphisms, as well as distributivity in rings and modules and derivations of rings, became also main streams of Miguel's research. As examples one can examine his remarkable contributions in the study of radicals, starting with his paper with K. Kishimoto and K. Motose [64] published in 1983, and in the related topic of the investigation of prime ideals in polynomial rings, including their skew and Laurent versions and more generally, in centralizing and normalizing extensions.

The latter theme had several developments, including the introduction of the concept of a (principal) closed ideal in Miguel's 1990 paper [53], and its application to the study of prime ideals in polynomial rings in the same paper, as well as in Ore extensions in two other articles published in the same year, one of which with E. Cisneros and M. I. Gonzáles [51], and another one with J. Matczuk [54]. In fact, Miguel's 1990 paper on closed ideals induced a series of articles on the subject in which the idea was further developed resulting in a powerful method with many applications. In particular, the technique was successfully used in more general case of free centered extensions in [48] (1992), which include such examples as semigroup rings, matrix rings and tensor products. The next notable step was the passage to non-necessarily free centered extensions via use of more general closed submodules in centered bimodules with prime base ring in [39] (1995). Soon afterwards the semi-prime base ring case was also handled in [29] (1996).

Many results and applications were obtained as a consequence of the method. Thus in the above mentioned 1995 paper valuable applications were given to non-singular modules, strongly prime rings and strongly closed submodules, as well as to the torsion-free rank and the Goldie dimension of a submodule. In a paper with E. Puczilowski [31] (1996) important applications were obtained to radicals of centered extensions, including the prime, strongly prime, locally nilpotent, nil, singular Jacobson and Brown-McCoy radicals, as well as to the study of prime ideals in tensor products. Further results and developments were obtained in [33](1996, with R. Wisbauer), [24] (1998), [18] (2001) and more recently in [10] (2004, with R. R. Steffenon).

Beside the general facts on closed and prime ideals, in the case of polynomial rings more specific valuable information was obtained in [36] and [28]. The results in the latter paper are especially elegant: It is a basic fact that an ideal P of the polynomial ring $K[x]$ over a field K is prime exactly when $P = (f)$ for some irreducible polynomial f. Miguel gave a generalization of this for the polynomial ring $R[x]$ over an arbitrary ring R with identity: it was shown that there exists a one-to-one correspondence between the prime ideals in $R[x]$ and the pairs (Q, f), where $Q = P \cap R$, a prime ideal in R, and f is a "Γ_Q-completely irreducible polynomial" in $R[x]$. This fact was used then to describe the prime ideals in the polynomial ring $R[x_1, \ldots x_n]$ over arbitrary R. In particular, any prime ideal P of $R[x_1, \ldots x_n]$ is determined by its intersection with R plus n polynomials.

One of the famous problems in ring theory is the Köthe's Conjecture. It is well-known that if I and J are two-sided nil ideals in a non-necessarily unital ring R then $I+J$ is nil. It is also true that if I and J are nilpotent left ideals then so too is $I+J$. However it is far from being known whether $I+J$ is nil, provided that I and J are left nil ideals. The Köthe's Conjecture says that this is the case, and it has a number of equivalent formulations. Some of them were given by Miguel Ferrero and Edmund Puczylowski in [55] and they are interesting enough to being recalled here. Assume that $R = R_1 + R_2$, where R_1, R_2 are subrings. Then the following assertions are equivalent to the Köthe's Conjecture: (1) R is nil, provided that R_1 is nilpotent and R_2 is nil. (2) R is nil, provided that R_1 is right (or left) T-nilpotent and R_2 is nil. These facts were obtained as consequences of more general results concerning radicals. Assume that R_1 is right T-nilpotent. Then for many radicals S, including the Jacobson, locally nilpotent, the prime, the right strongly prime and the generalized nil radicals, it was proved that if R_2 is an S-radical ring then R is also an S-radical ring. The corresponding statement for the nil radical is equivalent to Köthe's Conjecture.

Another article related to Köthe's Conjecture was done in collaboration with R. Wisbauer [12] and deals in particular with radicals of polynomial rings. According to a result by Jan Krempa (1972), the Köthe's Conjecture is equivalent to the following statement: If R is a nil ring then $R[x]$ is a Jacobson radical ring. E. Puczylowski and A. Smoktunowicz (1998) proved that if R is nil then $R[x]$ is a Brown-McCoy radical ring. Recall that the Brown-McCoy radical contains the Jacobson radical. J. Krempa determined the Brown-McCoy radical of $R[x]$ and M. Ferrero and R. Wisbauer went on to polynomial rings with various indeterminates. More precisely they study the Brown-McCoy radical and the unitary strongly prime (u-strongly prime, for short) radical. The u-strongly prime radical $S(R)$ of a ring R is defined as the intersection of all prime ideals P of R such that R/P is a u-strongly prime ring, i. e. the central closure of R/P is a simple ring with 1. It is proved that if R is any ring and X is a finite or infinite set of either commuting or non-commuting indeterminates, then $S(R[X]) = S(R)[X]$. It is also shown that if R is an arbitrary ring and X is an infinite set of either commuting or non-commuting indeterminates, then the Brown-McCoy radical of $R[X]$ is $S(R)[X]$. The latter conclusion still holds with finite X provided that R is a PI-ring.

Between 1993 and 2005 Miguel, with several other collaborators, produced a series of articles dedicated to another topic of systematic interest, namely that of chain rings, distributive rings, the more general so-called rings with comparability, as well as Bezout rings: [45], [44], [42], [38], [26], [23], [17], [11], [8] (see also [7]). The main attention was paid to right distributive rings and right Bezout rings wich contain a completely prime ideal in their Jacobson radical. In particular, a sort of comparability was used, envolving elements, which turned out to be rather efficient in this study, permitting to describe the structure of ideals of these rings, as well as to obtain known results with more direct and simple proofs. As an effect, the structure of such rings below the Jacobson radical was completely determined. On the other hand, very little is known about their structure above the Jacobson radical. Information about these rings above the Jacobson radical corresponds to information about distributive (Bezout) J-semisimple domains, which are not well undestrood. In this direction, in [26] the authors ask the following question: does there exist a J-semisimple right distributive domain which is not left distributive?

Miguel's research production is not limited to the articles cited above and includes other relevant papers on the above mentioned topics, as well as on derivations and higher derivations, some interesting isolated publications, and the intensive recent production on partial actions, a new attractive topic in algebra which comes from the theory of operator algebras, and which involves several of his former and present students, as well as other collaborators. The list of publications by Miguel Ferrero is given in the references below.

Miguel Ferrero supervised 10 Master Degree students and 10 PhD students which are working nowadays at several universities in the south of Brazil. He published 78 scientific articles, was an editor of *Communications in Algebra* since 1992 till 2006, and at the moment he is member of editorial boards of *The East-West Journal of Mathematics* (since 1998) and *Journal of Algebra and its Applications* (since 2001). Miguel Ferrero was honored by the award "FAPERGS - Pesquisadores Destaque 2001", in area of Informatics, Mathematics and Statistics in recognition of his scientific work.

In 1998, Miguel Ferrero organized the XV Escola de Álgebra (XV Brazilian School of Algebra) which was held in Canela (Brasil), from 26 of July till 1 of August, and edited its proceedings. This conference had approximately 200 participant, many of them from abroad.

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