THE CONSEQUENCES OF THE MODULATION INSTABILITIES

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The paper demonstrates the consequences of modulation instability of intense periodic structures in wave and non-wave media. In the case of a large dissipation level, near and above the threshold, the instability leads to the excitation of spectra whose width narrows, forming narrow spectral lines and self-similar structure of the big spatial clearness. At an insignificant level of dissipation, far from the threshold of modulation instability, the wave motion (initiated by the source) forms anomalous amplitude waves and envelopes exceeding the average amplitude by at three times. The shape of the envelope or wave packet is similar to the shape of Peregrine breather, and the dynamics over time is also similar. The formation of self-similar spatial structures in the developed convection of a thin liquid or gas layer due to the development of modulation instability is presented. In this case, toroidal convection vortices generate poloidal vortices of large scale – the effect of a hydrodynamic dynamo. Experimental results of the investigation of emerging self-similar structures on the graphite surface are presented. The features of the development of parametric instabilities are discussed.

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INTRODUCTION

There are various cases of the development of the modulation instability of intense periodic structures in wave and non-wave media (see, for example [1]). The peculiarity of the modulation instability is the appearance of the perturbation spectrum, which is practically symmetric with respect to large amplitude wave vector [2 - 6]. The modes of the perturbation spectrum are improper for a given medium, as a rule. The cases of different dissipation levels of large amplitude wave, in the presence of a source that supports it existence, are considered.

In the case of a large dissipation level, near and above the threshold, the instability leads to the excitation of spectra whose width narrows, forming narrow spectral lines [7]. The line spectrum creates the conditions for the development of a more large-scale modulation [8]. Thus, the modulation instabilities near the threshold represent a cascade of processes with an increasing characteristic time of development and a larger characteristic scale [9, 10]. The perturbation spectrum in the developed regime turns out to be practically linear. Forming thus self-similar structure of the big spatial clearness.

At an insignificant level of dissipation, far from the threshold of modulation instability, the wave motion (initiated by the source) forms anomalous amplitude waves and envelopes exceeding the average amplitude by at three times [7]. The asymmetry of the modulation instability spectrum can lead to the formation of wave packets of anomalous amplitude with different steepness of the leading and trailing fronts. The shape of the envelope or wave packet is similar to the shape of Peregrine breather [11], and the dynamics over time is also similar. Breathers are autowaves in conservative wave systems. In the nonequilibrium medium of the real ocean the formation of a wave packet (similar to the breather) occurs due to the interference caused by pumping. One can observe the interference (analytically [7], numerically [7, 12-13] and experimentally [12, 13]) of standing waves of different lengths arising in this neighborhood, the velocity of which is greater or less than the velocity of the main wave motion. It is shown that the formation of a given autowave-breather is the result of the development of a modulation instability in a nonequilibrium medium in the presence of wave motion of large amplitude.

The development of convection in a thin liquid or gas layer with a temperature gradient with poorly conductive heat boundaries is considered. Similar conditions are realized in thin clouds. Near the threshold of convective instability, a field of spatially homogeneous convective cells is formed [14 - 16], which turns out to be unstable [17]. It is the line spectrum of convective toroidal vortex cells that creates the conditions for the development of large-scale modulation instability. The formation of self-similar spatial structures in the developed convection of a thin liquid or gas layer due to the development of modulation instability presents in [17-19]. In this case, toroidal convection vortices generate poloidal vortices of large scale – the effect of a hydrodynamic dynamo.

Experimental results of the investigation of emerging self-similar structures on the graphite surface present in [20]. Important in the experimental data obtained is not so much the fact of the presence of a vertical component of the modulation, but rather an undeniable similarity of the primary structure – the unit cell and the secondary structure – of the modulation of the electron density surface. The nature of the modulation instability is discussed, which leads to the formation of similar self-similar surface structures [1].

The features of the development of parametric instabilities under the influence of fields homogeneous in space are considered. The nature of self-consistent parametric instabilities is similar to the processes of modulation instability. The spectra of parametric instabilities are symmetric in the space of wave numbers, respectively, in the positive and negative regions. As a result of the development of parametric instabilities, the parameters of the medium are modulated. This gives grounds for considering parametric instabilities to be similar to modulation instabilities and vice versa [21, 22].

1. SELF-SIMILAR STRUCTURES

With increasing wave energy in media with cubic nonlinearity, the frequency dependence of its amplitude appears, that is, the wave spectrum is broadened. This process of spectral broadening a monochromatic wave is a modulation instability. Consider the case of a balanced source and drain (absorption or dissipation) of the wave energy. The Lighthill equation [2], describing the slow evolution of the envelope of oscillations under these conditions, takes the form

$$\frac{\partial A}{\partial t} = -\delta A - i\frac{\partial^2 A}{\partial x^2} - iA|A|^2 + g, \qquad (1)$$

where δ – is the absorption decrement and g – is the external source of wave energy. Assuming the time and the coordinates dependence in the form $\exp\{-i\Omega\tau + iK\zeta\}$, we represent the dispersion equation of the process

$$D(\Omega, K) = (\Omega + i\delta + K^2 - |u_0|^2)(\Omega + i\delta - K^2 + |u_0|^2) + |A_0|^4 = 0,$$

from which we obtain that the absolute instability in the reference frame, which moves with the group velocity of the wave relative to the laboratory one, has an increment equal to

$$\mathrm{Im}\,\Omega = -\delta + \sqrt{K^4 - 2K^2|u_0|^2}.$$

With a maximum growth rate

$$(\mathrm{Im}\,\Omega)_{MAX} = -\delta + |u_0|^2,$$

perturbations whose wave number $K^2 = K_0^2 = |u_0|^2 = 1$, grows. The width of the spectrum determines the localization of this modulation. The value $(K_0 - \delta) = 2\pi / L$ corresponds to the localization area of the modulation L. The position of the maximum of the increment determines the average spatial period T of modulation, that is $K_0 = 2\pi / T$. It was shown in [8, 9] that near the threshold, a cascade of modulation instabilities forming self-similar structures occurs due to the narrowing of the spectra of each such process and the creation of conditions for the development of a new, larger scale (Figs. 1 and 2).

In addition, the narrow spectra of each cascade instability form a self-similar spatial structure, that is clearly observable on each scale.

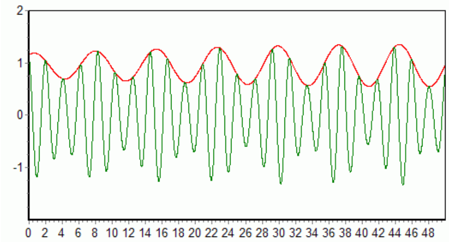

Fig. 1. Formation of self-similar field structures in a numerical experiment [7]: k_0=3 – the wave number of the main wave, K_{OPT}=0.8 – the wave number of the envelope of the first order, ΔK = 0.05 – the wave number of the second-order envelope

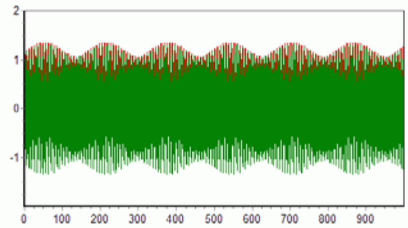

Fig. 2. Formation of self-similar field structures in a numerical experiment [7]: K_{OPT}=0.8 – the wave number of the envelope of the first order, $\Delta K = 0.05$ – the wave number of the second-order envelop

2. WAVES OF ANOMALOUS AMPLITUDE

For small values of dissipation, the appearance of large amplitude envelopes is possible. The most interesting case is the case of gravitational surface waves in deep water, for which the following expression for the frequency of waves of large amplitude is valid

$$\omega = \sqrt{g \cdot k} \cdot \{1 + |A|^2 k^2 / 2\}.$$

Anomalously high waves are considered to be waves whose height is more than twice the significant height of the waves. The significant wave height is calculated for a given period in a given region. For this, one third of all recorded waves having the greatest height is selected and their average height is found. Most modern vessels can withstand up to 15 tons per square meter and in case of even strong waves this corresponds to more than twice the safety margin, however, anomalously large waves can cause pressure up to hundreds of tons per square meter.

In paper [7], we compare the results (Fig. 3) of calculations for two approaches: S-theory and direct calculation of equation (1).

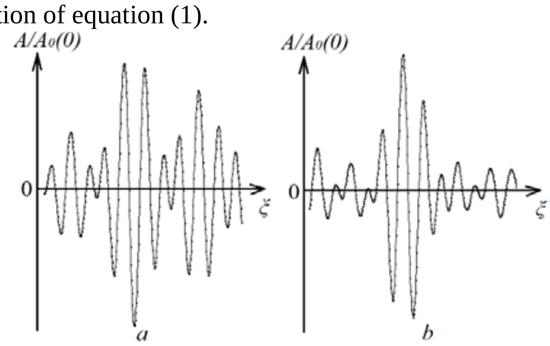

Fig. 3. A characteristic type of anomalous waves in the composition of wave groups in the case of the S-theory (a) and in the case of a direct solution of equation (1) (b) [7]

However, a single short-lived breather, the Peregrin soliton [11] (Fig. 4), is similar to the solution considered in Fig. 3, describing a single wave of anomalous amplitude.

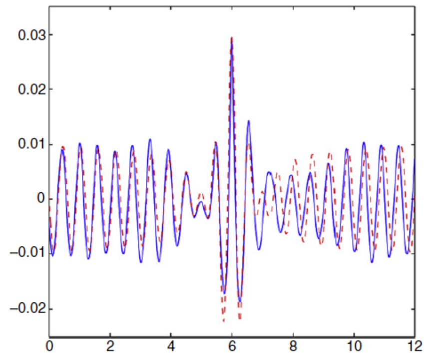

Fig. 4. Comparison of the experimentally observed wave of anomalous amplitude (solid curve) with the envelope filling, which is the Peregrina soliton (dotted line) [12]. The ordinate is the amplitude in meters, the abscissa is the time in seconds

Such a perturbation arises in the field of wave motion and then disappears, which corresponds to the appearance and disappearance of a short-lived anomalous wave, due to the interference of a stack of standing waves of the modulation instability spectrum moving at different speeds under the action of the fundamental wave.

It can be shown, that according to the S-theory, the wave packet of the spectrum can be represented in the form [7]

$$a - u_0 \exp[i\pi/4] \propto 2\sum_{m>0}^{N} u_m \cdot \mathrm{Cos}\{-K_m\xi + 2(K_n^2 - u_0^2)t + \alpha_n\}$$

where $\alpha_n = (\frac{\Phi_n^* - \pi/2}{2})$. It is a set of standing waves. The phases of longer perturbations are located in the negative region and move in the positive direction, and the phases of the shorter ones are in the positive region and move in the negative direction. That is, longer standing waves move towards shorter ones, and with decreasing amplitude of the main wave, energy is more concentrated in the long-wave part of the envelope spectrum. The more different is the length of the standing wave formed by a pair of modes from the length of the perturbation growing with the maximum increment, the greater the rate of change of its phase. The interference of these standing waves, imposed by the main wave, is forced [1] and is accelerated with a change in the amplitude of the main wave. The interference process of a set of standing waves forming an anomalous envelope can be seen (Fig. 5 in [12]).

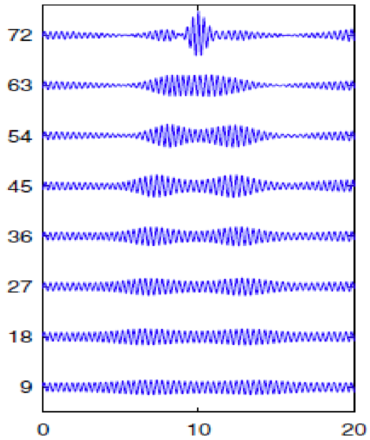

Fig. 5. Evolution of the wave profile of the anomalous amplitude in the experiment [13]. On the ordinate axis, estimate the distance (in meters), along the abscissa, the time of each segment (in seconds)

It is important to note that in the one-dimensional case under discussion the maximum amplitude of the anomalous wave (breather) is three times higher than the average wave amplitude.

3. HYDRODYNAMIC DYNAMO EFFECT

For the first time, the possibility of the occurrence of a modulation instability of a convective cell system in the extremely productive Proctor-Sivashinsky-Pis'men [16] model was stated in the report [14, 15]. This modulation of a system of developed convective cells in a thin layer of fluid between poorly conducting heat by horizontal surfaces is caused by the generation of vortices of a completely different nature than those that form a convective structure.

$$\dot{\Phi} = \varepsilon^2 \Phi - (1-\nabla^2)^2 \Phi + \frac{1}{3} \nabla \left(\nabla \Phi |\Phi|^2 \right) + \gamma_{\Pr} \nabla \Phi \times \nabla \Psi, \quad (2)$$

$$\nabla^2 \Psi = \nabla \nabla^2 \Phi \times \nabla \Phi, \quad (3)$$

The Proctor-Sivashinsky-Pismen model [16] is the result of the modification [14, 15], describes a convection, but taking into account the toroidal velocity, where $\gamma_{\Pr}$ – is the inverse value of the Prandl number $\Pr = \nu/\kappa$, which characterizes the nonequilibrium state of the liquid, ν – is the kinematic viscosity, κ – here is the specific thermal diffusivity, $\varepsilon \ll 1$ in this case. As a result of the primary instability, which is accompanied by a number of structural phase transitions, is the field of convective cells-toroidal vortices. A small excess of the threshold of the instabilities determines the high spatial clarity of the vortex structures, which formed line spectra that provided the development of subsequent cascades of processes.

As a result of the secondary – modulation instability, large-scale structures (Figs. 6 and 7) consisting of convective vortices and poloidal vortices [17 - 19] are formed (similar vortices are considered in [23, 24]). The latter are of the greatest interest – this is the effect of a regular hydrodynamic dynamo, predicted by S.S. Moiseev.

4. STRUCTURE ON THE SURFACE OF GRAPHITE

Modulation of the surface of a solid can also be described within the framework of a modulation instability (Fig. 8) that develops in such a non-wave medium due to the potential energy of the stresses.

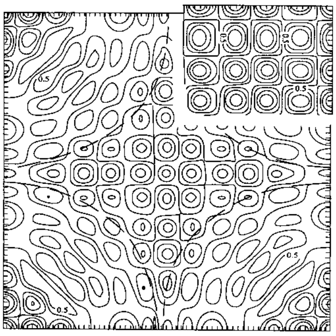

Fig. 6. It is a regular defect in the convective structure. In the upper corner is a fragment of the primary unperturbed structure. The dashed lines show the characteristic lines of the current of large-scale vortices [17 - 19]

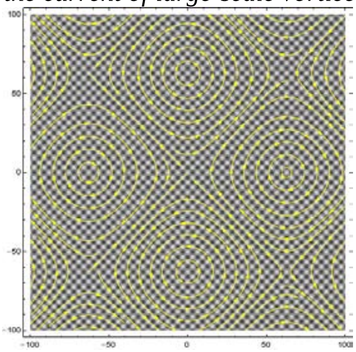

Fig. 7. In the background of a field of modulated convective cells, is the appearance of large poloidal vortices [17 - 19]

We consider a simple two-dimensional case, with the X axis directed along an inextensible layer, and the OY axis defined perpendicular to the boundary of the sample. Then the wave number of the corrugation inside the graphite sample can be written as $k_0 = k_{00} + a_0^2 k_0^3 / 4$, where the ratio between the spatial period λ and the wave number of the periodic structure $k = 2\pi/\lambda$, , k_{00} – is the wave number in the absence of corrugation, a_0 – is the amplitude of the corrugation, and the expression given above is valid for $(k_0 a_0)^2 < 1$. For a perturbed system, we can write equation

$$(k - i\frac{\partial}{\partial y})\cdot a = k_{00} + \frac{k^3 |a|^2}{4} a. \qquad (4)$$

Let the corrugation perturbations have wave numbers $k_{\pm} = k_0 \pm K$ and amplitudes $a_{\pm}$, then for these perturbations we can write equation

$$\frac{\partial}{\partial y} a_{\pm} \pm iKa_{\pm} = i\frac{k_0^3 a_0^2}{4} a_{\mp}^*, \qquad (5)$$

from which it is not difficult to find a solution increasing to the surface

$$\sim \exp\{-iKy\}\cdot \exp\{k_0^3 a_0^2 y/4\},$$

where, because of the oscillating factor, the growth of the amplitudes of large-scale perturbations of the corrugation is limited. An approximate equality

$$(k_0 a_0)^2 \approx (ka)^2 + (a_{\pm} K)^2$$

is performed, where $(k_0 a_0)^2$ – is the value in the depth of the sample. On a surface $(ka)^2 \approx \alpha \cdot (a_{\pm} K)^2$.

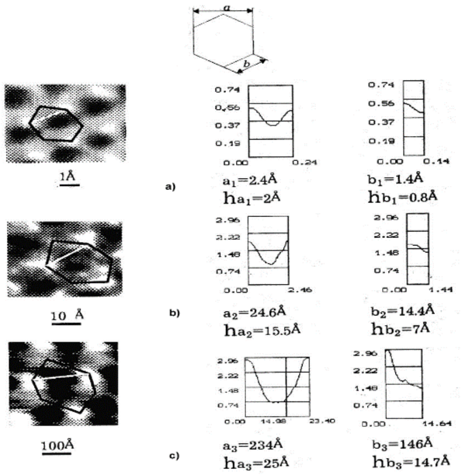

Fig. 8. Topographical images of highly oriented graphite sections at various magnifications, obtained by scanning tunneling microscopy [20]

5. PARAMETRIC OR MODULATION INSTABILITIES

In the process of development of instability of monochromatic intensive oscillations (excited by electron beams [25 - 27]), resonant perturbations get advantages, for which the relations $2\omega_0 = \omega_1 + \omega_2$ and $2k_0 = k_1 + k_2$ are satisfied. With increasing spectrum, the interaction of the perturbations with each other and with a wave of large amplitude is described by the following relations $2\omega_0 = \omega_1' + \omega_2' = \omega_1'' + \omega_2''$ and $2k_0 = k_1' + k_2' = k_1'' + k_2''$. It was used by the authors of [27, 28] to describe the excitation of the spin waves by an oscillatory magnetic field, homogeneous in space. This approach allowed the authors [28, 29] to construct a parametric instability of spin waves. Indeed, as in the theory of parametric instabilities described by the Mathieu and Hill equations, the multiplicative action of a variable parameter or noise is able to provide an exponential growth of the perturbation.

For this reason, in the book [3], V.P. Silin called the decay processes of intense Langmuir oscillations, which are homogeneous in space, as a parametric. A generalization of the Silin model was in papers whose detailed bibliography in the review [20]. At one time, with the representation of a model in a cold plasma, V.P. Silin [3], V.E. Zakharov, investigating the instability of a high-intensity Langmuir field in a nonisothermal plasma, discovered an extremely important and extremely efficient mechanism for the absorption of field energy by plasma particles [6]. Here, too, the intense field of Langmuir oscillations at the initial instant was homogeneous in space. Therefore, the model of V.E. Zakharov (Zakharov's equation) can also be attributed to parametric instability. On the other hand, instabilities in the models of V.E. Zakharov and V.P. Silin are often called modulation instabilities, since its spectrum is similar to the modulation instability spectrum with the only difference that the wave vector of intense oscillations is zero. The result of the instability is a strong modulation of the plasma density (Figs. 9 and 10) and heating of ions, not so much because of the Landau damping, but to a greater extent because of stochastic scattering by field in homogeneities (see example [30]).

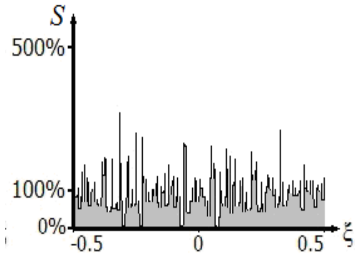

Fig. 9. The distribution of ions in space in the regime of developed instability in the model of Zakharov [21]

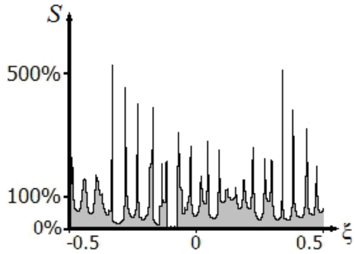

Fig. 10. Distribution of plasma ions in space in the regime of developed instability in the Silin model [21]

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ПОСЛЕДСТВИЯ МОДУЛЯЦИОННЫХ НЕУСТОЙЧИВОСТЕЙ

В.М. Куклин

Демонстрируются последствия модуляционной неустойчивости интенсивных периодических структур в волновых и неволновых средах. В случае большого уровня диссипации, вблизи и выше порога неустойчивость представляет собой каскад процессов с увеличивающимся временем развития и все большим характерным масштабом, формируя при этом самоподобную структуру большой пространственной четкости. При незначительном уровне диссипации, вдали от порога волновое движение формирует волны и огибающие аномальной амплитуды, в максимуме превышающие среднюю амплитуду волнения в три раза. Форма огибающей или волнового пакета подобна форме бризера Перегрина, причем динамика во времени так же подобна. Показано формирование самоподобных пространственных структур в развитой конвекции тонкого слоя жидкости или газа вследствие развития модуляционной неустойчивости. При этом, тороидальные вихри конвекции генерируют полоидальные вихри большого масштаба – эффект гидродинамического динамо. Представлены экспериментальные результаты исследования возникающих самоподобных структур на поверхности графита. Обсуждаются особенности развития параметрических неустойчивостей.

НАСЛІДКИ МОДУЛЯЦІЙНИХ НЕСТІЙКОСТЕЙ

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Демонструються наслідки модуляційної нестійкості інтенсивних періодичних структур у хвильових і не хвильових середовищах. У разі великого рівня дисипації, поблизу і вище порога формується каскад процесів зі зростаючим часом розвитку і все більшим характерним масштабом, формуючи при цьому самоподібну структуру великої просторової чіткості. При незначному рівні дисипації, далеко від порога хвильовий рух формує хвилі і огинаючі аномальної амплітуди, які в максимумі перевищують середню амплітуду в три рази. Форма огинаючої або хвильового пакету подібна формі брізера Перегріна, при чому динаміка в часі також подібна. Показано формування самоподібних просторових структур у розвиненій конвекції тонкого шару рідини або газу внаслідок розвитку модуляційної нестійкості. При цьому, тороїдальні вихори конвекції генерують полоїдальні вихори великого масштабу – ефект гідродинамічного динамо. Представлено експериментальні результати дослідження самоподібних структур на поверхні графіту. Обговорюються особливості розвитку параметричних нестійкостей.