Український математичний вісник
Том 13 **(2016)**, № 3, 376 – 387

On the recursive sequence $x_{n+1} = \frac{x_{n-(4k+3)}}{1+\prod_{t=0}^{2} x_{n-(k+1)t-k}}$

Dağistan Simsek and Fahreddin Abdullayev

Abstract. In this paper a solution of the following difference equation was investigated

$$x_{n+1} = \frac{x_{n-(4k+3)}}{1+\prod_{t=0}^{2} x_{n-(k+1)t-k}}, \; n = 0, 1, 2, \ldots$$

where $x_{-(4k+3)}, x_{-(4k+2)}, \ldots, x_{-1}, x_0 \in (0, \infty)$ and $k = 0, 1, \ldots$.

2010 MSC. 39A10, 39A14, 39A20.

Key words and phrases. Difference equation, period 4k+4 solution.

1. Introduction

Difference equations appear naturally as discrete analogues and as numerical solutions of differential and delay differential equations having applications in biology, ecology, physics, etc [26].

Recently there has been a lot of interest in studying the periodic nature of nonlinear difference equations. For some recent result concerning among other problems, the periodic nature of scalar nonlinear difference equations see, for examples [1–26].

Cinar, studied the following problems with positive initial values

$$x_{n+1} = \frac{x_{n-1}}{1 + a x_n x_{n-1}}$$

$$x_{n+1} = \frac{x_{n-1}}{-1 + a x_n x_{n-1}}$$

$$x_{n+1} = \frac{a x_{n-1}}{1 + b x_n x_{n-1}}$$

Received 28.08.2016

ISSN 1810 – 3200.

for $n = 0, 1, 2, \ldots$, in [2, 3, 4], respectively.

In [18] Stevic assumed that $\beta = 1$ and solved the following problem

$$x_{n+1} = \frac{x_{n-1}}{1 + x_n} \text{ for } n = 0, 1, 2, \ldots$$

where $x_{-1}, x_0 \in (0, \infty)$. Also, this result was generalized to the equation of the following form:

$$x_{n+1} = \frac{x_{n-1}}{g(x_n)} \text{ for } n = 0, 1, 2, \ldots$$

where $x_{-1}, x_0 \in (0, \infty)$.

In [19, 20, 21] studied the following problems with positive initial values:

$$x_{n+1} = \frac{x_{n-3}}{1 + x_{n-1}},$$

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}},$$

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-1}x_{n-3}}$$

for $n = 0, 1, 2, \ldots$, respectively.

In this paper we investigate the following nonlinear difference equation

$$x_{n+1} = \frac{x_{n-(4k+3)}}{1 + \prod_{t=0}^{2} x_{n-(k+1)t-k}} \text{ for } n = 0, 1, 2, \ldots \tag{1}$$

where $x_{-(4k+3)},\ x_{-(4k+2)}, \ldots,\ x_{-1},\ x_0$ and $k = 0, 1, \ldots$.

2. Main result

Theorem 2.1. *Consider the difference equation* (1). *Then the following statements are true.*

a) *The sequences* $(x_{(4k+4)n-(4k+3)}), (x_{(4k+4)n-(4k+2)}), \ldots, (x_{(4k+4)n})$ *are decreasing and there exists* $a_1, a_2, \ldots, a_{4k+4} \geqslant 0$ *such that*

$$\lim_{n\to\infty} x_{(4k+4)n-(4k+3)+t} = a_{1+t}, \qquad t = 0, 1, \ldots, (4k+3).$$

b) $(a_1, a_2, \ldots, a_{4k+4}, a_1, a_2, \ldots, a_{4k+4}, \ldots)$ *is a solution of equation* (1) *of period 4k+4.*

c) $\prod_{t=0}^{3} \lim_{n\to\infty} x_{(4k+4)n-(k+1)t-k} = 0, \ldots, \prod_{t=0}^{3} \lim_{n\to\infty} x_{(4k+4)n-(k+1)t} = 0$ *or*

$\prod_{t=0}^{3} a_{(k+1)t+1} = 0, \ldots, \prod_{t=1}^{4} a_{(k+1)t} = 0.$

d) *If there exists* $n_0 \in N$ *such that*

$$\prod_{t=0}^{2} x_{n-(k+1)t-k} \geq x_{n+1} \prod_{t=0}^{1} x_{n-(k+1)t-k}$$

for all $n \geqslant n_0$, *then*

$$\lim_{n\to\infty} x_n = 0.$$

e) *The following formulas*

$$x_{(4k+4)n+r(k+1)+s+1} = x_{(r-4)(k+1)+s+1} \left(1 - \frac{\left(\prod_{m=1}^{4} x_{-(mk+m-1)+s}\right) / x_{(r-4)(k+1)+s+1}}{1 + (\prod_{m=1}^{3} x_{-(mk+m-1)+s})} \times \sum_{j=0}^{n} \prod_{i=1}^{4j+r} \frac{1}{1 + \prod_{m=1}^{3} x_{(k+1)i-(mk+m-1)+s}} \right),$$

where $r = 0, 1, 2, 3$ *and* $s = 0, 1, \ldots, k$ *hold.*

f) *If* $x_{(4k+4)n+(t-1)k+t} \to a_t \neq 0$, $(t = 1, 2, 3)$ *then* $x_{(4k+4)n+3k+4} \to 0$ *as* $n \to \infty, \ldots$. *If* $x_{(4k+4)n+tk+t} \to a_{tk+t} \neq 0$, $(t = 1, 2, 3)$ *then* $x_{(4k+4)n+4k+4} \to 0$ *as* $n \to \infty$.

Proof. **a)** Firstly, from the equation (1), we obtain

$$x_{n+1}(1 + x_{n-k}x_{n-(2k+1)}x_{n-(3k+2)}) = x_{n-(4k+3)}.$$

If $x_{n-k}, x_{n-(2k+1)}, x_{n-(3k+2)} \in (0, +\infty)$, then

$$(1 + x_{n-k}x_{n-(2k+1)}x_{n-(3k+2)}) \in (1, +\infty).$$

Since $x_{n+1} < x_{n-(4k+3)}$, $n \in N$, we obtain that there exist

$$\lim_{n\to\infty} x_{(4k+4)n-(4k+3)+t} = a_{1+t}, t = 0, 1, \ldots, (4k+3).$$

b) $(a_1, a_2, \ldots, a_{4k+4}, a_1, a_2, \ldots, a_{4k+4}, \ldots)$ is a solution of equation (1) of period $4k + 4$.

c) In view of the equation (1), we obtain

$$x_{(4k+4)n+1} = \frac{x_{(4k+4)n-(4k+3)}}{1 + \prod_{t=0}^{2} x_{(4k+4)n-(k+1)t-k}}.$$

Taking limits as $n \to \infty$ on both sides of the above equality, we get

$$\lim_{n\to\infty} x_{(4k+4)n+1} = \lim_{n\to\infty} \frac{x_{(4k+4)n-(4k+3)}}{1 + \prod_{t=0}^{2} x_{(4k+4)n-(k+1)t-k}}.$$

Then

$$\prod_{t=0}^{3} \lim_{n\to\infty} x_{(4k+4)n-(k+1)t-k} = 0 \text{ or} \prod_{t=0}^{3} a_{(k+1)t+1} = 0.$$

Similarly,

$$\prod_{t=0}^{3} \lim_{n\to\infty} x_{(4k+4)n-(k+1)t} = 0 \text{ or} \prod_{t=1}^{4} a_{(k+1)t} = 0.$$

d) If there exists $n_0 \in N$ such that $x_{n-k}x_{n-(2k+1)}x_{n-(3k+2)} \geq x_{n+1}x_{n-k}x_{n-(2k+1)}$ for all $n \geqslant n_0$, then $a_1 \leq a_{k+2} \leq a_{2k+3} \leq a_{3k+4} \leq a_1, \ldots, a_{k+1} \leq a_{2k+2} \leq a_{3k+3} \leq a_{4k+4} \leq a_{k+1}$. Since

$$\prod_{t=0}^{3} a_{(k+1)t+1} = 0, \ldots, \prod_{t=1}^{4} a_{(k+1)t} = 0,$$

we obtain the result.

e) Subracting $x_{n-(4k+3)}$ from the left and right-hand sides in equation (1) we obtain

$$x_{n+1} - x_{n-(4k+3)} = \frac{1}{1 + x_{n-k}x_{n-(2k+1)}x_{n-(3k+2)}} \left(x_{n-k} - x_{n-(5k+4)}\right),$$

and the following formula

$$\text{for } n \geq k+1 \begin{cases} x_{(k+1)n-((k+1)^2-1)} - x_{(k+1)n-[(k+2)^2+2k]} \\ = \left(x_1 - x_{-(4k+3)}\right) \prod_{i=1}^{n-(k+1)} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}} \\ \vdots \\ x_{(k+1)n-((k+1)^2-(k+1))} - x_{(k+1)n-[(k+2)^2+k]} \\ = \left(x_{k+1} - x_{-(3k+3)}\right) \prod_{i=1}^{n-(k+1)} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}} \end{cases} \quad (2)$$

holds. Replacing n by $4j$ in (2) and summing from $j=0$ to $j=n$ we obtain

$$x_{(4k+4)n+1}-x_{-(4k+3)}=(x_1-x_{-(4k+3)})\sum_{j=0}^{n}\prod_{i=1}^{4j}\frac{1}{1+\prod_{t=0}^{2}x_{(k+1)i-(k+1)t-k}} \tag{3}$$

.
.
.

$$x_{(4k+4)n+k+1}-x_{-(3k+3)} = (x_{k+1}-x_{-(3k+3)})\sum_{j=0}^{n}\prod_{i=1}^{4j}\frac{1}{1+\prod_{t=0}^{2}x_{(k+1)i-(k+1)t}}.$$

Also, replacing n by $4j+1$ in (2) and summing from $j=0$ to $j=n$ we obtain

$$x_{(4k+4)n+k+2}-x_{-(3k+2)}=(x_1-x_{-(4k+3)})\sum_{j=0}^{n}\prod_{i=1}^{4j+1}\frac{1}{1+\prod_{t=0}^{2}x_{(k+1)i-(k+1)t-k}}, \tag{4}$$

.
.
.

$$x_{(4k+4)n+2k+2}-x_{-(2k+2)}=(x_{k+1}-x_{-(3k+3)})\sum_{j=0}^{n}\prod_{i=1}^{4j+1}\frac{1}{1+\prod_{t=0}^{2}x_{(k+1)i-(k+1)t}}.$$

Also, replacing n by $4j+2$ in (2) and summing from $j=0$ to $j=n$ we obtain

$$x_{(4k+4)n+2k+3}-x_{-(2k+1)}=(x_1-x_{-(4k+3)})\sum_{j=0}^{n}\prod_{i=1}^{4j+2}\frac{1}{1+\prod_{t=0}^{2}x_{(k+1)i-(k+1)t-k}}, \tag{5}$$

.
.
.

$$x_{(4k+4)n+3k+3}-x_{-(k+1)}=(x_{k+1}-x_{-(3k+3)})\sum_{j=0}^{n}\prod_{i=1}^{4j+2}\frac{1}{1+\prod_{t=0}^{2}x_{(k+1)i-(k+1)t}}.$$

Also, replacing n by $4j+3$ in (2) and summing from $j=0$ to $j=n$ we obtain

$$x_{(4k+4)n+3k+4} - x_{-(k)} = (x_1 - x_{-(4k+3)}) \sum_{j=0}^{n} \prod_{i=1}^{4j+3} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}, \tag{6}$$

$$\vdots$$

$$x_{(4k+4)n+4k+4} - x_0 = (x_{k+1} - x_{-(3k+3)}) \sum_{j=0}^{n} \prod_{i=1}^{4j+3} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}}.$$

Now, we obtained of the above formulas;

$$\begin{aligned} &x_{(4k+4)n+r(k+1)+s+1} \\ &= x_{(r-4)(k+1)+s+1} \left(1 - \frac{\left(\prod_{m=1}^{4} x_{-(mk+m-1)+s}\right) / x_{(r-4)(k+1)+s+1}}{1+(\prod_{m=1}^{3} x_{-(mk+m-1)+s})} \right. \\ &\qquad \left. \times \sum_{j=0}^{n} \prod_{i=1}^{4j+r} \frac{1}{1+\prod_{m=1}^{3} x_{(k+1)i-(mk+m-1)+s}} \right) \end{aligned}$$

where $r = 0,1,2,3$ and $s = 0,1,\ldots,k$ hold.

f) Suppose that $a_1 = a_{k+2} = a_{2k+3} = a_{3k+4} = 0$. By (e) we have

$$\begin{aligned} \lim_{n\to\infty} x_{(4k+4)n+1} &= \lim_{n\to\infty} x_{-(4k+3)} \left(1 - \frac{\prod_{t=0}^{2} x_{-(k+1)t-k}}{1+\prod_{t=0}^{2} x_{-(k+1)t-k}} \right. \\ &\qquad \left. \times \sum_{j=0}^{n} \prod_{i=1}^{4j} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}} \right). \end{aligned}$$

$$a_1 = x_{-(4k+3)}\left(1 - \frac{\prod_{t=0}^{2} x_{-(k+1)t-k}}{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}\sum_{j=0}^{\infty}\prod_{i=1}^{4j}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}\right),$$

$$a_1 = 0 \Rightarrow \frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{\prod_{t=0}^{2} x_{-(k+1)t-k}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}. \qquad (7)$$

Similarly,

$$\lim_{n\to\infty} x_{(4k+4)n+k+2} = \lim_{n\to\infty} x_{-(3k+2)}\left(1 - \frac{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(3k+2)}}{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}\right.$$
$$\left.\times\sum_{j=0}^{n}\prod_{i=1}^{4j+1}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}\right).$$

$$a_{k+2} = 0 \Rightarrow \frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(3k+2)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j+1}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}. \qquad (8)$$

Similarly,

$$a_{2k+3} = x_{-(2k+1)}\left(1 - \frac{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(2k+1)}}{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}\right.$$
$$\left.\times\sum_{j=0}^{\infty}\prod_{i=1}^{4j+2}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}\right),$$

$$a_{2k+3} = 0 \Rightarrow \frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(2k+1)}}$$

$$= \sum_{j=0}^{\infty} \prod_{i=1}^{4j+2} \frac{1}{1 + \prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}. \tag{9}$$

Similarly,

$$\lim_{n\to\infty} x_{(4k+4)n+3k+4} = \lim_{n\to\infty} x_{-(k)} \left(1 - \frac{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-k}}{1 + \prod_{t=0}^{2} x_{-(k+1)t-k}} \times \sum_{j=0}^{n} \prod_{i=1}^{4j+3} \frac{1}{1 + \prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}} \right)$$

$$a_{3k+4} = x_{-(k)} \left(1 - \frac{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-k}}{1 + \prod_{t=0}^{2} x_{-(k+1)t-k}} \times \sum_{j=0}^{\infty} \prod_{i=1}^{4j+3} \frac{1}{1 + \prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}} \right),$$

$$a_{3k+4} = 0 \Rightarrow \frac{1 + \prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-k}}$$

$$= \sum_{j=0}^{\infty} \prod_{i=1}^{4j+3} \frac{1}{1 + \prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}. \tag{10}$$

From the equation (7) and (8),

$$\frac{1 + \prod_{t=0}^{2} x_{-(k+1)t-k}}{\prod_{t=0}^{2} x_{-(k+1)t-k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{4j} \frac{1}{1 + \prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}} \tag{11}$$

$$> \frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(3k+2)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j+1}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}$$

thus $x_{-(4k+3)} > x_{-(3k+2)}$.

From the equation (8) and (9),

$$\frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(3k+2)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j+1}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}$$

$$> \frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(2k+1)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j+2}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}, \quad (12)$$

thus, $x_{-(3k+2)} > x_{-(2k+1)}$.

From the equation (9) and (10),

$$\frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-(2k+1)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j+2}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}}$$

$$> \frac{1+\prod_{t=0}^{2} x_{-(k+1)t-k}}{(\prod_{t=0}^{3} x_{-(k+1)t-k})/x_{-k}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j+3}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t-k}} \quad (13)$$

thus, $x_{-(2k+1)} > x_{-k}$, $x_{-(4k+3)} > x_{-(3k+2)} > x_{-(2k+1)} > x_{-k}$. We arrive at a conradiction.

Suppose that $a_{k+1} = a_{2k+2} = a_{3k+3} = a_{4k+4} = 0$.

From that the equation (14) in (e) follows, Proof of the equation (11) is similar and will be omitted.

$$\frac{1+\prod_{t=0}^{2} x_{-(k+1)t}}{\prod_{t=0}^{2} x_{-(k+1)t}} = \sum_{j=0}^{\infty}\prod_{i=1}^{4j}\frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}}$$

$$> \frac{1+\prod_{t=0}^{2} x_{-(k+1)t}}{(\prod_{t=0}^{3} x_{-(k+1)t})/x_{-(2k+2)}} = \sum_{j=0}^{\infty} \prod_{i=1}^{4j+1} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}}, \tag{14}$$

thus, $x_{-(3k+3)} > x_{-(2k+2)}$.

From that the equation (15) in (e) follows, the proof of the equation (12) is similar and will be omitted.

$$\frac{1+\prod_{t=0}^{2} x_{-(k+1)t}}{(\prod_{t=0}^{3} x_{-(k+1)t})/x_{-(2k+2)}} = \sum_{j=0}^{\infty} \prod_{i=1}^{4j+1} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}}$$

$$> \frac{1+\prod_{t=0}^{2} x_{-(k+1)t}}{(\prod_{t=0}^{3} x_{-(k+1)t})/x_{-(k+1)}} = \sum_{j=0}^{\infty} \prod_{i=1}^{4j+2} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}}, \tag{15}$$

thus, $x_{-(2k+2)} > x_{-(k+1)}$.

From that the equation (16) in (e) follows, the proof of the equation (13) is similar and will be omitted.

$$\frac{1+\prod_{t=0}^{2} x_{-(k+1)t}}{(\prod_{t=0}^{3} x_{-(k+1)t})/x_{-(k+1)}} = \sum_{j=0}^{\infty} \prod_{i=1}^{4j+2} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}}$$

$$> \frac{1+\prod_{t=0}^{2} x_{-(k+1)t}}{(\prod_{t=0}^{3} x_{-(k+1)t})/x_{0}} = \sum_{j=0}^{\infty} \prod_{i=1}^{4j+3} \frac{1}{1+\prod_{t=0}^{2} x_{(k+1)i-(k+1)t}} \tag{16}$$

thus, $x_{-(k+1)} > x_0$, $x_{-(3k+3)} > x_{-(2k+2)} > x_{-(k+1)} > x_0$. We arrive at a conradiction which completes the proof of theorem. □

References

[1] A. M. Amleh, E. A. Grove, G. Ladas, D. A. Georgiou, *On the recursive sequence* $y_{n+1} = \alpha + \frac{y_{n-1}}{y_n}$ // J. Math. Anal. Appl., **233** (1999), 790–798.

[2] C. Cinar, *On the positive solutions of the difference equation* $x_{n+1} = \frac{x_{n-1}}{1+ax_nx_{n-1}}$ // Appl. Math. Comp., **158** (3) (2004), 809–812.

[3] C. Cinar, *On the positive solutions of the difference equation* $x_{n+1} = \frac{x_{n-1}}{-1+ax_nx_{n-1}}$ // Appl. Math. Comp., **158** (3) (2004), 793–797.

[4] C. Cinar, *On the positive solutions of the difference equation* $x_{n+1} = \frac{ax_{n-1}}{1+bx_n x_{n-1}}$ // Appl. Math. Comp., **156** (3) (2004), 587–590.

[5] E. M. Elabbasy, H. El-Metwally, E. M. Elsayed, *On the difference equation* $x_{n+1} = ax_n - \frac{bx_n}{cx_n - dx_{n-1}}$ // Advances in Difference Equation, Vol. 2006, Article ID 82579, (2006), 1–10.

[6] E. M. Elabbasy, H. El-Metwally, E. M. Elsayed, *Qualitative behavior of higher order difference equation* // Soochow Journal of Mathematics, **33** (4) (2007), 861–873.

[7] E. M. Elabbasy, H. El-Metwally, E. M. Elsayed, *Global attractivity and periodic character of a fractional difference equation of order three* // Yokohama Mathematical Journal, **53** (2007), 89–100.

[8] E. M. Elabbasy, H. El-Metwally, E. M. Elsayed, *On the difference equation* $x_{n+1} = \frac{\alpha x_{n-1}}{\beta + \gamma \prod_{i=0}^{k} x_{n-i}}$ // J. Conc. Appl. Math., **5**(2) (2007), 101–113.

[9] E. M. Elabbasy, E. M. Elsayed, *On the Global Attractivity of Difference Equation of Higher Order* // Carpathian Journal of Mathematics, **24** (2) (2008), 45–53.

[10] E. M. Elsayed, *On the Solution of Recursive Sequence of Order Two* // Fasciculi Mathematici, **40** (2008), 5–13,.

[11] E. M. Elsayed, *Dynamics of a Recursive Sequence of Higher Order* // Communications on Applied Nonlinear Analysis, **16**(2) (2009), 37–50.

[12] E. M. Elsayed, *Solution and atractivity for a rational recursive sequence* // Discrete Dynamics in Nature and Society, Vol. 2011, Article ID 982309, 17 p., 2011.

[13] E. M. Elsayed, *On the solution of some difference equation* // Europan Journal of Pure and Applied Mathematics, **4** (3) (2011), 287–303.

[14] E. M. Elsayed, *On the Dynamics of a higher order rational recursive sequence* // Communications in Mathematical Analysis, **12** (1) (2012), 117–133.

[15] E. M. Elsayed, *Solution of rational difference system of order two* // Mathematical and Computer Modelling, **55** (2012), 378–384.

[16] C. H. Gibbons, M. R. S. Kulenović, G. Ladas, *On the recursive sequence* $x_{n+1} = \frac{\alpha + \beta x_{n-1}}{\chi + x_n}$ // Math. Sci. Res. Hot-Line, **4** (2000), No. 2, 1–11.

[17] M.R.S. Kulenović, G. Ladas, W. S. Sizer, *On the recursive sequence* $x_{n+1} = \frac{\alpha x_n + \beta x_{n-1}}{\chi x_n + \delta x_{n-1}}$ // Math. Sci. Res. Hot-Line, **2** (1998), No. 5, 1–16.

[18] S. Stevic, *On the recursive sequence* $x_{n+1} = \frac{x_{n-1}}{g(x_n)}$ // Taiwanese J. Math., **6** (2002), No. 3, 405–414.

[19] D. Şimşek, C. Çınar, I. Yalçınkaya, *On the recursive sequence* $x_{n+1} = \frac{x_{n-3}}{1+x_{n-1}}$ // Int. J. Contemp. Math. Sci., **1** (2006), No. 9-12, 475–480.

[20] D. Şimşek, C. Çınar, R. Karataş, I. Yalçınkaya, *On the recursive sequence* $x_{n+1} = \frac{x_{n-5}}{1+x_{n-2}}$ // Int. J. Pure Appl. Math., **27** (2006), No. 4, 501–507.

[21] D. Şimşek, C. Çınar, R. Karataş, I. Yalçınkaya, *On the recursive sequence* $x_{n+1} = \frac{x_{n-5}}{1+x_{n-1}x_{n-3}}$ // Int. J. Pure Appl. Math., **28** (2006), No.1, 117–124.

[22] D. Şimşek, C. Çınar, I. Yalçınkaya, *On The Recursive Sequence x(n+1) = x[n-(5k+9)] / 1+x(n-4)x(n-9) ... x[n-(5k+4)]* // Taiwanese Journal of Mathematics, **12** (2008), No.5, 1087–1098.

[23] D. Şimşek, A. Doğan , *On A Class of Recursive Sequence* // Manas Journal of Engineering, **2** (2014), No. 1, 16–22.

[24] I. Yalcinkaya, B. D. Iricanin, C. Cinar, *On a max-type difference equation* // Discrete Dynamics in Nature and Society, Vol. 2007, Article ID 47264, 10 p., doi: 1155/2007/47264, 2007.

[25] H. D. Voulov, *Periodic solutions to a difference equation with maximum* // Proc. Am. Math. Soc., **131** (7) (2002), 2155–2160.

[26] X. Yang, B. Chen, G. M. Megson, D. J. Evans, *Global attractivity in a recursive sequence* // Applied Mathematics and Computation, **158** (2004), 667–682.

Contact information

Dağıstan Simsek
Kyrgyz-Turkish Manas University,
Bishkek, Kyrgyzstan
&
Selcuk University
Konya, Turkey
E-Mail: dagistan.simsek@manas.edu.kg
dsimsek@selcuk.edu.tr

Fahreddin Abdullayev
Kyrgyz-Turkish Manas University,
Bishkek, Kyrgyzstan
&
Mersin University
Mersin, Turkey
E-Mail: fabdul@mersin.edu.tr
fahreddin.abdullayev@manas.edu.kg