

# DISPERSIVE DISTORTIONS OF THE FRACTAL ULTRA-WIDEBAND SIGNALS IN PLASMA MEDIA

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The results of numerical modeling of dispersive distortions of the model high-frequency fractal ultra-wideband (UWB) signals propagating in linear and parabolic plasma layers are considered. The character of the dispersive distortions appeared is described and the corresponding numerical characteristics are estimated. Special attention is paid to the comparison of the results with similar ones obtained for non-fractal ultra-short UWB signals.

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## INTRODUCTION

Being proposed by D.L. Moffatt [1] and E.M. Kennaugh [2], in the last thirty years, the ultra-wideband (UWB) signals have revolutionized many branches in science and technologies [3].

The main advantage of the UWB signals over the narrowband and wideband ones is that the first of them carry the volume of information, which is  $\mu / \mu_n \gg 1$  times more than the second of them are able to carry ( $\mu$  and  $\mu_n$  are the relative bandwidths for the UWB and narrowband signal respectively). The relative bandwidth of a signal is given by the relation  $\mu = 2(f_{\max} - f_{\min}) / (f_{\max} + f_{\min})$ , where  $f_{\min}$  and  $f_{\max}$  are the minimal and the maximal frequencies of spectral density function (SDF) of the one-dimensional Fourier transform (OFT) [4]. The relative bandwidth is  $0.2 \leq \mu < 2$  for the UWB signals, is  $0 < \mu \leq 0.01$  for the narrowband signals and is  $0.01 < \mu < 0.2$  for the wideband signals [4].

Solving the problems of the UWB signal application in radar, communication and remote radio sounding of different objects and media, it is necessary to estimate the dispersive distortions appeared in plasma media. In the late 1990s, the authors had shown that such distortions for the ultra-short UWB (USUWB) signals can be significant in many cases [4, 5] and, therefore, should be taken into account.

In the mid-2000s, the authors had proposed a new UWB signal class named as the fractal UWB (FUWB) signals [4, 6, 7]. Such signals unite the advantages of the UWB and the fractal signals and, for example, have very high noise immunity. Thus, the idea to estimate the dispersive distortions of the FUWB signals appeared during their propagation in plasma layers seems to be actual and useful.

The purpose of the paper is to calculate the set of numerical characteristics, which are able to describe the dispersive distortions of the model FUWB signals appeared during their propagation in the model linear and parabolic plasma layers, and to compare the results obtained with ones for USUWB model signals.

## 1. FUWB SIGNAL MODEL

As long as the paper volume is very limited, only one FUWB signal model was chosen. There are no limitations to perform the calculations listed below for other FUWB signal models, for example, for those proposed in [8]. In this paper, we consider the FUWB signal model based on the Weierstrass function [4, 7]:

$$s(t) = \left[ 1 - b^{2D-4} \right] \times \frac{\sum_{n=0}^M b^{(D-2)n} \cos(2\pi s b^n t + \psi_n)}{1 - b^{(2D-4)(M+1)},}$$

where  $t$  is the dimensionless time, normalized on the signal duration,  $b$  is the time scale parameter,  $s$  is the frequency scale parameter,  $D$  is the fractal dimension of the signal,  $1 < D < 2$ ,  $\psi_n$  is the phase distributed randomly at the interval  $[0, 2\pi]$ ,  $M$  is the harmonics number (if  $M \rightarrow \infty$ , a mathematical fractal is obtained). The main advantage of this model is the possibility to build the signals with different fractal dimension values. At the Fig. 1 the model FUWB signal with  $D = 1.5$  and model USUWB signal are shown.

The fractal dimension value  $D$  for the model FUWB signal travelling in the plasma medium will be estimated with the Hurst exponent ( $H$ ) calculation technique usage. In bounds of the generalized Brownian motion model, they are connected with  $D = 2 - H$  [9].

## 2. DISPERSIVE DISTORSION MODELING

In this paper, only the dispersive distortions appeared due to phase velocity dispersion existence are considered. Moreover, only the high-frequency (HF) FUWB signals are used. A UWB signal is called as a HF UWB signal, if  $f_{\min} \gg f_p$ , where  $f_p$  is the plasma frequency [4, 5].

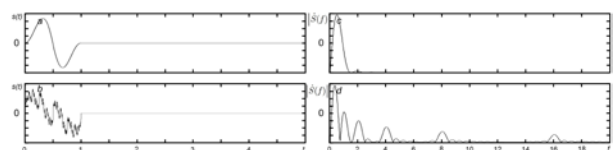


Fig. 1. Model UWB (a) and FUWB (b) signals and their OFT SDF modules (c and d) correspondently

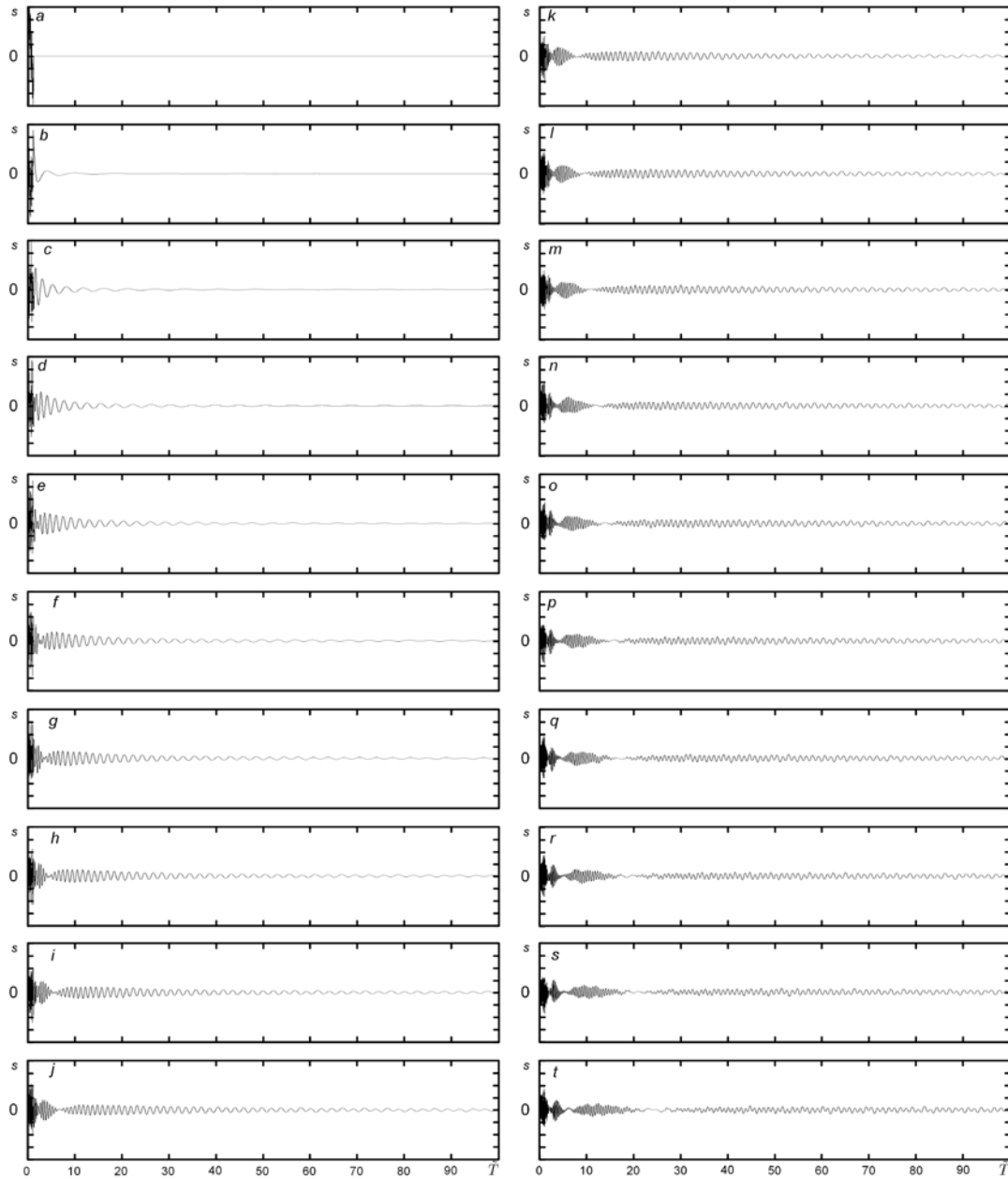


Fig. 2. Dispersive distortions of the FUWB signal ( $D = 1.5$ ,  $f_0 = 10^9$  Hz,  $\mu = 1.84$ ) appeared in the parabolic plasma layer with  $f_p(z_{\max}) = 10^7$  Hz,  $z_{\max} = 200$  km for different distances: a –  $z = 0$  km; b –  $z = 10$  km; c –  $z = 20$  km; d –  $z = 30$  km; e –  $z = 40$  km; f –  $z = 50$  km; g –  $z = 60$  km; h –  $z = 70$  km; i –  $z = 80$  km; j –  $z = 90$  km; k –  $z = 100$  km; l –  $z = 110$  km; m –  $z = 120$  km; n –  $z = 130$  km; o –  $z = 140$  km; p –  $z = 150$  km; q –  $z = 160$  km; r –  $z = 170$  km; s –  $z = 180$  km; t –  $z = 200$  km

In such case, the amplitude of the electric field in a semi-infinite ( $z \geq 0$ ), isotropic plasma medium at the distance  $z$  from the bound is given by the relation

$$E(t, z) = \int_{-\infty}^{+\infty} \dot{S}(f) \dot{K}(f, z) \exp(i2\pi ft) df,$$

where the OFT SDF of the signal  $s(t) = E(t, 0)$  is given as

$$\dot{S}(f) = \int_{-\infty}^{+\infty} s(t) \exp(-i2\pi ft) dt.$$

The function  $\dot{K}(f, z)$  given by the relation

$$\dot{K}(f, z) = K(f, z) \exp(-i\phi(f, z))$$

describes the effect of the plasma media, in which a signal  $s(t)$  propagates. As far as the absorption disper-

sion effects are not considered in this paper, we have  $K(f, z) = 1$ . The phase is given as

$$\phi(f, z) = 2\pi \frac{f}{c} \int_0^z n(f, z') dz'.$$

For the HF UWB signals, the dispersion law given by the relation

$$n^2(f, z) = 1 - \frac{f_p^2(z)}{f^2}$$

is applied. We use two plasma layer models, namely, the linear layer model given by the relation

$$f_p^2(z) = f_p^2(z_{\max}) \frac{z - z_{\min}}{z_{\max} - z_{\min}}$$

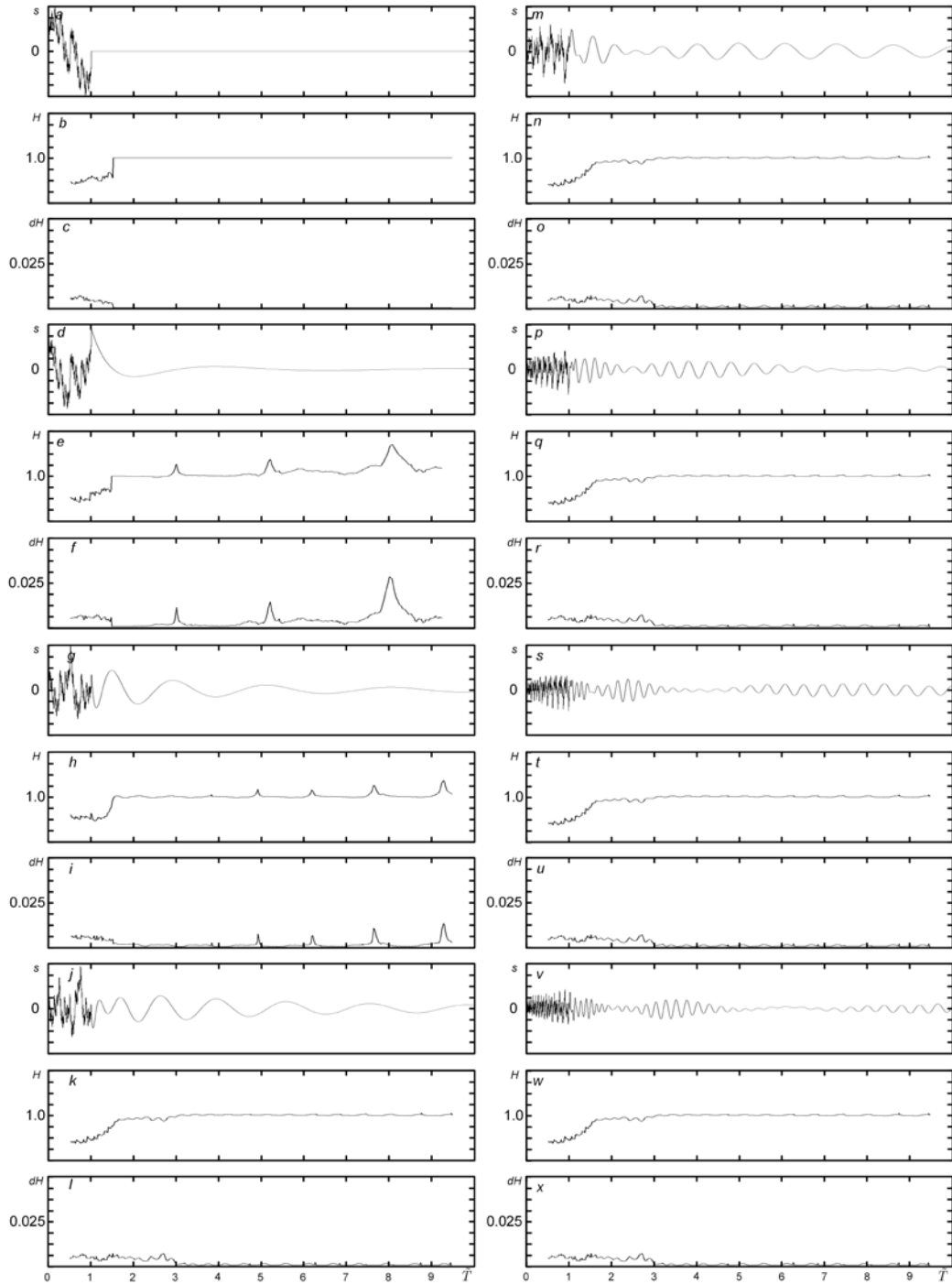


Fig. 3. Dispersive distortions of the FUWB signal ( $D = 1.5$ ,  $f_0 = 10^9$  Hz,  $\mu = 1.84$ ) appeared in the parabolic plasma layer with  $f_p(z_{\max}) = 10^7$  Hz,  $z_{\max} = 200$  km, dynamical Hurst exponent  $H(t)$  and its error  $dH(t)$  for different distances: a, b, c –  $z = 0$  km; d, e, f –  $z = 10$  km; g, h, i –  $z = 20$  km; j, k, l –  $z = 30$  km; m, n, o –  $z = 50$  km; p, q, r –  $z = 100$  km; s, t, u –  $z = 150$  km; v, w, x –  $z = 200$  km

and parabolic layer model given by the relation

$$f_p^2(z) = f_p^2(z_{\max}) \left[ 1 - \left( \frac{z_{\max} - z}{z_{\max} - z_{\min}} \right)^2 \right],$$

where  $z \in [z_{\min}, z_{\max}]$ .

If we investigate the UWB signal propagation in the Earth's ionosphere, we should use the following parameters:  $z_{\min} = 100$  km,  $z_{\max} = 300$  km,  $f_p(z_{\max}) = 10$  MHz for the daytime ionosphere and  $f_p(z_{\max}) = \sqrt{10}$  MHz for the night ionosphere.

### 3. ANALYSIS RESULTS

For non-fractal HF USUWB signals [6 - 8], the dispersive distortions appeared during their propagation in plasma media with phase dispersion only contain the delays of the signal leading edge  $\tilde{T}_f$  ( $\tilde{T} = t / \tau_{s0}$  is a dimensionless time) and envelope maximum of a signal  $\tilde{T}_m$  (when such envelope has been formed), the increasing of the relative signal duration  $\tau_s / \tau_{s0}$  ( $\tau_s$  and  $\tau_{s0}$  are current and initial signal durations) and the decreasing of the relative amplitude of the signal envelope

$E_{\max} / E_{\max 0}$  ( $E_{\max}$  and  $E_{\max 0}$  are current and initial signal envelope amplitudes). The values of the effects appeared depend on the traveling distance  $z$ , the plasma layer model and the relations between  $m$ ,  $f_0 = (f_{\min} + f_{\max}) / 2$  and  $f_p$ . Such dispersive distortions called as ‘traditional’ ones can be very significant. For example, for daytime ionosphere described above for  $f_0 = 10^9$  Hz and  $\mu = 0.2 \dots 1.6$  we have  $\tau_s / \tau_{s0} = 1.4 \dots 6.8$ ,  $\check{T}_f = 3.4 \dots 6.6$ ,  $\check{T}_m = 3.8 \dots 16.8$ ,  $E_{\max} / E_{\max 0} = 0.75 \dots 0.21$  and for  $f_0 = 10^{10}$  Hz and  $\mu = 0.2 \dots 1.6$  we have  $\tau_s / \tau_{s0} = 1.0 \dots 1.6$ ,  $\check{T}_f = 0.0 \dots 1.0$ ,  $\check{T}_m = 0.3 \dots 2.0$ ,  $E_{\max} / E_{\max 0} = 1.00 \dots 0.55$ .

For FUWB signals, the character of dispersive distortions has some differences. As far as their OFT SDF is appeared to be more complex than ones for USUWB signals (see Fig. 1,c), and most importantly, to be fractal (see Fig. 1,d), the new peculiarities of the dispersive distortions occur. At the Fig. 2 they are clearly shown at the sample of the model FUWB signal described on the Fig. 1,b. First, there are all dispersive distortions appeared for USUWB signal. But being formed by high-frequency part of the OFT SDF, the main fractal structure of the FUWB signal is appeared to be almost not changed. Moreover, as it is shown more clearly at the Fig. 3, remaining fractal ( $0 < H < 1$ ), it has the same place in the signal and its Hurst exponent  $H$  is slightly decreasing, when the traveling distance  $z$  increases. Other parts of the signal are appeared to be non-fractal ( $H \geq 1$ ). Being estimated in the sliding window with the width  $\Delta \check{T} = 1$ , the Hurst exponent becomes a dependence on time  $H(t)$  and can be called as dynamical one. Second, the distorted signal is appeared to be amplitude modulated in time domain. This explains by the fractal structure of the OFT SDF for FUWB signal. Third, for FUWB signal (see Fig. 1,b) the ‘traditional’ dispersive distortions described above are appeared to be more significant than for corresponding USUWB signal (see Fig. 1,a). This can be explained by the fact that having the same  $f_0$ , the FUWB signal has a bigger relative bandwidth  $\mu$  ( $\mu_{UWB} = 1.57$ ,  $\mu_{FUWB} = 1.84$ ).

## CONCLUSIONS

Main fractal structure of the HF FUWB signal traveled in plasma media with phase dispersion only is appeared to be almost not changed.

Due to fractal OFT SDF, the distorted HF FUWB signal is appeared to be amplitude modulated.

As far as having the same mean frequency, the FUWB signals have bigger relative bandwidth, than the same USUWB signals have, the ‘traditional’ dispersive distortions of the HF FUWB signals in plasma media are appeared to be more significant.

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## ДИСПЕРСИОННЫЕ ИСКАЖЕНИЯ ФРАКТАЛЬНЫХ СВЕРХШИРОКОПОЛОСНЫХ СИГНАЛОВ В ПЛАЗМЕННЫХ СРЕДАХ

Л.Ф. Черногор, О.В. Лазоренко, А.А. Онищенко

Рассматриваются результаты численного моделирования дисперсионных искажений модельных высокочастотных фрактальных сверхширокополосных (СШП) сигналов, распространяющихся в линейном и параболическом плазменных слоях. Описывается характер возникающих дисперсионных искажений и оцениваются соответствующие числовые характеристики. Особое внимание уделяется сравнению данных результатов с полученными ранее аналогичными результатами для нефрактальных высокочастотных ультракоротких СШП-сигналов.

## ДИСПЕРСІЙНІ ВИКРИВЛЕННЯ ФРАКТАЛЬНИХ НАДШИРОКОСМУГОВИХ СИГНАЛІВ У ПЛАЗМОВИХ СЕРЕДОВИЩАХ

Л.Ф. Черногор, О.В. Лазоренко, А.А. Онищенко

Розглядаються результати числового моделювання дисперсійних викривлень модельних високо частотних фрактальних надшироко смугових (НШС) сигналів, що поширюються в лінійному та параболическому плазмових шарах. Описується характер дисперсійних спотворень, що виникають, оцінюються відповідні числові характеристики. Особлива увага приділяється порівнянню даних результатів з отриманими раніше аналогічними результатами для нефрактальних високо частотних ультракоротких НШС-сигналів.