DISCRETIZED COLLISION OPERATOR FOR SIMULATIONS OF FUSION NON-MAXWELLIAN PLASMA RELAXATION

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The plasma observed in modern fusion devices very often exhibits strongly non-Maxwellian distribution. This is the result of magnetic field lines reconnection with formation of magnetic resonant structures like magnetic islands and stochastic layers. Along with that, the plasma heating by means of neutral beam injection (NBI) and ion/electron cyclotron resonance frequency (ICRF/ECRF) heating induce the non-Maxwellian fast ions. In order to get the comprehensive description of plasmas one should take care of plasma particles interaction, i.e. Coulomb collisions in non-Maxwellian environment. In present paper the expression for the discretized collision operator of a general Monte Carlo equivalent form in terms of expectation values and standard deviation for the non-Maxwellian bulk distribution function is derived for a magnetized plasma assuming distribution function isotropy. The simulation for relaxation of fusion product fractions like α-particles, protons and deuterium ions on background plasma particles is performed with the use of presented collision operator. On this purpose the δ-function distribution for the bulk plasmas is assumed.

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INTRODUCTION

In modern studies the plasma observed in fusion devices like Tokamak and Stellarator is very often characterized by strongly non-Maxwellian distribution. The transition from Maxwellian to non-Maxwellian distribution is caused by the reconnection of the magnetic field lines with the further formation of magnetic islands and magnetic stochastic layers. Another reason for the transition is the plasma heating by means of neutral beam injection (NBI) and ion/electron cyclotron resonance frequency (ICRF/ECRF) heating that induce the non-Maxwellian fast ions, which interact with bulk and thermal ions. This phenomenon significantly modifies the characteristics of plasma in general that is clearly demonstrated on Tokamak JET [1, 2]. At present time the variety of numerical techniques to simulate the transition from Maxwellian to non-Maxwellian distribution is developed [3 - 5]. At the same time in order to get the comprehensive description of plasma dynamics one should take care of plasma particles interaction, i.e. Coulomb collisions in Maxwellian/non-Maxwellian environment. The crucial point is the fact that the approach to describe the non-Maxwellian plasma relaxation through collisions should be introduced. That could be done via discretized collision operator developed for the test particle tracing approach. This operator was introduced in the paper [6] for the pitch-angle scattering and the energy slowing down and scattering. Later it was extended to different plasma species [7], and its validity to trace heavy impurities in fusion plasmas was shown in [8]. The significant constraint put in this operator is the isotropic Maxwellian distribution of the background plasmas.

The objective of our work is to extend the applicability of the discretized collision operator to non-Maxwellian plasma. Starting from the Fokker-Planck collision operator, which includes Rosenbluth potentials, we derive new expressions for the discretized operator of a general Monte Carlo equivalent form in terms of expectation value and standard deviation including an arbitrary shape of distribution function for bulk plasma.

The operator is used to simulate slowing down of fusion products like α-particles, protons and deuterium ions on background plasma particles. The initial kinetic energies for each test particle are chosen as 3.52 MeV for fusion alphas, 3.02 and 14.7 MeV for protons and 9.5 MeV for deuterons. Under these conditions the bulk plasma is assumed to have δ-function distribution and the criteria of using the operator under mentioned conditions is presented. The applicability of the operator to reproduce the time scale for fusion products slowing down in bulk plasmas is shown.

1. TEST PARTICLE APPROACH

A test particle approach is an idealized model of an object whose physical properties are considered to be negligible except of those sufficient to impact the rest of the system. Concerning plasma physics, in simulations with electromagnetic fields and Coulomb collisions the most important characteristics of a test particle become its electric charge and its mass. As to the fusion magnetized plasma, in order to describe test particle motion [9] one could integrate Newton's equation with the Lorentz force to trace the exact particle trajectory

$$m_a \frac{d\mathbf{v}}{dt} = Z_a e\mathbf{E} + \frac{Z_a e}{c}[\mathbf{v}\times\mathbf{B}], \qquad \frac{d\mathbf{x}}{dt} = \mathbf{v}, \tag{1}$$

where, m_a is the test particle mass, $Z_a e$ is its charge, $\mathbf{E}$ and $\mathbf{B}$ are the electric field vector and magnetic induction respectively. On the other hand, the guiding center equation of the form

$$\mathbf{v}_g = \mathbf{v}_{||}\frac{\mathbf{B}}{B} + \frac{c}{B^2}[\mathbf{E}\times\mathbf{B}] + \frac{m_a c\left(2\mathrm{v}_{||}^2 + \mathrm{v}_{\perp}^2\right)}{2Z_a eB^3}[\mathbf{B}\times\nabla B] + \frac{m_a c\,\mathrm{v}_{||}^2}{Z_a eB^4}[[\mathbf{B}\times rot\,\mathbf{B}]\times\mathbf{B}], \tag{2}$$

could be used to calculate guiding center trajectory [10] in the assumption that the Larmor radius is small comparing to the characteristic lengths of the inhomogeneity of the background plasma. Here $\mathrm{v}_{||}$ and $\mathrm{v}_{\perp}$ are the parallel and perpendicular components of test particle velocity with respect to the magnetic field line direction. The equation (2) should be backed with the particle energy conservation low

$W \equiv m_a\left(\mathrm{v}_\perp^2+\mathrm{v}_{||}^2\right)/2+Z_a\, e\,\Phi = const$ and conservation of perpendicular invariant of motion $\mu \equiv \mathrm{v}_\perp^2/B = const$. To complete the test particle motion description, the Coulomb collisions should be included. The idea is that each integration time step the particle suffers a number of collisions that leads to modification of kinetic energy and additional modification of velocity vector direction. This effect could be described by discretized collision operator acting on the test particle after each integration step.

2. DISCRETIZED COLLISION OPERATOR

The Fokker-Planck collision operator acting on distribution function $f_a(\mathbf{v})$ of an arbitrary test particle species (a) under the assumption of isotropy for the distribution function $f_b(\mathbf{v}')$ of background plasma species (b) could be rewritten as follows

$$\frac{d\,f_a}{d\,t} = \nu_d(\mathrm{v})\,\mathrm{L_C} + \frac{1}{\mathrm{v}^2}\frac{\partial}{\partial \mathrm{v}}\left[\mathrm{v}^3\left(\frac{m_a}{m_a+m_b}\nu_S(\mathrm{v})\, f_a + \frac{1}{2}\nu_{||}\,\mathrm{v}\frac{\partial f_a}{\partial \mathrm{v}}\right)\right], \qquad (3)$$

where $\mathrm{L_C} = \frac{1}{2}\frac{\partial}{\partial\lambda}\left[\left(1-\lambda^2\right)\frac{\partial f_a}{\partial\lambda}\right]$ is Lorenz collision operator, $\mathrm{L^{ab}} = \ln\Lambda\left(4\pi\, Z_a Z_b\, e^2/m_a\right)^2$ is function of Coulomb logarithm $\ln\Lambda$, charge numbers Z_a and Z_b, and the mass m_a. $\nu_S = \mathrm{L^{ab}}/\mathrm{v}\left(1+m_a/m_b\right)\partial\varphi_b/\partial\mathrm{v}$, $\nu_d = -2\frac{\mathrm{L^{ab}}}{\mathrm{v}^3}\frac{\partial\psi_b}{\partial\mathrm{v}}$ and $\nu_{||} = -2\frac{\mathrm{L^{ab}}}{\mathrm{v}^2}\frac{\partial^2\psi_b}{\partial\mathrm{v}^2}$ are slowing down, deflection and parallel velocity diffusion frequencies respectively. The Rosenbluth potentials

$$\varphi_b = -\frac{1}{4\pi}\int\frac{1}{u}f_b(\mathbf{v}')d^3\mathrm{v}', \quad \psi_b = -\frac{1}{8\pi}\int u\, f_b(\mathbf{v}')d^3\mathrm{v}', \qquad (4)$$

are functions of the relative velocity of particles $u = |\mathbf{v}-\mathbf{v}'|$ and distribution function f_b.

The Monte Carlo equivalent of the collision operator of the general form expressed in terms of time derivative of expectation values and the square of the standard deviation reads

$$F_n = F_o + d\langle F\rangle/dt\;\Delta\tau \pm \sqrt{\left(d\sigma_F^2/dt\right)\Delta\tau}\,, \qquad (5)$$

where $\Delta\tau$ is integration time step and function F could be replaced either by the kinetic energy of test particle $K = m_a\mathrm{v}^2/2$ or by its pitch angle $\lambda = \mathrm{v}_{||}/\mathrm{v}$. The sign $\pm$ is to be chosen randomly but with the equal probability [6]. By definition, the expectation value is $\langle F\rangle = \int F\, f_a\, dF$, and its time derivative reads as $\frac{d\langle F\rangle}{dt} = \int F\frac{d\,f_a}{dt}dF + \int f_a\frac{d\,F}{dt}dF$, where the second term vanishes due to the fact of constant random variable values. Then the time derivative of square of standard deviation becomes $\frac{d\sigma_F^2}{dt} = \frac{d\langle F^2\rangle}{dt} - 2\langle F\rangle\frac{d\langle F\rangle}{dt}$. Taking into account the derivative $d\,f_a/d\,t$ of the form (3) the analytical treatment under assumption of an arbitrary isotropic distribution function leads us to general expression for the energy slowing down and scattering operator

$$K_n = K_0 - 2\,K_0\,\Delta\tau\left(\frac{m_a}{m_a+m_b}\nu_S - \frac{5}{2}\nu_{||} - K_0\frac{\partial\nu_{||}}{\partial K}\bigg|_{K=K_0}\right) \pm 2K_0\sqrt{\nu_{||}\,\Delta\tau}\;. \qquad (6)$$

Now we are out of the possibility to derive the analytical form for the collision operator basing on arbitrary distribution function for the background plasma particles. Nevertheless, we can integrate numerically the expression (6) for any shape of bulk distribution and are able to study some special cases of the background plasma distribution function f_b.

3. FAST PARTICLES RELAXATION ON "δ-FUNCTION" BACKGROUND

The discretized collision operator for isotropic Maxwellian distribution of the background plasma is fairly well described and its validity approval is presented in literature in details [6-8].

We have to point that Maxwellian operator [6-8] could be derived analytically, as a special case, from the newly obtained expression (6) including the transformation of Rosenbluth potentials (4).

Since the non-Maxwellian plasma is the point of our interest, to test the applicability of the operator (6), we consider a simplified case assuming the bulk ions distribution to be in the form of δ-function $f_b(\mathrm{v}') = n_b\,\delta(\mathrm{v}')$. This approach is valid for the fast test particles colliding with the bulk ions since the initial test particle velocity is much higher than the bulk thermal velocity. For example, the slowing down of fusion α-particles with birth energy of 3.52 MeV, which are the result of D + T fusion, is of interest for our test approach. Along with that the relaxation of fast protons with energies 3.02 and 14.7 MeV that are the result of D + D and D + He^3 fusion respectively and deuterium ions of 9.5 MeV that are the result of He^3+T reaction is studied further in details.

In the case of δ-function distribution for the bulk ions the Fokker-Planck collision operator (3) becomes $\frac{d\,f_a}{d\,t} = \mathrm{L^{ab}}\frac{n_b}{4\pi}\frac{1}{\mathrm{v}^2}\left(\frac{m_a}{m_b}\frac{\partial f_a}{\partial\mathrm{v}} + \frac{1}{\mathrm{v}}\mathrm{L_C}\right)$. In accordance to the general formula (5) the discretized operator (6) reduces to the form

$$K_n = K_o - 2\,K_0\,\nu_K\,\Delta\tau \qquad (7)$$

in terms of collision frequency $\nu_K = \frac{\mathrm{L^{ab}}}{\mathrm{v}^3}\frac{n_b}{4\pi}\frac{m_a}{m_b}$. The fact that time derivative of square of standard deviation is $d\sigma_K^2/dt = 0$ excludes the broadening of kinetic energy.

On the Fig. 1 the relaxation of different fusion products calculated by means of the operator (7) is compared to the same processes calculated by the collision operator based on Maxwellian distribution [6-8] for different bulk plasmas with density 10^{14} cm^{-3} and the temperature 5 keV for all cases.

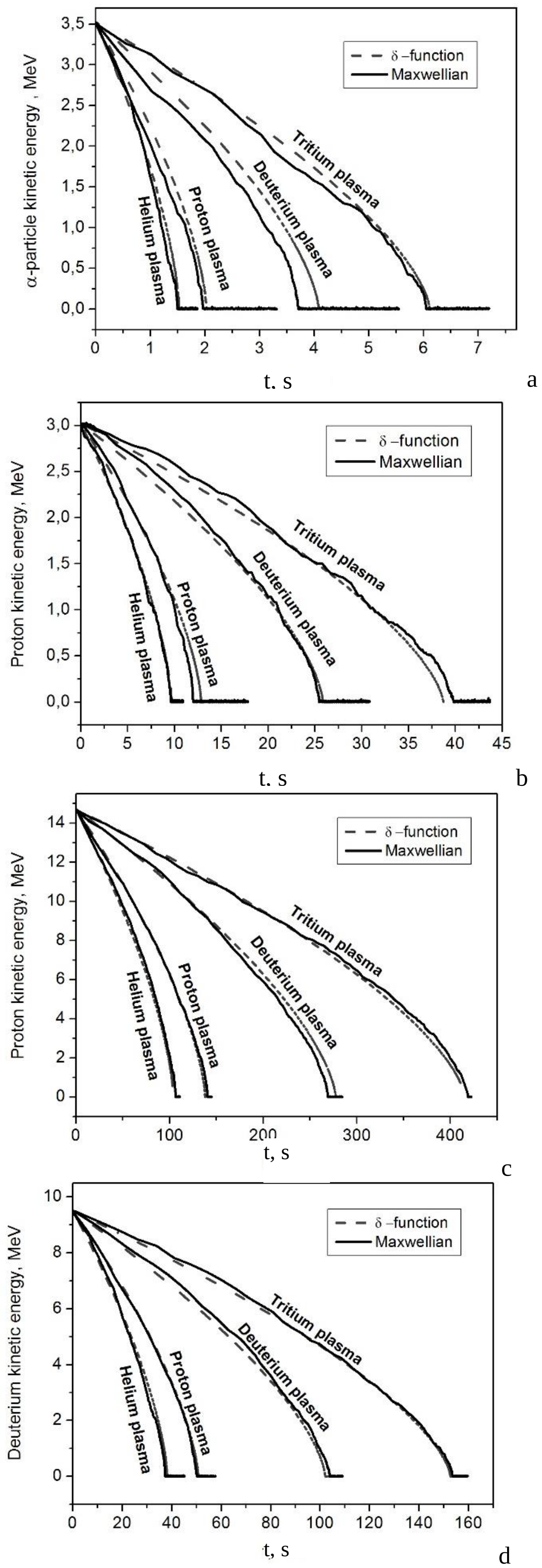

Fig. 1. Slowing down of fusion α-particles with initial energy 3.52 MeV (a), fast protons with initial energies 3.02 MeV and 14.7 MeV (b and c), deuterium with initial energy 9.5 MeV (d) on Maxwellian bulk plasmas and on bulk with δ-function distribution

The Fig. 2 is the extension of Fig. 1,a. It shows in different scale the fluctuations of kinetic energy of *α-particle* colliding with the helium Maxwellian plasma.

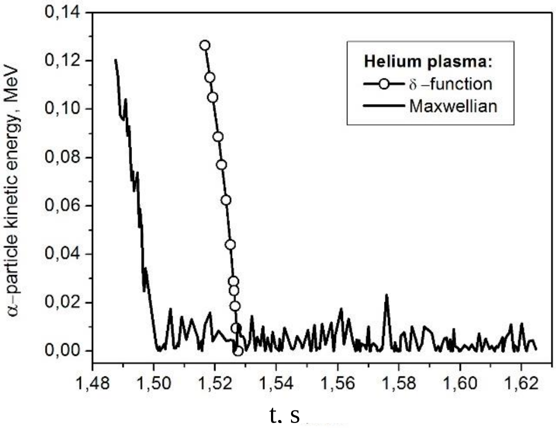

Fig. 2. Extension of figure 1,a: the fluctuations of α-particle velocity caused by collisions with the Maxwellian background

The fluctuation peaks represent the interaction with the bulk particles existing in the tail of Maxwellian distribution function.

Good agreement of the results for the typical time scale for the relaxation process proofs the validity of the operator (7) to simulate fast test particles slowing down in the energy frame starting from the high initial energies down to thermal energies of bulk plasmas. The advantages of this operator are the simplicity and the possibility to reduce significantly the calculation time for the certain plasma scenarios.

CONCLUSIONS

Current research is devoted to the derivation of general comprehensive form of the Monte Carlo equivalent of slowing down and scattering collision operator. The integration of equations of motion (1) and (2) together with the collision operator that is called discretized collision operator (5) completes the description of test particle motion in fusion collisional plasma.

Starting from the Fokker-Planck collision operator (3) the general form of discretized collision operator for kinetic energy slowing down and scattering (6) is obtained. It includes the collision frequencies expressed via Rosenbluth potentials in general integral form (4).

The crucial point is the fact that the only assumption made for this derivation is the isotropy of the distribution function of bulk plasma in velocity space. Hence the distribution depends only on the value of the velocity but not on its direction $f_b(\mathbf{v}') \rightarrow f_b(\mathrm{v}')$. No any other assumption on the shape of bulk distribution function is made in operator (6). The test of newly derived operator was performed in two steps.

Firstly, the ability to pass from the general form (6) to the form that is well known and described in [6-8] assuming the isotropic Maxwellian distribution of bulk plasma was successfully checked.

As the second step basing on bulk particles distribution in the form of δ-function the simplified version of discretized collision operator is derived (7) and studied in details. Its validity to calculate relaxation reflected in energy slowing down for energetic particles like ion beams and fusion products is proved.

Generally, the numerical integration of the collision operator (6) gives us possibility to study comprehensive picture of particle transport for different magnetic field configurations and confinement scenarios characterized by non-Maxwellian distribution of plasma particles.

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ДИСКРЕТНЫЙ СТОЛКНОВИТЕЛЬНЫЙ ОПЕРАТОР ДЛЯ МОДЕЛИРОВАНИЯ РЕЛАКСАЦИИ ТЕРМОЯДЕРНОЙ НЕМАКСВЕЛЛОВСКОЙ ПЛАЗМЫ

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Плазма, которая наблюдается в современных установках темоядерного синтеза, очень часто характеризуется немаксвелловской функцией распределения. Такое распределение может быть результатом пересоединения силовых линий магнитного поля с последующим формированием резонансных структур, таких как магнитные острова и стохастические магнитные слои. Кроме того, нагрев плазмы методами ионного и электронного циклотронных резонансов приводит к появлению немаксвелловских быстрых ионов, которые, в свою очередь, взаимодействуют с основной плазмой и тепловыми ионами. Чтобы получить полное описание поведения плазмы в таких условиях, необходимо учесть взаимодействие между частицами, а именно – кулоновские столкновения в немаксвелловской среде. Представлено полное выражение для дискретного столкновительного оператора в общей эквивалентной форме Монте Карло с использованием ожидаемой величины и квадрата стандартного отклонения, а также в приближении изотропного распределения основной плазмы. При помощи данного оператора проведено численное моделирование релаксации продуктов термоядерной реакции, таких как альфа-частицы, протоны и ионы дейтерия на основной плазме. В данном моделировании использовалось приближение, где основная плазма может быть описана дельта-функцией.

ДИСКРЕТНИЙ ОПЕРАТОР ЗІТКНЕНЬ ДЛЯ МОДЕЛЮВАННЯ РЕЛАКСАЦІЇ ТЕРМОЯДЕРНОЇ НЕМАКСВЕЛІВСЬКОЇ ПЛАЗМИ

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Плазма, що спостерігається в сучасних пристроях керованого термоядерного синтезу, часто характеризується немаксвелівською функцією розподілу. Цей розподіл може бути результатом перез'єднання силових ліній магнітного поля з подальшим формуванням резонансних структур, таких як магнітні острови та магнітні стохастичні шари. Крім цього, нагрівання плазми методами іонного та електронного циклотронних резонансів призводить до появи немаксвелівських швидких іонів, які взаємодіють з основною плазмою та тепловими іонами. Щоб отримати повний опис поведінки плазми в таких умовах, необхідно урахувати взаємодію частинок плазми між собою, а саме – кулонівські зіткнення в немаксвелівському середовищі. Представлено повний вигляд дискретного оператора зіткнень у загальній еквівалентній формі Монте Карло із використанням величини очікування та квадрату стандартного відхилення. Також використано наближення ізотропної плазми. Із використанням цього оператора проведено числове моделювання релаксації продуктів термоядерної реакції, таких як альфа-частинки, протони та іони дейтерію на основній плазмі. У поданому моделюванні використовується наближення, де основна плазма може бути описана за допомогою дельта-функції.