CYCLOTRON RESONANCE CONDITIONS IN CURRENT-CARRYING PLASMAS

N.I. Grishanov[1,2], N.A. Azarenkov[1]

[1]*V.N. Karazin Kharkiv National University, Kharkiv, Ukraine;*
[2]*Ukrainian State University of Railway Transport, Department of Physics, Kharkiv, Ukraine*

The resonant cyclotron wave-particle interactions in the cylindrical current-carrying plasma and axisymmetric toroidal plasma models for large aspect ratio tokamaks with circular, elliptic and D-shaped cross-sections of the magnetic surfaces have been analyzed. The corresponding conditions are derived by solving the linearized Vlasov equations for perturbed distribution functions of plasma particles, accounting for the geometry of a confinement magnetic field in the zero-order over magnetization parameters. It is shown that the Doppler shift at the cyclotron resonance conditions in the current-carrying plasmas is entirely different from ones in uniform magnetic field.

PACS: 52.55.Fa, 52.50.Qt

INTRODUCTION

Effective schemes of plasma heating in tokamaks can be realized by collisionless wave dissipation in the range of ion-cyclotron and/or electron-cyclotron frequencies (fundamental cyclotron resonance: $\ell = 1$ for ions - ICR, $\ell = -1$ for electrons - ECR) and their harmonics $(|\ell| \geq 2)$. As is well known [1], the electromagnetic waves are always absorbed in the equilibrium plasma models, e.g. with the maxwellian distribution of charged particles. However, the presence of non-equilibrium energetic particles can lead to wave instabilities observed as ion-cyclotron and electron-cyclotron emissions under the ICR and ECR plasma heating.

To estimate the wave damping/growth rates in any plasma model we should know the conditions of the resonant wave-particle interactions there. The corresponding conditions can be derived automatically by solving the linearized Vlasov equation for perturbed distribution functions of plasma particles, accounting for the geometry of a confinement magnetic field $\mathbf{H}_0$.

In this paper we discuss the cyclotron wave-particle interactions in the cylindrical current-carrying plasma (i.e. with a helical magnetic field) and in the two-dimensional (2D) axisymmetric toroidal plasma models for tokamaks with circular, elliptic and D-shaped magnetic surfaces. The Vlasov equations are resolved in the zero-order over the magnetization parameters, using an approach developed in Refs. [2-6]. It is shown that the Doppler shifts at the cyclotron resonance conditions in the current-carrying plasmas are entirely different from ones for plasmas in uniform magnetic field [1]: $k_{\|}\upsilon_{\|} = \omega - \ell\Omega_\alpha$, where $\ell = \pm1, \pm2$, ... is the cyclotron harmonic number; $\Omega_\alpha = q_\alpha H_0 / M_\alpha c$ is the Larmor frequency of ions $(\alpha = i,\ \Omega_i > 0)$ and electrons $(\alpha = e,\ \Omega_e < 0)$; c is the speed of light, $k_{\|} = \mathbf{kh} = k_z$ is the parallel wave-number relative to confinement magnetic field $\mathbf{H}_0 = H_0\mathbf{e}_z,\ \mathbf{h} = \mathbf{H}_0 / H_0$.

1. CYLINDRICAL PLASMA MODEL

The simplest 1D model of tokamaks is a magnetized current-carrying plasma cylinder with identical ends in the helical magnetic field, where the longitudinal ohmic current generates the poloidal magnetic field $\mathbf{H}_{0\theta} = H_{0\theta}\mathbf{e}_\theta$ in addition to longitudinal $\mathbf{H}_{0z} = H_{0z}\mathbf{e}_z$. In this case the length of plasma cylinder is equal to $2\pi R_0$, where R_0 is the major tokamak radius. As a result, the field $\mathbf{H}_0 = \mathbf{H}_{0\theta} + \mathbf{H}_{0z}$ becomes helical with substantial rotational transformation, allowing to take into account the so-called shear effects and the radial profiles of ohmic current by the radial dependence of plasma safety factor $q(r) = rH_{0z}/R_0H_{0\theta}$.

To develop the kinetic theory for cyclotron waves in such plasma model one should resolve the Vlasov equation for perturbed distribution functions $f_\alpha(t, \mathbf{r}, \mathbf{v})$ by the Fourier-decomposition over the polar angle σ in velocity space:

$$f_\alpha(t, \mathbf{r}, \mathbf{v}) = \sum_{\ell}^{\pm\infty} f_\alpha^\ell(r, \upsilon_{\|}, \upsilon_\perp) \exp(-\mathrm{i}\omega t + \mathrm{i}m\theta + \mathrm{i}k_z z - \mathrm{i}\ell\sigma),$$

where we have used the usual standard notation for the radial coordinate r numbering the magnetic surfaces, θ is the poloidal angle. In velocity space we use the polar coordinates ($\upsilon_\perp, \sigma$) instead of the normal and binormal components (υ_n, υ_b) by the transformation:

$$\upsilon_n = \mathbf{vn} = \upsilon_\perp \cos\sigma,\quad \upsilon_b = \mathbf{vb} = \upsilon_\perp \sin\sigma,\quad \upsilon_{\|} = \mathbf{vh}.$$

The linearized Vlasov equation for cyclotron harmonics f_α^ℓ in the zero-order over the magnetization parameters can be reduced to algebraic equations

$$\mathrm{i}\left(\ell\Omega_\alpha - \omega + k_{\|}\upsilon_{\|} + \ell\kappa\upsilon_{\|}\right) f_\alpha^\ell = Q_\alpha^\ell, \qquad (1)$$

where
$$k_{\|} = \mathbf{kh} = \frac{mh_\theta}{r} + k_z h_z \approx \frac{m + nq}{R_0 q},$$

$$\kappa = \left(2 - \frac{r}{2q}\frac{dq}{dr}\right)\frac{1}{R_0 q}, \quad \frac{r}{h_\theta} \approx R_0 q, \text{ if } H_{0\theta} << H_{0z}, \qquad (2)$$

m and n are the poloidal and toroidal wave-numbers, $k_z = n / R_0$. The expressions of Q_α^ℓ depend substantially on the number of cyclotron harmonic ℓ and on the steady-state (unperturbed) distribution function of plasma particles F_α. For example, if F_α is maxwellian:

$$Q_\alpha^\ell = \frac{q_\alpha \upsilon_\perp F_\alpha}{M_\alpha \upsilon_{T\alpha}^2}(E_n + i\ell E_b), \qquad \ell = \pm1, \qquad (3)$$

$$F_\alpha = \frac{N_\alpha}{(\pi\upsilon_{T\alpha}^2)^{1.5}} \exp\left(-\frac{\upsilon_{\|}^2 + \upsilon_\perp^2}{\upsilon_{T\alpha}^2}\right), \quad \upsilon_{T\alpha} = \sqrt{\frac{2T_\alpha}{M_\alpha}},$$

E_n and E_b are the normal and binormal components of the perturbed electric field relative to $\mathbf{H}_0$. However, independently on the right-hand side of the Vlasov equation (and F_α. functions) the wave-particle resonance conditions in the current-carrying plasmas are defined by denominator of f_α^ℓ and can be rewritten as

$$(k_{\|} + \ell\kappa)\upsilon_{\|} = \omega - \ell\Omega_\alpha, \quad \ell = 0, \pm 1, \pm 2, \pm, \ldots . \quad (4)$$

If $\ell = 0$ we receive the well known Cherenkov resonance conditions: $\omega = k_{\|}\upsilon_{\|}$, where κ-corrections are absent. If $\ell \neq 0$ we have the cyclotron resonance conditions on the fundamental (first, $\ell = \pm 1$) harmonic of cyclotron frequency and their high harmonics if $|\ell| \geq 2$. As one can see the cyclotron resonance conditions in the current-carrying plasma are different from ones in uniform magnetic field by the $\ell\kappa$-terms, accounting for the rotation of helical magnetic field lines $\mathbf{H}_0 = \mathbf{H}_{0\theta} + \mathbf{H}_{0z}$ on the considered (by r) magnetic surface. These $\ell\kappa$-terms are very important to study the wave dissipation/excitation at the so-called rational magnetic surfaces, where $k_{\|}$ changes sign.

Of course, it is necessary to distinguish the resonances on the positive and negative cyclotron ℓ-harmonics. If $\ell = 1,2,3, \ldots$ we have the ICR conditions $\omega - \ell\Omega_i = (k_{\|} + \ell\kappa)\upsilon_{\|}$ under the normal Doppler effect for resonant ions (α=i) with the parallel velocities smaller than the wave phase velocity, $\upsilon_{\|} < \omega / k_{\|}$. In this case the resonant ions can effectively interact with the left-hand polarized waves, where rotation of the transverse electric field component $(E_n + i\ell E_b)$ coincides with the Larmor ion gyration. The ECR conditions (for $\ell > 0$), $\omega + \ell|\Omega_e| = (k_{\|} + \ell\kappa)\upsilon_{\|}$, are realized under the abnormal Doppler effect for resonant electrons (α=e) with the parallel velocities larger than the wave phase velocity, $\upsilon_{\|} > \omega / k_{\|}$. In this case, electrons cannot effectively interact with the left-hand polarized wave since their concentration is small and gyration is opposite to rotation of $(E_n + i\ell E_b)$.

If $\ell = -1,-2,-3, \ldots$ we have the ECR conditions $\omega - |\ell\Omega_e| = (k_{\|} - |\ell|\kappa)\upsilon_{\|}$ under the normal Doppler effect for electrons with $\upsilon_{\|} < \omega / k_{\|}$. In this case the resonant electrons can effectively interact with the right-hand polarized waves, where the rotation of transverse electric field component $(E_n - i|\ell|E_b)$ coincides with the electron gyration. In contrary, the ICR conditions (for $\ell < 0$), $\omega + |\ell|\Omega_i = (k_{\|} - |\ell|\kappa)\upsilon_{\|}$, are realized under the abnormal Doppler effect for resonant ions with $\upsilon_{\|} > \omega / k_{\|}$. In this case, ions cannot effectively interact with the right-hand polarized wave since their gyration is opposite to rotation of $(E_n - i|\ell|E_b)$.

The terms proportional to dq/dr in κ-corrections for cyclotron resonance conditions allow us to study the influence of ohmic current density profiles on the wave-particle interactions in the current-carrying plasmas:
a) if ohmic current is uniform: dq/dr=0;
b) if ohmic current decreases to plasma edge: $dq/dr > 0$;
c) if ohmic current increases to plasma edge: $dq/dr < 0$.

It should be noted, that the signs of dq/dr are opposite in the ICR and ECR conditions under the normal Doppler effects for ions and electrons, increasing the Doppler shift for ions and decreasing it for electrons.

2. AXISYMMETRIC D-SHAPED TOKAMAK

To describe an axisymmetric D-shaped tokamak we use the quasi-toroidal coordinates (r,θ,ϕ) connected with the cylindrical ones (ρ,ϕ,z) as [6]

$$\rho = R_0 + r\cos\theta - \frac{dr^2}{a^2}\sin^2\theta, \quad \phi = \phi, \quad z = -\frac{b}{a} r\sin\theta,$$

where R_0 is the radius of the magnetic axis; a and b are, respectively, the minor and major semiaxes of the cross-section of the external magnetic surface. In this model, all magnetic surfaces have the same elongation equal to b/a; their triangularity is small d/a<<1. The cylindrical components of an equilibrium magnetic field $\mathbf{H}_0$ are

$$H_{0\rho} = -H_{\theta 0}\frac{R_0}{\rho}\sin\theta\left(1 + 2\frac{dr}{a^2}\cos\theta\right),$$

$$H_{0\phi} = H_{\phi 0}\frac{R_0}{\rho}, \qquad H_{0z} = -H_{\theta 0}\frac{b}{a}\frac{R_0}{\rho}\cos\theta. \quad (5)$$

Here $H_{\phi 0}$ and $H_{\theta 0}$ are, respectively, the toroidal and poloidal magnetic field maximums at a given (by r) magnetic surface. Thus,

$$H_0(r,\theta) = |\mathbf{H}_0| = \sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}\, g(r,\theta),$$

$$g(r,\theta) = \frac{\sqrt{1 + \lambda\cos^2\theta + \nu\cos\theta\sin^2\theta}}{1 + \varepsilon\cos\theta - \delta\varepsilon\sin^2\theta}, \quad (6)$$

where

$$\varepsilon = \frac{r}{R_0}, \quad \delta = \frac{dr}{a^2}, \quad \lambda = h_\theta^2\left(\frac{b^2}{a^2} - 1\right), \quad \nu = 4\delta h_\theta^2,$$

$$h_\theta = \frac{H_{\theta 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}, \qquad h_\phi = \frac{H_{\phi 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}. \quad (7)$$

In tokamaks, in contrast to a cylindrical current-carrying plasma, the particle velocities $\upsilon_{\|}$ and $\upsilon_\perp$ are not constant. To reduce the number of derivatives in the Vlasov equation we use the standard method of switching to new variables associated with conservation integrals of energy and magnetic moment, introducing the variables υ and μ instead of $\upsilon_{\|}$ and $\upsilon_\perp$ as

$$\upsilon^2 = \upsilon_{\|}^2 + \upsilon_\perp^2, \qquad \mu = \frac{\upsilon_\perp^2}{\upsilon_{\|}^2 + \upsilon_\perp^2}\frac{1}{g(r,\theta)}. \quad (8)$$

In this case the linearized Vlasov equation for the perturbed distribution functions of ions and electrons,

$$f_\alpha(t,\mathbf{r},\mathbf{v}) = \sum_s^{\pm 1}\sum_\ell^{\pm\infty} f_\alpha^{\ell,s}(r,\theta,\upsilon,\mu)\exp(-\mathrm{i}\omega t + \mathrm{i}n\phi - \mathrm{i}\ell\sigma),$$

can be reduced to the first order differential equations with respect to the poloidal angle θ. For example [6], first harmonics ($\ell = \pm 1$) of $f_\alpha^{\ell,s}$ satisfy the equation:

$$\sqrt{\frac{1 - \mu g(\theta)}{1 + \lambda\cos^2\theta + \nu\cos\theta\sin^2\theta}} \times$$

$$\times\left(\frac{\partial f_\alpha^{\ell,s}}{\partial\theta}+\frac{\mathrm{i}nqf_\alpha^{\ell,s}}{1+\varepsilon\cos\theta-\varepsilon\delta\sin^2\theta}\right)- \quad (9)$$

$$-\mathrm{i}s\frac{R_0q}{\upsilon}\left[\omega-\ell\Omega_{\alpha0}g(\theta)\right]f_\alpha^{\ell,s}+\frac{\mathrm{i}\ell\sqrt{1-\mu g(r,\theta)}\tilde{\kappa}(r,\theta)f_\alpha^{\ell,s}}{1+\lambda\cos^2\theta+\nu\cos\theta\sin^2\theta}=$$

$$=s\frac{q_\alpha R_0 q}{M_\alpha\upsilon_{T\alpha}^2}F_\alpha\sqrt{\mu g(r,\theta)}E_\ell\ ,$$

where $E_\ell=E_n+\mathrm{i}\ell E_b$, $h_\phi\approx1$, $h_\theta<<1$, $r/h_\theta\approx R_0q$,

$$q=\frac{\varepsilon H_{\phi0}}{H_{\theta0}},\qquad \Omega_{\alpha0}=\frac{q_\alpha\sqrt{H_{\phi0}^2+H_{\theta0}^2}}{M_\alpha c}. \quad (10)$$

Account of centrifugal forces in Eq. (9) is reduced to

$$\tilde{\kappa}(r,\theta)=\frac{\frac{3}{2}\frac{a}{b}}{\cos^2\theta+\frac{b^2}{a^2}\sin^2\theta\left(1+2\delta\cos\theta\right)}-$$

$$-\frac{\frac{3}{2}\frac{b}{a}\varepsilon\cos\theta}{1+\varepsilon\cos\theta-\varepsilon\delta\sin^2\theta}+ \quad (11)$$

$$+\frac{a^3}{2b^3}\left(\frac{b^2}{a^2}-1\right)^2\frac{\cos^2\theta\sin^2\theta\left(1+2\delta\cos\theta\right)}{\cos^2\theta+\frac{b^2}{a^2}\sin^2\theta\left(1+2\delta\cos\theta\right)}+$$

$$+\frac{b}{2a}\left(1-\frac{r}{q}\frac{dq}{dr}\right)\left[\cos^2\theta+\frac{b^2}{a^2}\sin^2\theta\left(1+2\delta\cos\theta\right)\right].$$

By $s=\pm1$ we distinguish the perturbed distribution functions of particles, $f_\alpha^{\ell,s}$, with positive and negative parallel velocity $\upsilon_\parallel=s\upsilon\sqrt{1-\mu g(r,\theta)}$ relative to $\mathbf{H}_0$.

Describing the wave-particle interaction in tokamaks with one minimum of $\mathbf{H}_0$, i.e. when $\varepsilon>\lambda$, we should [5, 6] separate all particles on two groups of untrapped (u) and trapped (t) particles by the inequalities for μ and θ:

$0\le\mu\le\mu_u \quad -\pi\le\theta\le\pi$ - untrapped particles,

$\mu_u\le\mu\le\mu_t \quad -\theta_t\le\theta\le\theta_t$ - trapped particles,

analyzing the condition $\upsilon_\parallel(\mu,\theta)=0$. Here

$$\mu_u=1-\varepsilon-\frac{\lambda}{2},\qquad \mu_t=1+\varepsilon-\frac{\lambda}{2}, \quad (12)$$

and the angels $\pm\theta_t$ are the stop points of the trapped particles on the considered magnetic surface:

$$\pm\theta_t\approx\pm\arccos\frac{\mu-1+\varepsilon\delta}{\varepsilon-\mu\nu/2}\pm\frac{(\mu-1+\varepsilon\delta)^2}{\sqrt{1-\left(\frac{\mu-1+\varepsilon\delta}{\varepsilon-\mu\nu/2}\right)^2}}\times$$

$$\times\frac{\varepsilon\delta-\mu\frac{\lambda}{2}-\frac{\mu\nu}{2}\left(1-\frac{\mu-1+\varepsilon\delta}{\varepsilon-\mu\nu/2}\right)}{(\varepsilon-\frac{\mu\nu}{2})^3}. \quad (13)$$

To find the perturbed distribution functions of untrapped $f_{\alpha,u}^{\ell,s}$ and trapped $f_{\alpha,t}^{\ell,s}$ particles we should resolve Eq. (9) using the corresponding boundary conditions: the periodicity of $f_{\alpha,u}^{\ell,s}$ on θ, and continuity of $f_{\alpha,t}^{\ell,s}$ at the stop points $\pm\theta_t$; introducing the new time-like variable instead of poloidal angle θ as

$$\tau(\theta)=\int_0^\theta\sqrt{\frac{1+\lambda\cos^2\eta+\nu\cos\eta\sin^2\eta}{1-\mu\cdot g(r,\eta)}}d\eta\ . \quad (14)$$

In this case, the transit-time of u-particles and the bounce-period of t-particles are proportional to $T_u=2\tau(\pi)$ and $T_t=4\tau(\theta_t)$, respectively.

As a result, the cyclotron harmonics of the perturbed distribution functions of untrapped and trapped particles can be found in the forms:

$$f_{\alpha,u}^{\ell,s}=\sum_p^{\pm\infty}f_{\alpha,p,u}^{\ell,s}\exp\left[\mathrm{i}\Phi_{\alpha,p}^{\ell,0,s}(\theta,\upsilon,\mu)\right],$$

$$f_{\alpha,t}^{\ell,s}=\sum_p^{\pm\infty}f_{\alpha,p,t}^{\ell,s}\exp\left[\mathrm{i}\Psi_{\alpha,p}^{\ell,0,s}(\theta,\upsilon,\mu)\right], \quad (15)$$

where p is a number of the bounce resonances. After the bounce-averaging we have the following expressions for the bounce-resonant harmonics (if $\ell=\pm1$):

$$f_{\alpha,p,u}^{\ell,s}=-\mathrm{i}\sum_m^{\pm\infty}\frac{q_\alpha\upsilon\sqrt{\mu}F_\alpha}{M_\varepsilon\upsilon_{T\alpha}^2T_u}\frac{A_{\alpha,p}^{\ell,m,s}(\upsilon,\mu)}{Z_{\alpha,p,u}^{\ell,s}(\upsilon,\mu)}E_\ell^m\ ,$$

$$f_{\alpha,p,t}^{\ell,s}=-\mathrm{i}\sum_m^{\pm\infty}\frac{q_\alpha\upsilon\sqrt{\mu}F_\alpha}{M_\alpha\upsilon_{T\alpha}^2T_t}\frac{B_{\alpha,p}^{\ell,m,s}(\nu,\mu)}{Z_{\alpha,p,t}^{\ell,s}(\upsilon,\mu)}E_\ell^m. \quad (16)$$

Here

$$Z_{\alpha,p,u}^{\ell,s}=\frac{s2\pi\upsilon}{R_0qT_u}\left[p+nq_t+\ell\frac{I_\kappa(\pi)}{\pi}\right]-\omega+\ell\Omega_{\alpha0}\bar{g}_u\ ,$$

$$A_{\alpha,p}^{\ell,m,s}(\upsilon,\mu)=\int_{-\pi}^{\pi}\frac{\exp\left[\mathrm{i}\Phi_{\alpha,p}^{\ell,m,s}(\theta,\upsilon,\mu)\right]}{\sqrt{1-\mu g(r,\theta)}}d\theta\ ,$$

$$\Phi_{\alpha,p}^{\ell,m,s}(\theta,\upsilon,\mu)=2\pi(p+nq_t)\frac{\tau(\theta)}{T_u}-(m+nq_t)\bar{\theta}(\theta)+$$

$$+\ell\frac{R_0q}{\upsilon}\Omega_{\alpha0}\left[\bar{g}_u\tau(\theta)-I_g(\theta)\right]-\ell I_\kappa(\theta)+\ell\frac{2\tau(\theta)}{T_u}I_\kappa(\pi)\ ,$$

$$Z_{\alpha,p,t}^{\ell,s}=p\frac{s2\pi\upsilon}{R_0qT_t}-\omega+\ell\Omega_{\alpha0}\bar{g}_t\ ,$$

$$B_{\alpha,p}^{\ell,m,s}(\upsilon,\mu)=\int_{-\theta_t}^{\theta_t}\frac{\exp\left[\mathrm{i}\Psi_{\alpha,p}^{\ell,m,s}(\theta,\upsilon,\mu)\right]}{\sqrt{1-\mu g(r,\theta)}}d\theta+$$

$$+(-1)^p\int_{-\theta_t}^{\theta_t}\frac{\exp\left[\mathrm{i}\Psi_{\alpha,-p}^{\ell,m,s}(\theta,-\upsilon,\mu)\right]}{\sqrt{1-\mu g(r,\theta)}}d\theta\ ,$$

$$\Psi_{\alpha,p}^{\ell,m,,s}(\theta,\upsilon,\mu)=2\pi p\frac{\tau(\theta)}{T_t}-(m+nq_t)\bar{\theta}(\theta)+$$

$$+s\ell\frac{R_0q}{\upsilon}\Omega_{\alpha0}\left[\bar{g}_t\tau(\theta)-I_g(\theta)\right]-\ell I_\kappa(\theta)\ , \quad (17)$$

$$I_g(\theta)=\int_0^\theta g(r,\eta)\sqrt{\frac{1+\lambda\cos^2\eta+\nu\cos\eta\sin^2\eta}{1-\mu g(r,\eta)}}d\eta\ ,$$

$$I_\kappa(\theta)=\int_0^\theta\frac{\tilde{\kappa}(r,\eta)\cdot d\eta}{\sqrt{1+\lambda\cos^2\eta+\nu\cos\eta\sin^2\eta}}\ ,$$

$$\bar{g}_u=\frac{2}{T_u}I_g(\pi),\ \ \bar{g}_t=\frac{4}{T_t}I_g(\theta_t),\ \ q_t=q\left(1+\frac{\varepsilon(\varepsilon+\delta)}{2}\right),$$

$$E_\ell(\theta)\frac{\left(1+\lambda\cos^2\theta+\nu\cos\theta\sin^2\theta\right)^{3/4}}{\left(1+\varepsilon\cos\theta-\varepsilon\delta\sin^2\theta\right)^{1/2}}=\sum_m^{\pm\infty}E_\ell^m e^{im\bar{\theta}}\ ,$$

$$\bar{\theta}(\theta)\approx\theta-\varepsilon\sin\theta+\frac{\varepsilon}{4}(\varepsilon-\delta)\sin(2\theta)\ .$$

The zeros of denominators in (16) determine us the conditions of the cyclotron wave-particle interactions in D-shaped tokamaks:

$$\frac{s2\pi\upsilon}{R_0 qT_u}\left[p + nq_t + l\frac{I_\kappa(\pi)}{\pi}\right] - \omega + \ell\Omega_{\alpha 0}\bar{g}_u = 0 \qquad (18)$$

for the untrapped particles; and

$$p\frac{s2\pi\upsilon}{R_0 qT_t} - \omega + \ell\Omega_{\alpha 0}\bar{g}_t = 0 \qquad (19)$$

for trapped particles. These wave-particle resonance conditions in axisymmetric D-shaped tokamaks involve two energetic characteristics of particles (by υ and μ), the wave frequency ω, the integer numbers of cyclotron (by ℓ) and bounce (by p) resonances. For the low ℓ, as usual, we have the conditions of the:

- Cherenkov resonance, if $\ell = 0$;
- normal ICRs (α=i), if $\ell = 1,2,3,\ldots$;
- normal ECRs (α=e), if $\ell = -1,-2,-3\ldots$,

for both the untrapped and trapped particles.

Of course, analyzing the cyclotron wave-particle interactions in toroidal geometry we should take into account the coefficients $A_{\alpha,p}^{\ell,m,s}(\upsilon,\mu)$ and $B_{\alpha,p}^{\ell,m,s}(\upsilon,\mu)$ for untrapped and trapped particles, respectively.

CONCLUSIONS

Regarding the plasma response to perturbations in the current-carrying plasmas the kinetic wave analysis should take into account the so-called shear effects connected with the fact that the equilibrium magnetic field lines become helical and there are additional inertial (centrifugal) forces acting on the moving charged particles.

Specific features of the wave-particle interactions in D-shaped tokomaks are due to that i) the resonance conditions for untrapped and trapped particles are different, and ii) all *m*-harmonics of the perturbed electric field contribute to the perturbed distribution functions of untrapped and trapped particles.

If triangularity is absent, i.e. if $d \to 0$, the wave-particle resonant conditions for untrapped and trapped particles, Eq. (18) and Eq. (19), can be readily reduced to the corresponding expressions for tokamaks with elliptic magnetic surfaces. If elongation is absent (b=a), Eqs. (18), (19) have as limits the wave-particle resonance conditions for tokamaks with circular magnetic surfaces [2-4]. If $R_0 \to 0$, the cyclotron wave-particle resonant conditions for untrapped particles in tokamaks can be transformed to analogous conditions, Eq. (4), in the current-carrying plasma cylinder.

The $\ell\kappa$-corrections at ICR and ECR conditions for plasma systems in the helical magnetic field are very important analyzing the cyclotron wave dissipation/excitation at the rational magnetic surfaces, where $k_{||} \approx 0$. The terms proportional to *dq/dr* in $\ell\kappa$ – corrections allow us to study the influence of ohmic current density profiles on the cyclotron wave-particle interactions in the current-carrying plasma systems.

REFERENCES

1. A.I. Akhiezer, I.A. Akhiezer, R.V. Polovin, A.G. Sitenko, K.N. Stepanov // *Plasma Electrodynamics.* Oxford: “Pergamon”, 1975, v. I.
2. N.I. Grishanov, F.M. Nekrasov. Dielectric constant in toroidal plasmas // *Sov. J. Plasma Phys.* 1990, v. 16, p. 129-133.
3. N.I. Grishanov, C.A. de Azevedo, J.P. Neto. Dielectric characteristics of axisymmetric low aspect ratio tokamak plasmas // *Plasma Phys. Controlled Fusion*. 2001, v. 43, p. 1003-1022.
4. N.I. Grishanov, N.A. Azarenkov. Dispersion relations for field-aligned cyclotron waves in an axisymmetric tokamak plasma with anisotropic temperature // *Problems of Atomic Sci. and Techn. Series “Plasma Physics”.* 2011, № 1(17), p. 59-61.
5. N.I. Grishanov, G.O. Ludwig, C.A. de Azevedo, J.P. Neto. Wave dissipation by electron Landau damping in low aspect ratio tokamaks with elliptic magnetic surfaces // *Phys. Plasmas*. 2002, v. 9, p. 4089-4092.
6. N.I. Grishanov, N.A. Azarenkov. Cyclotron wave absorption in D-shaped tokamaks // *Problems of Atomic Sci. and Techn. Series “Plasma Physics”.* 2016, № 6(22), p. 52-55.

Article received 25.09.2018

УСЛОВИЯ ЦИКЛОТРОННЫХ РЕЗОНАНСОВ В ПЛАЗМЕ С ТОКОМ

Н.И. Гришанов, Н.А. Азаренков

Проанализированы условия резонансного взаимодействия заряженных частиц с волнами в плазменном цилиндре с током и в тороидальных моделях плазмы для токамаков с круговым, эллиптическим и D-образным сечениями магнитных поверхностей. Соответствующие резонансные условия получены путем решения линеаризованных уравнений Власова для возмущенных функций распределения частиц с учетом геометрии удерживающего магнитного поля в нулевом приближении по параметрам замагниченности. Показано, что доплеровская сдвижка в условиях циклотронных резонансов в токопроводящей плазме существенно отличается от аналогичных оценок в плазме с однородным магнитным полем.

УМОВИ ЦИКЛОТРОННИХ РЕЗОНАНСІВ У ПЛАЗМІ ЗІ СТРУМОМ

М.І. Гришанов, М.О. Азарєнков

Проаналізовано умови резонансної взаємодії заряджених частинок з хвилями в плазмовому циліндрі зі струмом та в тороідальних аксіально-симетричних моделях плазми для токамаків з круговим, еліптичним і D-подібним перерізами магнітних поверхонь. Відповідні резонансні умови отримано шляхом розв'язку лінеаризованих рівнянь Власова для збурених функцій розподілу плазмових частинок з урахуванням геометрії утримуючого магнітного поля в нульовому наближенні за параметрами замагніченості. Доведено, що доплерівський зсув в умовах циклотронних резонансів у плазмі зі струмом істотно відрізняється від аналогічних оцінок для плазми в однорідному магнітному полі.